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Algebraic Representation of Social Capital Matrix

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C O N T E N T S

ABSTRACT

This paper proposes a mathematical model based on a Boolean algebra involving a 4×4 social capital matrix [Shah (2008)], that emerges through interaction within and across individuals, communities, institutions and state. The framework provides a coding system for the existence or otherwise of various categories of social interaction. The model illustrates that social interaction can be neatly described in a format that facilitates the interpretation of social intra- and interactions among the four types of players in generating economic activity.

JEL classification: A13, D78, Z13

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1. INTRODUCTION

Accumulation of social capital has been viewed as investment of time and money (spending) for interaction with members of family, friends, community, ethnic groups, organisations, and state [Shah (2008)]. According to this study a crucial component of social capital is access to information. Castle (2003) states that a degree of trust, an expectation of reciprocity and exchange of information are expected to prevail in relationships (social capital). According to Carroll (2001), social capital is the trust, reciprocity, norms and networks of civic engagement in a society that facilitates coordinated action to achieve desired goals. Putnam (1993), state that working together is easier in a community blessed with a substantial stock of social capital and. Hence, social capital embodied in norms and networks of civic engagement can be regarded as an important precondition for economic development as well as for effective government. Bjornskov (2003), views that people trust each other and tend to cooperate for common causes. Robinson and Flora (2003) are of the view that individual utility-maximising behaviour cannot be pursued independent of the wellbeing of others. Cox (1995) contends that individuals' lives are about their relationships with others, but involve levels of trust and cooperation or anger and distrust. These comprise individuals' social capital, which makes democracy work, production rise and social cohesion develop. Grootaert and Bastelaer (2002) view social capital as the institutions relationship, attitudes, and values that govern interaction among people and contribute to economic and social development. Here social capital is assumed to be a relational capital that requires two or more than two individuals, one individual one community or group, one individuals vs. state etc. means at least there should be two interacting systems or partners. Shah (2008) points out that individuals maintain their social interactions on the basis of their actual and expected returns (welfare) from their relationships with others. The study develops the concept of social capital matrix of order sixteen [Shah (2008)], which represents interactions within and across state, organisations, communities and individuals.

It may be pointed out here that not all social interactions and the resulting accumulation of social capital are meant for socially acceptable goals. Ethnocenticism, corruption and even crime involve a great deal of social interaction. That is, social interaction is a vehicle for collecting useful information, which may be used for 'good' or 'bad' activities according to norms of society. Nevertheless, it may be noted that more often social capital is meant for socially desirable activities. Another point to be noted is that social capital may be exogenously given or endogenously accumulated through investment of time and money in social interactions. An example of the former is pre-existing level of social respect one acquires by birth in a family, society, etc., while the examples of the latter include participations in public meeting and charitable contributions to philanthropic activities.

Existing data sets contain a small number of variables and reflect only a few dimensions of social capital, while there is a natural need that one can identify a larger number of components of social capital in multiple dimensions. Since algebraic notion are simple to understand than any other branch of mathematics, the concept of vector space, along with the behaviour of its elements (vectors) regarding vector addition and scalar multiplication, provides a useful tool of analysis. The motivation for this kind of model construction at the conceptual level comes from the interaction of the systems in social capital matrix of [Shah (2008)]. The proposed mathematical construction is capable of representing social capital matrix in a formal way with a large number of components in multiple dimensions.

The state *S* is represented by finite Boolean algebra $Z_2 = \{ [0]_2, [1]_2 \} = \{0,1\}$, which has two active categories (vectors) 0,1 denoted as *S–vectors* or *S–categories*. The category 0 represents the investments/ spending and 1 represents the return/welfare indicator of the state. We assume that a higher order linear space Z_2^2 represents organisation *O* with four *O–vectors* (*O–categories)* (categories of organisations). Likewise the linear spaces Z_2^3 and Z_2^4 represent community *C* with eight *C–vectors* (*C–categories*) (categories communities) and individual *L* with sixteen *L–vectors* (*L–categories*) (categories of individuals) respectively. We consider systems *S*, *O*, *C* and *L* (linear spaces), and also other useful interactive systems of interest due to their productive nature. For example, the interaction of organisation with community could have the representation $Z_2^2 \times Z_2^3$ and it contains 32 different vectors (categories).

This algebraic representation of social capital matrix of [Shah (2008)] facilitates interpretation of social interaction in such a way that each possible choice (0 or 1) at the state level is embodied into the choices made by organisations, which in turn are embodies in the choices made at community level and so on.

The proposed algebraic representation of social capital matrix also helps in observing the specific behaviour of categories of each system during intra-action and across interactions regarding their economic activity and hence social capital formulation. It may be noted at this point that the algebraic structure proposed here identifies the existence and types of interactions or intra-actions and is not meant to quantify the level of interaction; a subject matter to be considered at a later stage of research. Thus, in the present context, the objective is to understand the structure of social interactions, hence, social capital and not to quantify or determine the optimal size of social capital or to analyse the process of depreciation (or appreciation) of social capital.

The paper is divided into three sections. Sections 2 and 3 give theoretical background of social capital and a short introduction of algebra which is used in representing social capital matrix [Shah (2008)] respectively. Section 4 contains the main analysis based on mathematical tools devised for interaction within and across individuals, communities, institutions and state.

2. SOCIAL CAPITAL MATRIX

The construction of social capital matrix here considers four levels: individuals, groups or communities, institutions or organisations and state, although other useful constructions are also possible including, for example, the global institutions that may lie above state. Social capital may exist in a number of interactive forms, which are individual vs. individual, individual vs. group or community, individual vs. institution or organisation, individual vs. state, group or community vs. group or community, group or community vs. institution or organisation, group or community vs. state, institution or organisation vs. institution or organisation, institution or organisation vs. state and state vs. state. The presence of social capital in different dimensions is reflected in a matrix form (here we must indicate that it does not fulfil the complete sense of a matrix as in algebra), we call *interactive social capital matrix*, as shown in the following table, which has 16 different interactions of the systems.

Stake holders	$Individual(L)$	$Community(C)$	$Organisation(O)$	$State(S)$
$Individual(L)$	L vs. L	L vs. C	L vs. O	L vs. S
$Community(C)$	C vs. L	C vs. C	C vs. O	C vs. S
$Organisation(O)$	O vs. L	O vs. C	O vs. O	O vs. S
$State(S)$	S vs. L	S vs. C	S vs. O	S vs. S

The following are the various components of social interaction in this matrix.

Individual vs. Individual

Social capital is accumulated between two individuals through their mutual interaction and reciprocity. Coleman (1990) points out that “social capital constitutes a capital asset for the individual and it consists of some aspect of social structure and facilitates certain action of the individuals who are within the structure”. This relationship in turn develops trust between individuals that enable them to generate returns in future. Multidimensional existence of social capital is viewed by Sobel (2002) as “these problems involve small numbers of agents who know each other and interact repeatedly”.

Individual vs. Group/Community

Interaction between individuals and communities also results in accumulation of social capital. Robinson and Flora (2003) confirm that individuals and groups can consciously work to strengthen social capital. An important characteristic of networks is their permeability. Castle (2003) and Sobel (2002) note that even though self-interest is an important motivator, it does not preclude, indeed it may require, participation in groups.

Individual vs. Institution/Organisation

Individuals interact with organisation through their members by allocation of resources that accumulate social capital between them. Individuals make investment through interaction and reciprocity with organisation that generates social capital. This develops a level of trust among individual and institutions or organisations. Sobel (2002) has the opinion that “studies of trust provide another example of the importance of institutions. Trust is the willingness to permit the decisions of your decisions or other to influence your welfare. Levels of trust determine the degree to which you are willing to extend credit or rely on the advice and actions of others”.

Individual vs. State

Social capital is accumulated between an individual and a state. Individuals make investment through reciprocity with state directly or indirectly through its institutions or organisation, which develops their mutual trust. Evidences in literature show that an individual with full trust in state will be more cooperative citizens. Social capital sustains reciprocity between individual and state. Cognitive social capital and structural social capital facilitates patterns of their interaction with each other.

Group/Community vs. Group/Community

Interaction between communities by allocation of time and money accumulates social capital among them. Robinson and Flora (2003) confirm that individuals and groups can consciously work to strengthen the social capital. Existence of such type of social capital has also been affirmed by Castle (2003) and Sobel (2002). Similarly, Woolcock (1998) is of the view that physical capital and human capital are essentially the property of individuals, while social capital and extension inheres in groups. Further he has argued that poor communities need to generate social ties extending beyond their primordial groups if developmental outcomes are to be achieved. The social capital is embodied within communities and according to Coleman (1990), it refers to the relations within a group, including social norms and sanctions, mutual obligations, trust, and information transmission, the same is viewed in Costa and Kahn (2003).

Group/Community vs. Institution/Organisation

A group or community interacts with institutions or organisations by allocation of resources that accumulates social capital among them. Castle (2003) and Sobel (2002) have pointed out existence of social capital among groups due to common interests. Similarly, Woolcock (1998) interprets that social capital is property of group. Furthermore critical aspect of effective group functioning is that the action of individuals when acting within or on behalf of the group contributes to group aims. Institutions serve as channels for collective action that is reinforced by diffused benefits, legitimations, and shared expectations.

Group/Community vs. State

Interaction between group or community and state or its institution or organisation through their members by allocation of resources accumulates social capital for them. Individuals directly or indirectly on behalf of group make investment through interaction and reciprocity with state or its institutions or organisation that accumulates social capital which develops their reciprocity and trust in each others. Evans (1996) is of the view that for development purposes it is not enough to scale up micro-level capital, but, contrary to most civil society advocates, the best effect results from state-society synergy. “Active government and mobilised communities can enhance each other’s development efforts”. While Evans admits that such a complementarity is mostly confined to egalitarian social structures and “robust, coherent state bureaucracies” he argues that synergy can be created even in the more adverse circumstances typical of some developing countries. Similarly, Harris (1997) support Putnam (1993) finding that interaction in civil society in different parts of Italy, to which he called ‘networks of civic engagement’ is a major determinant of government performance. A study on social capital and participation in developmental activities in a district (Faisalabad) of Pakistan is carried out by Beall (1997). Beall views that in many respects the interactions between state and civil society around urban services in Faisalabad had more in common with the vertical networks (social capital) described for southern Italy in Putnam (1993).

Institution/Organisation vs. Institution/Organisation

Institutions or organisations also interact with other institutions or organisations directly or indirectly through their members. Individuals on behalf of their institution or organisation make investments through interaction and reciprocity with other institutions or organisations. Social capital generates reciprocity between institutions or organisations in order to develop their mutual trust.

Cognitive social capital and structural social capital facilitate and regulate patterns of mutual interaction of institutions or organisations. Turner (1999) states that the term institution “denotes the way that members of a population are organised in order to face fundamental problems of coordinating their activities to survive within a given environment”. Ostrom’s (1990) contribution regarding common interests have been strengthened by Sobel (2002), that common-property resources, highlight the importance of institutions.

Institution/Organisation vs. State

Individuals on behalf of institutions or organisations make investment and develop reciprocity with a state directly or indirectly through its institutions or organisations that promote trust in each other. Cognitive social capital and structural social capital facilitate patterns of interaction of institutions or organisations with state. Evans (1996) is of the view that for development purposes, in addition to scaling up micro-level capital, state-society synergy can give better results. Social trust, norms of reciprocity networks of civic engagement and successful cooperation are mutually reinforcing. Putnam (1993) point out that for effective collaboration, institutions require interpersonal skills and trust. These skills and trust are also inculcated and reinforced by organised collaborations. Institutions, organisations and state may allocate resources for accumulation of social capital to enhance effectiveness.

State vs. State

Two or more states interact with each other and allocate resources or extend help, assistance or cooperation to each other. This exchange or reciprocity accumulates social capital among states. The accumulated social capital is used as a means to get returns in future. The states retain their reciprocal relations with each other by extending different forms of reciprocity to each others. The quantum and form of exchange may be heterogeneous like in barter trade model. The exchange may depend upon need and demand of one state and supplying capability of other states.

3. ALGEBRAIC STRUCTURES FOR SOCIAL CAPITAL MATRIX

Social capital has wide range, number of dimensions, therefore need to be coded for analysis. The special algebraic structures are used to codify the concepts, type and mode of transaction of social capital amongst different players. We selected algebraic structures of particular interest, that is, groups, rings, integral domains, fields, vector spaces, homeomorphisms of rings and linear transformation. But with giving this we emphasise on the finite nature structures.

We begin with the following definitions.

Let G be a non empty set. We say $*$ is a binary operation on G if $a*b \in G$ for $a,b \in G$. The representation $(G,*)$ is called groupoid, which means G is a non empty set and $*$ is a binary operation on G. A groupoid $(G,*)$ is a semigroup if the binary operation $*$ is associative. A semigroup $(G,*)$ is said to be monoid if there exist $e \in G$ such that $e*\tau = \tau*e=\tau$, we call e, the identity element in G with respect to the binary operation $*$. A monoid $(G,*)$ is said to be a group if for each $g \in G$, there exist $h \in G$ such that $g*h = h*g = e$, whereas we call g and h, the inverses of each other.

A non-empty set R with two binary operations, say "+" and "×" is said to be a ring if $(R,+)$ is abelian group (i.e., $a+b=b+a$), $(R,.)$ is semigroup and "×" is distributive over "+". A ring R is commutative if $a.b=b.a$, for all $a, b \in R$. A ring R is with identity if $(R,.)$ is monoid. A commutative ring R with identity is said to be an integral domain if $ab = 0$, where $a, b \in R$, then either $a = 0$ or $b = 0$. Alternatively a commutative ring R with identity is said to be an integral domain if it has no zero divisors.

Let R be a commutative ring with identity. An element $a \in R$ is said to be invertible or unit in R if there exist an element $b \in R$ such that $ab = ba = 1$. We represent $U(R)$, the set of all unit elements in R. A commutative ring F with identity is said to be field if $U(F) = F \setminus \{0\}$. Obviously Q, R, C and $Q[i] = \{p + iq : p,q \in Q\}$ are fields but the integral domains Z and $Z[i] = \{a + ib : a, b \in Z\}$ are not fields. A field is an integral domain but converse is not true in general. A finite integral domain is a field.

Let R be a commutative ring with identity. A non-empty subset I of R is said to be an ideal of R if $a - b \in I$ and $ra \in I$, for all $a, b \in I$ and $r \in R$. $nZ = \{na : a \in Z\}$, $n \in Z^+$ are ideals in the ring of integers Z. Let I be an ideal of a commutative ring R with identity 1. $R/I = \{a + I \ : \ a \in R\}$ is a ring known as the factor ring under the binary operations $(r + I) + (r' + I) = r + r' + I$ and $(r + I) + (r' + I) = rr' + I$, where $r, r' \in R$. I is the additive identity and $1 + I$ is the multiplicative identity in R/I respectively.

Let R and S be commutative rings. A ring homomorphism is a map $\varphi : R \to S$ if for all $x, y \in R$, $\varphi(x + y) = \varphi(x) + \varphi(y)$ and $\varphi(xy) = \varphi(x) + \varphi(y)$. A ring homomorphism φ is said to be a monomorphism (respectively epimorphism, isomorphism) if e is one-one (respectively onto, bijective).

Given non-negative integers $0 < a$ and b, there exist $q \geq 0$ and r with $0 \leq r \leq a$ such that $b = aq + r$, where q is quotient and r is remainder which are unique (Division algorithm is stated). If $r = 0$, we say a divides b (that is $a \mid b$).

For a fixed $m \in Z^+$, we say $a, b \in Z$ are congruent modulo m, written $a = b$ (mod m) if $m \mid a - b$ or equivalently, if $a = b + mt$, where $t \in Z$. Here m is called the modulus (plural; moduli). $a = 0$ (mod m) means $m \mid a$, $a = b$ (mod1) for all $a, b \in Z$, therefore we consider the positive integer $m > 1$ and $\{b + mt : t \in Z\}$ is the set of integers to which $b \in Z$ is congruent modulo m.

Every integer is congruent modulo m to exactly one of the numbers in the set $\{0, 1, 2,.., m-1\}$.

Let $2 \leq m \in Z^{+}$ be the modulus, which is fixed. Define the congruence class of b (mod m), written $[b]_m$ as

$$[b]_m = \{a \in Z : a \equiv b \,(\text{mod}\, m)\} = \{a \in Z : m \text{ divides } a-b\}$$
$$= \{a \in Z : a = b + mt, \text{ where } t \in Z\}.$$

$[a]_m = [b]_m$ if and only if $a \equiv b$ (mod m).

Every congruence class mod m is equal to one of $[0]_m$, $[1]_m$, $[2]_m$,.., $[m-1]_m$. Obviously all these classes are different. Thus there are only m congruence classes modulo m. We represent the set of all congruence classes modulo m by Z_m. So

$$Z_2 = \{[0]_2, [1]_2\},$$
$$Z_3 = \{[0]_3, [1]_3, [2]_3\}$$
.
.
$$Z_m = \{[0]_m, [1]_m, [2]_m,.., [m-1]_m\}.$$

As $[b]_m = b + mZ$. So $[0]_m = mZ$, $[1]_m = 1 + mZ$ and $[m]_m = mZ$. Thus there is an isomorphism between Z_m and Z / mZ, the factor ring of Z by its ideal mZ.

In $Z_m = \{[0]_m, [1]_m, [2]_m,..,[m-1]_m\}$ we define the binary operations $\oplus_m$ and $\otimes_m$ (or just take $\oplus$ and $\otimes$)

If $n = 2$, then $Z_2 = \{[0]_2, [1]_2\}$ and we define the binary operations as $\oplus_2$ and $\otimes_2$ as follow

$\oplus_2$	[0]	[1]
[0]	[0]	[1]
[1]	[1]	[0]

and

$\otimes_2$	[0]	[1]
[0]	[0]	[0]
[1]	[0]	[1]

.

If $n = 4$, then $Z_4 = \{[0]_4, [1]_4, [2]_4, [3]_4\}$ and we define the binary operations as $\oplus_4$ and $\otimes_4$ as follow

$\oplus_4$	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]

and

$\otimes_4$	[0]	[1]	[2]	[3]
[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]
[2]	[0]	[2]	[0]	[2]
[3]	[0]	[3]	[2]	[1]

.

$[1]_m$ is the identity in Z_m with respect to binary operation $\otimes_m$ and it is unique.

$(Z_m, \oplus_m, \otimes_m)$ is a commutative ring with identity for any $m = 2$. Indeed; as$[0]_m$, $[1]_m \in Z_m$ and $[-a]_m = -[a]_m$, so it is no hard to confirm that Z_m is a commutative ring with identity. $(Z_m, \oplus_m, \otimes_m)$ is an integral domain if m is prime integer. Z_4 is not an integral domain as $[2]_4 \neq [0]_4$ but $[2]_4 \otimes [2]_2 = [0]_4$. $U(Z_m)$ represent the set of unit elements of Z_m. It is easy to verify that $U(Z_m) = \{[a] \in Z_m : (a,m) = 1\}$. Moreover Z_m is an integral domain (and hence a field) if and only if m is prime integer.

An additive abelian group V is said to be a vector space or linear space over the field F if the scalar multiplication map $F \times V \to V$, defined as $(\alpha, \upsilon) \to \alpha, \upsilon$ satisfies

(i) $\alpha(\upsilon + m) = \alpha\upsilon + \alpha w$;

(ii) $(\alpha + \beta)\upsilon = \alpha\upsilon + \beta\upsilon$;

(iii) $(\alpha\beta)\upsilon = \alpha(\beta\upsilon)$;

(iv) $1.\upsilon = \upsilon$, for all $\alpha, \beta \in F$, $\upsilon, w \in V$.

A vector space V is said to be an algebra over the field F if V is ring and $\alpha(\upsilon w) = (\alpha\upsilon)w = (\alpha w)$ for all $\alpha \in F$, υ, $w \in V$. A field is not only a vector space over itself with dimension 1 but it is an example of algebra. Furthermore, for a positive integer n, $F^n = \{(\alpha_1, \alpha_2, \ldots, \alpha_n) : \alpha_1, \alpha_2, \ldots, \alpha_n \in F\}$ is an algebra over F with dimension n. If p is prime integer and n be any positive integer, then Z_p is a one dimensional algebra over the field Z_p and Z_p^n is n dimensional algebra over the field Z_p, particularly we may take $p = 2$. Interestingly Z_2 is a Boolean algebra, as $a^2 = a$ and $a + a = 0$, for all $a \in Z_2$.

Let V and W be finite dimensional vector spaces over the same field F. A vector space homomorphism (linear transformation) is a map $\varphi: V \to W$ which satisfies $\varphi(x + y) = \varphi(x) + \varphi(y)$ and $\varphi(\alpha y) = \alpha\varphi(y)$, for all x, $y \in V, \alpha \in F$. A vector space homomorphism is an isomorphism if it is bijective. If φ is isomorphism, then we say V is isomorphic to W and we represent it as $V \cong W$. For more details one can consult [Dummit and Foote (2002); Durbin (1992); Hungerford (1974) and Wallace (1998)].

The finite nature and simplicity of these algebraic notions make it applicable. Here assumption that works is, the state has finite number of resources, algebraically labeled these as vectors of the system and socially and economically we shall call the categories or economic indicators of the system. In the similar manner we may correlate organisation, community and individuals with different finite structures.

Shah (2008) study social capital theory through a framework of social capital matrix but in the next section we articulate this with algebraic structures so as to make it estimatable and predictable.

4. ALGEBRAIC REPRESENTATION OF SOCIAL CAPITAL MATRIX

The algebraic representation is devised in view of the systems namely state, organisation, community and individuals and their interactions as described in [Shah (2008)] but we assumed that behind these interactions the economics of spending and welfare work, ultimately which cause to create social capital amongst different systems. The framework in [Shah (2008)] limited to one period information regarding social capital but this algebraic representation have capability to provide multi period analysis.

4.1. The Model

Four types of players including state, organisations, communities and individuals are interacting and constitute their respective economic and social environment while interacting with each other, the new environment emerge to formulate social capital and start the economic activity. By different interactions we obtain the 4×4 matrix, which contains 16 different interactions of the systems, known as social capital matrix [Shah (2008)].

We start by recall a characterisation of the algebraic structure under consideration.

$$Z_2^m = \{(a_1, a_2, .., a_m) = a_1 a_2 .. a_m \ : \ a_1, a_2, .., a_m \in Z_2\} \text{ and}$$
$$Z_2^l = \{(a_1, a_2, .., a_l) = a_1 a_2 .. a_l \ : \ a_1, a_2, .., a_l \in Z_2\}$$

are m and l dimensional linear spaces over the field Z_2 respectively. So

$$Z_2^l \times Z_2^m \cong Z_2^{l+m}$$

is $l+m$ dimensional linear space over the field Z_2.

4.1.1. *Adjustments of Algebraic Structures with Social Capital Matrix*

The state S is represented by finite Boolean algebra $Z_2 = \{[0]_2, [1]_2\} = \{0,1\}$, which have two active categories (vectors) $0,1$ denoted as $S-vectors$ or $S-categories$. Furthermore the category 0 represents the investments/spending and 1 represents the return/welfare indicator of the state. We assume that a higher order linear space Z_2^2 represents organisation O with four O – *vectors* or O – *categories* (categories of organisations). Likewise the linear spaces Z_2^3 and Z_2^4 represent community C with eight $C-vectors$ or $C-categories$ (categories communities) and individual L with sixteen $L-vectors$ or $L-categories$ (categories of individuals) respectively.

Now we have the following adjustments.

$Z_2 \leftrightarrow$ State (S)

$Z_2^2 \leftrightarrow$ Institution/Organisation (O)

$Z_2^3 \leftrightarrow$ Group/Community (C)

$Z_2^4 \leftrightarrow$ Individual (L).

This formation will lead to the format of Social Capital Matrix, that is State-Organisation-Community and then individual (abbreviated as *SOCL*), which may be interpreted as the state leading the all types of activities through organisation, community and finally, individual. Obviously, this format can be criticized on the basis of the presumption that the individuals constitute the communities, the communities constitute the organisation and the organisation constitute the state, which would require the reverse format individual-Community-Organisation and then State (abbreviated as *LCOS*) [Shah (2008)]. But in our case, the business of a state depending on two indicators is running all other systems by its authoritative position. It is partially similar to [Shah, Khalid, and Shah (2006)] which addressed principal agent model in Pakistani local government systems. So it is essential that we have to consider the reverse order for social capital matrix than [Shah (2008)] and it may be observed as follow and here after we shall call it *SOCL*.

$$\begin{array}{ccccc} \times & S & O & C & L \\ S & SS & SO & SC & SL \\ O & OS & OO & OC & OL \\ C & CS & CO & CC & CL \\ L & LS & LO & LC & LL \end{array} \equiv \begin{array}{ccccc} \times & Z_2 & Z_2^2 & Z_2^3 & Z_2^4 \\ Z_2 & Z_2 \times Z_2 & Z_2 \times Z_2^2 & Z_2 \times Z_2^3 & Z_2 \times Z_2^4 \\ Z_2^2 & Z_2^2 \times Z_2 & Z_2^2 \times Z_2^2 & Z_2^2 \times Z_2^3 & Z_2^2 \times Z_2^4 \\ Z_2^3 & Z_2^3 \times Z_2 & Z_2^3 \times Z_2^2 & Z_2^3 \times Z_2^3 & Z_2^3 \times Z_2^4 \\ Z_2^4 & Z_2^4 \times Z_2 & Z_2^4 \times Z_2^2 & Z_2^4 \times Z_2^3 & Z_2^4 \times Z_2^4 \end{array}$$

4.1.2. *Components in Categories of Systems*

The following provide a picture of the components of the categories of four systems.

$S-categories$	$O-categories$	$C-categories$	$L-categories$
0	00	000	0000
			0001
		001	0011
			0010
	01	010	0100
			0101
		011	0110
			0111
1	10	101	1010
			1011
		100	1000
			1001
	11	110	1100
			1101
		111	1110
			1111

On the basis of size of the systems we may call *S* is smaller than *O*, *O* is smaller than *C* and *C* is smaller than *L*.

As $Z_2^l \times Z_2^m \cong Z_2^{l+m}$, where $1 \le l, m \le 4$, is $l+m$ dimensional linear space over the field Z_2, so the interaction of any two systems can be represented as like *S*, *O*, *C* and *L*.

The System V	Z_2	Z_2^2	Z_2^3	Z_2^4	$Z_2 \times Z_2^4$..
No. of categories of V	2	4	8	16	32..
No. of components in the categories of V	1	2	3	4	5..

This table causes the following three findings.

4.1.3. *The Top Row and First Column of SOCL*

The state Z_2, Organisation Z_2^2, Community Z_2^3 and Individual Z_2^4 have 2,4,8 and 16 categories respectively, which are representing the investments/spending and return/welfare indicator.

In some sense this is an established ground, i.e. State, Organisation, Community and Individual. We may call them Stable (Canonical/Natural) Systems (N-Systems), that is these are on the top row and first column of *SOCL*.

	Z_2	Z_2^2	Z_2^3	Z_2^4
Z_2				
Z_2^2				
Z_2^3				
Z_2^4				

4.1.4. *The Main Diagonal of SOCL*

These are the interactions of a system with itself, i.e., State vs. State, Organisation vs. Organisation, Community vs. Community, Individual vs. Individual. We may call all of 4, the Intra-action of the systems or D-Interactions Diagonal-Interactions, that is these activities are on main diagonal of SOCL. It may be observed as follow.

$$\begin{matrix} Z_2 \times Z_2 & & & \\ & Z_2^2 \times Z_2^2 & & \\ & & Z_2^3 \times Z_2^3 & \\ & & & Z_2^4 \times Z_2^4 \end{matrix}$$

Define a function $\delta : Z_2^m \times Z_2^m \to Z_2^m$, where $1 \le m \le 4$ by

$\delta(a_1..a_m) + (b_1..b_m) = (c_1..c_m) \in Z_2^m$, for any $(a_1..a_m),(b_1..b_m) \in Z_2^m$, whereas $c_i = a_i + b_i$, $1 \leq \quad i \leq 4.$ We call δ intra-active function, which is interpreted as the economic trade off among the categories of an N-system. However in resulting one can obtain again a category of the same system, which may have *m*! number of possibilities regarding its status in respect of economic activity or formulation of social capital of categories of the systems.

If we take $m = 3$, then $\delta : Z_2^3 \times Z_2^3 \to Z_2^3$, defined as

$$\delta(a_1a_2a_3, b_1b_2b_3) = c_1c_2c_3 \in Z_2^3, \text{ where } c_i = a_i + b_i.1 \leq i \leq 3.$$

As Z_2^3 represent the community. The δ explains the community vs. community. In this type of interaction all components of two categories of the community is doing business with all of their corresponding components. This also reflects that the total assets of interactive categories of the community are fully operationalised and no part left for substance for its own survival. Hence this indicates the case, that is in favour to this finding that categories of the community that consumes/spend all of its assets/resources in one period. This also indicates that intra-action of any system provide a high level of trust among the categories of the same system, which causes economic activity and creates social capital of categories and hence to the system under consideration.

4.1.5. *Lower and Upper Diagonal of SOCL*

1. The interaction of State Z_2 with Individual Z_2^4, and it has the representation $Z_2 \times Z_2^4$ (respectively the interaction of Individual Z_2^4 with State Z_2 , and it has the representation $Z_2^4 \times Z_2$)·
2. The interaction of State Z_2 with Community Z_2^3, and it has the representation $Z_2 \times Z_2^3$ (respectively the interaction of Community Z_2^3 with State Z_2, and it has the representation $Z_2^3 \times Z_2$)·
3. The interaction of State Z_2 with Organisation Z_2^2, and it has the representation $Z_2 \times Z_2^2$ (respectively the interaction of Organisation Z_2^2 with State Z_2, and it has the representation $Z_2^2 \times Z_2$)·
4. The interaction of Organisation Z_2^2 with Individual Z_2^4, and it has the representation $Z_2^2 \times Z_2^4$ (respectively the interaction of Individual Z_2^4 with Organisation Z_2^2, and it has the representation $Z_2^4 \times Z_2^2$)·
5. The interaction of Organisation Z_2^2 with Community Z_2^3, and it has

the representation $Z_2^2 \times Z_2^3$ (respectively the interaction of Community Z_2^3 with Organisation Z_2^2, and it has the representation $Z_2^3 \times Z_2^2$).

6. The interaction of Community Z_2^3 with Individual Z_2^4, and it has the representation $Z_2^3 \times Z_2^4$ (respectively the interaction of Individual Z_2^4 with Community Z_2^3, and it has the representation $Z_2^4 \times Z_2^3$).

These are representing 12 numbers of across interactions of the systems, i.e. State vs. Organisation and vice versa, State vs. Community and vice versa, Community vs. Individual and vice versa. We may call these Lower and Upper Diagonal Interactions (LUD-Interactions), that is these are not on the main diagonal of SOCL. It may be represented as

$$\begin{array}{cccc} & Z_2 \times Z_2^2 & Z_2 \times Z_2^3 & Z_2 \times Z_2^4 \\ Z_2^2 \times Z_2 & & Z_2^2 \times Z_2^3 & Z_2^2 \times Z_2^4 \\ Z_2^3 \times Z_2 & Z_2^3 \times Z_2^2 & & Z_2^3 \times Z_2^4 \\ Z_2^4 \times Z_2 & Z_2^4 \times Z_2^2 & Z_2^4 \times Z_2^3 & \end{array}$$

Lower Diagonal interactions and Upper Diagonal interactions having symmetries due to this model, for example $Z_2^l \times Z_2^m$ and $Z_2^m \times Z_2^l$, $1 \le l, m \le 4$, are same in nature in algebraic perspective.

First it is noticed that if $0 = \{0\}$ is zero vector space, consisting on 0 only. So, for $l \le m$, $Z_2^l \to Z_2^m$ is imbedding of Z_2^l in Z_2^m, i.e. $Z_2^l \cong Z_2^l \times 0 \times .. \times 0 \subset Z_2^m$, this means $a_1..a_l = a_1..a_l 0_{l+1}..0_m \in Z_2^m$. Similarly $m \le l$, $Z_2^m \to Z_2^l$ is imbedding of Z_2^m in Z_2^l , i.e. $Z_2^m \cong 0_1 \times .. \times 0_{l-m} \times Z_2^m \subset Z_2^l$, this means $a_1..a_l = 0_1..0_{l-m} a_1..a_l \in Z_2^l$.

Now we define functions $\delta_{m \leftarrow l}$ and $\delta_{l \to m}$ as follow:

$$\delta_{l \leftarrow m} : Z_2^l \times Z_2^m \to Z_2^m, \text{ where } 1 \le m \le 4, \text{ and } m \le l$$

by $\delta_{l \leftarrow m}(a_1..a_l, b_1..b_m b_{m+1}..b_l) = c_1..c_m c_{m+1}..c_l \in Z_2^m$

for any $a_1..a_l \in Z_2^l, b_1..b_m \in Z_2^m$ and $b_{m+1} = .. = b_l = 0$.

and

$$\delta_{l \to m} : Z_2^l \times Z_2^m \to Z_2^m, \text{ where } 1 \le m \le 4, \text{ and } l \le m$$

by $\delta_{l \to m}(a_1..a_l a_{l+1}..a_m, b_1..b_m) = c_1..c_l c_{l+1}..c_m \in Z_2^m$,

for any $a_1..a_l \in Z_2^l, b_1..b_m \in Z_2^m$ and $a_{l+1} = .. = a_m = 0$.

Whereas $c_i = a_i + b_i$, $1 \leq \; i \leq 4$. We call $\delta_{m \to l}$ and $\delta_{l \leftarrow m}$, the across inter-active functions, which are interpreted as the economic trade off among the categories of different N-systems. However, in result of this trade off, again a category is obtained, which is in fact belongs to the larger system of across inter-active systems.

By across inter-active function $\delta_{l \to m}$, $l \leq m$ we conclude that interaction of the system Z_2^l with Z_2^m provided that the $l+1, l+2,.., m$ components of the larger system (in size and dimension) Z_2^m remains inactive during interaction, i.e., the only first l number of components interact with their corresponding l members in the smaller system (in size and dimension) Z_2^l. Similarly, by across inter-active function $\delta_{l \leftarrow m}$, $m \leq l$ we conclude that interaction of the system Z_2^l with Z_2^m provided that the $1, 2,.., m$ components of the larger system (in size and dimension) Z_2^l remains inactive during interaction, i.e., the only last m number of components interact with their corresponding m members in the smaller system (in size and dimension) Z_2^m.

This can be illustrated by an example, for instance if we consider $\delta_{2 \to 3} : Z_2^2 \times Z_2^3 \to Z_2^3$, then

$$\delta_{2 \to 3}(a_1 a_2 0, b_1 b_2 b_3) = c_1 c_2 c_3 \in Z_2^3, \text{ where } c_3 = b_3.$$

By across inter-active function $\delta_{m \to l}$, $m \leq l$ we conclude that interaction of the system Z_2^m with Z_2^l provided that the $m+1, l+2,.., l$ components of the larger system (in size and dimension) Z_2^l remains inactive during interaction, i.e. the only first m number of components interact with their corresponding m members in the smaller system (in size and dimension) Z_2^m.

Now it can be interpreted if we consider $\delta_{3 \leftarrow 2} : Z_2^3 \times Z_2^2 \to Z_2^3$, then $\delta_{3 \leftarrow 2}(a_1 a_2 a_3, 0 b_2 b_3) = c_1 c_2 c_3 \in Z_2^3$, where $c_1 = a_1$.

Recall that Z_2^2 and Z_2^3 represent organisation and community respectively. The $\delta_{2 \to 3}$ explains the organisation vs. community. In organisation vs. community the first two components of the category of community are doing business with all two components of the organisation. This means the total assets are not operationalised by the community rather a part is left for subsistence for its own survival. This also reflects extreme case, that is in contradiction to this finding that community or organisation that consumes/spend all of its assets/resources in one period does not survive for next period. Furthermore, it is not like the intra-action of an N-System.

In similar fashion if $\delta_{3\to 2} \ : \ Z_2^3 \times Z_2^2 \to Z_2^3$, then

$$\delta_{3\to 2}(a_1a_2a_3, b_1b_2b_3) = c_1c_2c_3 \in Z_2^3, \text{ where } c_3 = b_3.$$

It is of course one could genuinely ask that; where should be the status of a category of a smaller system after interaction?

A simple response can be derive by the following comments.

This algebraic model of social capital matrix provided that the economic activity and hence creation of social capital of a category of individual reflects the presence of three indicators of a category of the community. As well in the category of community there is a reflection of two components of a category of organisation. Similarly, in the category of organisation there is a reflection of one component of a category of state.

These findings strengthened our format of Social Capital Matrix, that is *SOCL*, which compels for the leading role of state in all types of activities of categories of organisation, community and finally, individual.

The following represent the *SOCL*.

L ....
$\cup$ ... ...
C
$\cup$
O .. ..
$\cup$
S . .

5. CONCLUSION

The social capital matrix [Shah (2008)], emerges through interaction of State, Organisation, Community and the individual, we identified as the system *S*, the system *O*, the system *C* and the system *L* respectively. Through algebraic representation of social capital matrix with the assumption that *S*, *O*, *C* and *L* have 2, 4, 8 and 16 categories respectively, the paper proposes a mathematical framework for understanding the process of social interaction in generating economic activity. It is observed that in each category of individuals there is a reflection of the presence of three economic indicators (i.e., from $\{0,1\}$) of a category of the community. As well in each category of community there is a reflection of two economic indicators of a category of organisation. Similarly, in each category of organisation there is a reflection of one economic indicator of the state.

Interactions across the systems, given that not all the components of a category of the larger system are doing business with the components of the smaller system, shows that the total assets/resources are not operationalised by the larger system rather a part is left for its own survival. This also reflects extreme case, that is in contradiction to this finding that community or organisation that consumes/spend all of its assets/resources in one period do not survive for next period.

In the process of intra-action of a system all components of two interactive categories are doing business with all of their corresponding components, which reflects that the total assets of interactive categories of the system under consideration are fully operationalised and no part is left for its own survival. This also indicates that intra-action of any system provides a high level of trust among the categories of the same system, which causes formulation of social capital of categories and hence of the system concerned. It can also be observed that it is not like across interaction of different systems.

This study may be generalised by considering Z_n as a state S with any positive integer n. If n is prime then Z_n behave as a field and almost same algebraic construction applies as considered in this paper and the behaviour of *SOCL* can be characterised with complexities. On the other hand, if n is not prime then Z_n behaves as a commutative ring with identity, which is not an integral domain. This would of course be more suitable option in analysing the systems in a rational way. This extended approach may provide a rationale regarding non-availability of smooth environment for interaction of categories.

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