Quasi-Stable Estimates of the Vital Rates of Pakistan

by

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Introduction

The emergence of stable and quasi-stable population theory has been one of the most interesting developments of demographic theory in the recent past. Zelnik and Rahman Khan writing in the Spring 1965 issue of this *Review* have applied this methodology to Pakistani data with rather interesting results[1]. The purpose of this present note is two-fold. First, is to discuss the assumptions and procedures of the Zelnik-Rahman Khan use of the quasi-stable model. In other words, accepting the validity of the model and also its applicability to Pakistan, have the authors made good use of it? Second, and more basically, is the quasi-stable methodology in fact relevant for Pakistan, given the data available?

Let us begin with a brief review of the stable and quasi-stable population models as they have developed in the last ten years.

The Quasi-Stable Model Summarized

Stable population theory builds on the simple mathematical relationship which exists among the birth rate, the death rate, the growth rate and the age distribution of a population. A stable population is one in which constancy of the birth and death rates over some time has produced a stable (that is, unchanging) age distribution and, of course, a constant growth rate. In recent years, Coale and others [2; 6] have extended the basic model to cases of quasi-stability—that is, populations with constant fertility, but changing (usually falling) mortality. In these cases too, given knowledge of some of the population parameters the remaining parameters can be derived. More recently yet, the Office of Population Research at Princeton University has compiled and published a set of model life tables based on an exhaustive collection of life tables for many European (and some Asian) populations going back a hundred years and more [3]. The Princeton study also presents the parameter values which would be generated by a stable population exposed to the mortality experience described in each of these different model life tables and stipulated levels of fertility. In supplementary work, the same group of researchers have derived techniques for

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[1] [19]. A shorter version of this same paper was presented under Zelnik's name alone at the World Population Conference in Belgrade September 1965, and will appear in its proceedings.

estimating the probable error of stable estimates derived from a population which is actually quasi-stable [6].

The result of these efforts is a kind of "do-it-yourself" reference book of estimated families of stable population parameters. Armed with the knowledge of any two of the parameters, an interested researcher can find the appropriate family of stable population parameters, read the other population parameters and then adjust for whatever changes are believed to have occurred in mortality.

This, in brief, is what Zelnik and Rahman Khan have done. Starting with the preliminary Population Growth Estimation (PGE) Project's 1962 age distribution and the 1951-61 intercensal growth rate they have used these reference tables and correction factors to derive estimates of Pakistan's current vital rates.

Age Data Used

The Zelnik-Rahman Khan article was evidently based on preliminary and incomplete PGE age data published in 1963. Final PGE age data have now been tabulated and, although differences between the two are of only minor magnitude, differences do exist (Table I). However, in terms of cumulative age groups, the preliminary PGE figures, the final PGE figures and the 1961 Census distribution are all rather similar. Only in the case of under 10 years in East Pakistan is there as much as over three percentage point difference in the two cumulative distributions. Thus, one basic premises of the Zelnik-Rahman Khan article, that the PGE and Census age distributions are strikingly different, is somewhat doubtful (Table II). Moreover, holding other factors constant, the three age distributions yield almost similar quasi-stable estimates.

The goal of the Zelnik-Rahman Khan article is nonetheless a wholly worthwhile one. Quasi-stable estimates of the vital rates implied by the PGE age distribution can provide a kind of check on the PGE vital rates themselves. Unfortunately, the authors have been less than thorough in presenting their results. Their estimates of the birth rates derived from the various cumulative age groups are not shown in tabular form and their estimates of the death rates are not shown at all. Table IV presents their apparent birth rate estimates "read" from their Figure 1 [19, p. 67]. Similarly, the exact method by which the "best estimates" (of their Table I) were reached is not stated by the authors. Presumably, some judgement was applied to a plot of the estimates. But it is difficult to reconstruct the logic. The authors clearly state that their "best estimate" is not an average of the rates estimated from the six cumulative age groups [19, footnote 9].

On the other hand, after implying that the rates generated from the under 30 group are the least affected by errors in the age data [19, p. 69] they appear to

have, in fact, produced a "best estimate" by very roughly averaging a number of the cumulative groups. Thus, for East Pakistan females it almost appears that the "best estimate" (64) is an average of the under 10 and under 30 estimates (68 and 60 respectively). But, for East Pakistan males this cannot have been the case since the "best estimate" (58) is above both the 0 to 9 and 0 to 29 estimates (55 and 51). Indeed, it is above all the estimates except that for the under 15 group. For West Pakistan males the "best estimate" again appears to be some rough average of the estimates for the under 10 and under 30 cumulative age groups[2].

The Estimated Vital Rates

There are, as noted, some differences between the PGE preliminary age data (with which Zelnik and Rahman Khan worked) and the final PGE age data. Since the full range of birth and death rate estimates generated by Zelnik and Rahman Khan are not available to us, we have chosen to generate a new series of estimated vital rates employing the final PGE age data and based on the same assumptions as used by Zelnik and Rahman Khan insofar as it was possible to determine their assumptions[3]. Table III presents the birth rates estimated by cumulative age groups under 10 and under 30 (the groups in which the authors indicate they have the greatest faith) according to the procedure outlined in the appendix of this review.

Table IV provides a range of quasi-stable birth rates estimated by Zelnik and Rahman Khan using different cumulative age groups. These birth rates vary from 38 to 68 for East Pakistan and from 37 to 51 for West Pakistan.

The article in question does not present estimates of the death rate and the authors suggest that the death rate generated by their analysis should be considered no more than a dubious residual [19, pp. 80-81]. In fact, the method employed by the authors yields estimates of the death rate quite as much as it does

[2] There appears to be some discrepancy between the authors' Figure 1 and their textual discussion [19, p. 68] for the ranges indicated by the figure are much wider than those mentioned in the text.

[3] The authors, as noted above, made it difficult to retrace their steps. For example, they do not indicate the source of the PGE data which they employ. Of the references they cite, Hashmi [10] does contain the cumulative percent under 5, under 10 and under 15, and also a graphic plot of the other percents. It is to be assumed that they obtained their values by "reading" this graph. Hashmi's data were based on returns received from six out of the ten PGE areas in each Province and, as noted above, differ slightly from the final PGE age distributions employed in this note.

The growth rates employed in deriving Table III came from 1951 and 1961 Census data and are as follows:

	East Pakistan	West Pakistan
Male	1.9	2.4
Female	2.1	2.5

Presumably Zelnik and Rahman Khan also used these rates.

estimates of the birth rate. While the estimated death rates may be subject to large errors they are nevertheless the logical consequence of the conditions assumed and the analysis undertaken. The entire model thus developed—birth rate, death rate, growth rate and other demographic consequences of the analysis—must be evaluated *in toto*. The death rates generated by their analysis were not presented except in the most general terms [19, p. 80] but the ones yielded by our analysis of the final PGE age data may be assumed to be close to theirs[4].

The death rates which result are well above those of the PGE (ranging from 36 to 46 for East Pakistan and 20 to 27 for West Pakistan) and looking at these figures it is easy to understand how the authors could have felt moved to argue that the PGE rates were too low. But the argument can be put the other way around. Instead of asking whether the PGE death rates are too low, we can ask whether overall crude death rates of 30 to 40 are plausible for Pakistan in 1962.

[4] To repeat, the birth and death rates of Table III above represent our quasi-stable estimates prepared using, insofar as possible, the same assumptions and techniques employed by the authors. The table below summarizes the two series (I=Zelnik-Rahman Khan from Table IV above; II=represent estimates from Table III above) of birth rate estimates:

Cumulative age group	East Pakistan				West Pakistan			
	Male		Female		Male		Female	
	I	II	I	II	I	II	I	II
Under 10	55.0	59.4	68.0	64.3	47.0	42.5	49.0	48.6
Under 30	51.0	52.1	60.0	62.5	48.0	46.2	51.0	53.1

Differences in the two series arise presumably in the main because of differences in the preliminary and final PGE age data. In all the instances in which the two estimates differ substantially, one finds corresponding differences in the cumulative age groups of the preliminary and final PGE data. The differences in the male and female rates are not as large in the estimates based on the final PGE data, and thus are more plausible than those from the preliminary data.

It is also possible that the adjustment for declining mortality accounts for some of the difference in the two sets of estimates. Our correction factors were borrowed from Demeny [6, Appendix Tables A-1 to A-5]. These factors were prepared by Demeny for "East" model life table females but we use them with reference to both sexes and in a "West" model life table context. Zelnik-Rahman Khan indicate that their correction factors were "derived from" Demeny's paper too and it seems likely that they did, in fact, employ these same correction factors. If they did, however, actually employ different correction factors, discrepancies between the two sets of estimates could arise on this account. In any case the two sets of estimates are reasonably close to one another and this encourages us to think that our death rate estimates must approximate theirs as well.

Trends in Mortality

There is little external evidence on the question of trends in the death rate in Pakistan. But, there is some evidence and this has not been examined by Zelnik and Rahman Khan. If one is willing to take the prewar registration figures in the Punjab as being reasonably valid index of mortality changes, one finds that by 1940 the crude death rates for both males and females were in the middle and low twenties, having declined almost continuously from the high thirties around the turn of the century [7; 8]. (By way of contrast, the birth rates shown in the Punjab series are nearly constant in the middle forties in the period 1900 to 1940.) For India as a whole (before Partition) Kingsley Davis found that the death rate between 1931 and 1941 averaged about 31, with a declining trend evident [5]. Thus, by 1941 it was presumably 25 or below. Numerous sample surveys (other than those of the PGE) have also shown death rates in the middle and low twenties in the recent past [1; 11; 12; 13]. One can argue, therefore, with some cogency that the long term down-trend in the death rates in both East and West Pakistan had resulted in rates somewhere in the high twenties by the beginning of World War II. Death rates rose because of the 1943 famine and also because of a lessened hold on public health due to the disruptions of war. These disruptions continued during the days just prior to and after Partition and were added to by the actual losses connected with the violence which accompanied Partition. By the 1950s however, the death rate resumed its downward trend, making up the ground it had lost during the 1940s and slowly falling even further. By 1960 a rate as high as 30 or above seems very unlikely.

Turning to life expectancies as our measure of mortality, we also find that the levels implied by the quasi-stable approach are very high relative to all recent estimates for Pakistan. Table V presents an array of life table values which have a bearing on this question. Here, as in the case of the crude birth rate estimates, a pattern emerges. Life expectancies at birth in the low thirties in the prewar period change very little during the war and immediate postwar years, but begin increasing by the early 1950s and reached the high forties or low fifties by the present decade.

Another point which the authors note but do not explain in satisfactory fashion is the wide differences between the male and female rates, especially in East Pakistan. They note [19, p. 71] that given the very high overall sex ratio in the population, the female rates should be higher than the male. However, this argument can also be used against them, for the sex ratio is lower in East (108) than in West Pakistan (116) whereas the gap between their female and male rates is the highest in the East Wing. These differences are reduced by using the final data but even so the sex differences are suspiciously large in both estimated rates

(Table III). These differences almost certainly stem from the model life tables employed (by the present authors and also by Zelnik and Rahman Khan). The tables are based on Western mortality experience (only a handful of non-European life tables went into the preparation of the model) and assume, among other things a "normal" situation of female mortality below male mortality at corresponding mortality levels. There is reason to doubt this assumption in Pakistan. Female mortality in the early years (under 10) and again during child-bearing years (20-29) appears to be higher than male mortality. The age pattern of mortality may also be unlike those of Western populations, present or past.

This does not mean, of course, that the underlying family of life tables is "wrong" but what it does perhaps mean is that this family of life tables is inappropriate for use in Pakistan in conjunction with the assumed growth rates and the cumulative age groups. In other words, the differences in the female-male age distributions in Pakistan (particularly in the East Wing) and the assumed (intercensal) growth rates for the separate sexes when considered within the framework of the "model" life tables imply very improbable female-male mortality differentials. The life table selected, in turn, in conjunction with the age proportions determines the birth rate estimates and the observed birth rate differentials by sex result therefrom[5].

The Growth Rate Chosen

Stable (and quasi-stable) population theory permits the use of a fairly substantial number of different pairs of demographic variables as input parameters to the analysis and the pair chosen may greatly affect the result. As the authors [19, p. 80] and others [2; 6] have pointed out estimates based on different pairs vary widely with respect to their sensitivity to errors in the parameters themselves. The authors chose as input parameters the growth rate and the age distribution. This pair of variables is among the most sensitive to errors in the input data [19, p. 80; 6]. (While the authors felt their choice was guided by the availability of data, other alternatives do exist. They could have employed as a parameter, for example, the ratio of children under 5 to women 15-49 and to population 5-14 which would not be more biased than the cumulative age proportion to estimate other parameters. Use of the ratio of children under 5 to women 15-49 (instead of cumulative proportions of age) and the rates of growth may well yield birth rates which are at variance with the birth rates obtained by using cumulative proportions of age and growth rates).

[5] It is of course, quite possible to employ quasi-stable theory without using the OPR model life tables. Rahman Khan, in another paper [14] constructed life tables based on preliminary PGE mortality data and estimated the birth rate for females in West Pakistan for 1962. Even after Rahman Khan increased the PGE death rates by 50 per cent, he obtained lower estimated birth rates than those of Zelnik and Rahman Khan.

Use in this case of the intercensal growth rate (2.2 on the average) in conjunction with an age distribution showing a very large percentage in the young age groups inevitably results in very high birth rates and death rates. But, different growth rates would give different results for both birth and death rates. The authors note this [19, pp. 69-70] but do not indicate the magnitude of the change which such an adjustment would introduce.

To illustrate the responsiveness of this model to changes in the parameters used, we have developed estimates of birth and death rates by using an overall growth rate of 2.6 and the 1962 PGE age distribution (*see*, Table VI). The use of 2.6 as the *current* annual rate of population growth is of course not inconsistant with the authors' use of an *intercensal* average of 2.2 per year. This 2.6 rate is the official Third Five Year Plan's estimate and, in fact, may actually understate the current growth rate [15]. Our revised estimates employing the 2.6 growth rate lead to fairly substantial reductions in the estimated birth rate. Whether the under 10 or under 30 age group is used, the birth rates are well below the earlier (Table III) estimates. A higher growth rate implies lower death rates but also, in this case, lower birth rates as well.

In Tables III and VI (and in Zelnik-Rahman Khan) mortality was assumed to have been declining for 20 years. This is probably too long. These are 1962 data after all, and the middle and late 1940s were not notoriously healthy years on the Indo-Pakistan subcontinent. Assuming a 10-year mortality decline seems more reasonable. This results in higher birth rate estimates and lower death rates. Thus, adjusting both growth rate and mortality decline, we obtain the final estimates shown in Table VII. If the under 35 cumulative age group is used instead of the under 30 group, then the resulting quasi-stable birth rates are even lower than shown in Table VII. These results suggest that the PGE death rate is correct but that the PGE birth rates estimated by the Chandrasekar-Deming technique [15] may be slightly high.

While the rates shown in Table VII may appear more reasonable on *a priori* grounds than those derived by Zelnik and Rahman Khan, neither set of rates has a greater claim of scientific validity if the underlying assumptions of the quasi-stable approach are not met. In the remainder of this review we will attempt to set forth some of the factors that, in our view, make the quasi-stable approach a tool of limited analytical value in the current Pakistan setting.

The Applicability of the Quasi-Stable Model

The quasi-stable approach assumes certain conditions: a constancy of fertility over time, a closed population, smooth changes in mortality and reasonably accurate input parameters. How well are these assumptions met in Pakistan?

In the case of a completely stable population, the birth rate estimates yielded by any cumulative age group would agree with those yielded by any other group. For example, rates estimated from the under 5 age group would not differ from those of the under 30 age group except for differences caused by errors in the age distribution (age heaping and the like). Now, if one is sure of the stability of the population, such deviations in the birth rate estimates which result can fairly be deduced to indicate errors in the age distribution. Likewise in the case of quasi-stable populations, if mortality is changing regularly and by some known rate, the effect on the actual age distribution and the birth rate estimates can be worked out. Where unexpected results occur one can again deduce errors in the underlying age distributions—provided one can be certain that the conditions of quasi-stability do in fact hold. However, if fertility has not been constant, or if mortality changes have not been regular and monotonic, or if migration has been important, then the results can be an age distribution which is not smooth and which does not yield the regular series of quasi-stable birth rate estimates for the various cumulative age groupings.

In other words, the cause of disturbances in the birth rate estimates are "kinks" in the age distribution. And these "kinks" may be spurious or they may be real. But, nothing in the quasi-stable techniques tells us which kind we are confronting in any given instance. To insist that all "kinks" are pure error in census reporting is to insist that all populations are stable or quasi-stable when we know this is not the case. This, in essense, is what Zelnik and Rahman Khan have done. Faced with the problem of explaining the curious pattern of estimates yielded by the consecutive cumulative age groups (rising, then falling, then rising again as one goes from under 5 to under 30) they resort to ingenious (one might even say imaginative) arguments concerning possible allocative errors in the age distribution. There is, indeed, ample evidence that age misreporting is not negligible in the PGE data. In order to illustrate the magnitude of the errors in the reporting of age data in Pakistan we have applied Myers' blended method [16] to the final 1962 PGE single year age distribution. The results in terms of the Myers index (zero equals no heaping) are shown below along with index values computed for the 1961 Census of India (no single year age distribution was tabulated from the 1961 Census of Pakistan)[6]. These indices (computed over the age 20-59 years) show that the quality of the PGE age data

[6] The "blended" method introduced by Myers is one of the several methods of measuring digital preference and evaluating quality of ages reported in single years. The extent of error in a population is shown by the extent of deviation of the index from zero. In the developed countries where age reporting is reasonably accurate this index usually does not exceed 20.

is poorer than the age data obtained in the 1961 Indian Census.

	Male	*Female*
PGE, East Pakistan 1962	76.8	75.2
PGE, West Pakistan 1962	97.0	95.2
India 1961 Census	70.7	75.1

Another important aspect of the age-misreporting difficulty is not touched on by these authors. Ages in Pakistan, as well as in other developing countries, are not only erroneously reported but there is a definite tendency to report an age lower than the true one, except for older persons and very young children. This bias is of substantial magnitude especially in countries with low levels of literacy. Due to this persistant under-stating of ages, use of cumulative age proportions would tend to produce erroneous demographic parameters. The authors of the article under review have briefly dealt with the questions of quality of age data produced by the 1961 Census and PGE, but the question of understating of ages and its effect on the obtained birth rates have not been considered. Zelnik and Rahman Khan think "estimates based on cumulative proportions under 15, under 20 and possibly under 25 would be too low because of over and understatement of age" [19, p. 69]. Thus, they put more reliance in using cumulative proportions under age 30 and suggest that this ends the matter. We do not agree with this argument for two reasons. Firstly, understatement of ages would increase the cumulative proportion under age 15, 20, or 25 and would provide a higher rather than a lower estimate. Secondly, cumulative proportions under age 30 would also be biased by understatement of ages. This is because this group contains age 25 which has a pronounced attraction for reporting ages in Pakistan. There is a definite preference for those who are in their early thirties to report their ages in late twenties. Thus the authors should have also considered the use of cumulative proportions under ages 35 or 40.

Because Pakistan's fertility is high and the population is known to have little knowledge of modern contraceptive techniques many analysts, including Zelnik and Rahman Khan, have rather uncritically accepted the proposition that fertility is high and unvarying. The lack of usable data on year to year changes in the birth rate no doubt encourages such a view. Yet there is indirect evidence that fertility patterns in Pakistan may have changed somewhat in recent years. According to Census data, the proportion of women married in each age group has risen between 1951 and 1961. In addition, the declines in mortality (and associated morbidity) discussed earlier may also have had a direct effect on fertility through the reduction of foetal losses. While such factors may not have had a controlling influence on fertility in Pakistan they should be carefully examined prior to accepting the assumption that fertility has remained unchanged.

All attempts to apply stable or quasi-stable techniques to Pakistan data quickly run up against the quantitative question mark of migration. The birth of Pakistan as a nation was associated with an historically significant but statistically undocumented migration of millions of persons involving the entire subcontinent. Subsequent to Partition migration has continued, but again, with little usable statistical documentation. Given the volume of gross migration in the subcontinent since 1947 and in the absence of concrete evidence to the contrary it seems improbable to us that net migration, either for the total population of Pakistan or for detailed subgroups, equals zero.

Users of quasi-stable techniques must, of necessity, assume that the populations under study are and have been closed[7]. Failure to meet the conditions of a population unaffected by migration can result in erroneous quasi-stable estimates for a number of reasons. If the intercensal growth rate is used as an input parameter to the analysis (as in Zelnik and Rahman Khan) then to the extent that total net migration differs from zero the intercensal growth rate will fail to be an adequate approximation of the natural growth rate. As demonstrated earlier, even modest differences in the growth rate used has a startling effect on the resulting quasi-stable estimates of vital rates. Another source of error can arise when net migration for a specific age-sex group of the population differs from zero. In other words, the age distribution may have been formed, in part, by factors other than constant fertility as modified by monotonically changing mortality. Age-sex specific net migration and age-misreporting operate together to produce "kinks" in the age distribution. The effets of these 'kinks on quasi-stable estimates has already been discussed above[8].

In view of these difficulties with regard to even the PGE age distribution, one cannot help but question the meaning of vital rates estimated on the basis of these age data. To put the matter another way, at some point adjustments and judgments become rather too many and too complex and the end result dubious. The series of birth rates generated by quasi-stable techniques show great variability and we cannot be sure of why. It may be due to errors in the age data or it may be because one or more of the basic conditions of quasi-stable do not hold. In either case, one has reason for being skeptical about the meaning of the final estimates.

Summary and Overview

We have shown that the quasi-stable estimates of the birth rates generated by the article in question are too high; and that this error stems chiefly from

[7] Possibly, future modifications of these techniques may permit one to adjust for certain types of known departures from the closed population hypothesis.

[8] While Zelnik and Rahman Khan speculated on possible effects of migration on their estimates [19, p. 69] their comments covered only total net migration. They did not deal with the problem of age specific migration.

the use of the low intercensal growth rate. By using an estimate of the current growth rate and also by making more realistic assumptions about the duration of the mortality decline, one arrives at somewhat different birth and death rate estimates, rates which tend to confirm the PGE death rate and suggest that the PGE birth rate estimated by the Chandrasekar-Deming technique may be slightly too high.

There are, however, serious objections to the use of the quasi-stable model at all. Fertility changes and migration may have affected the age distribution. Age misreporting remains as serious a problem for the PGE data as for the census data and the use of even very wide cumulative age groups can not wholly eliminate the problem.

The quasi-stable approach is (as Coale stresses [2, pp. 186-187]) an estimating technique, not a substitute for registration and/or enumeration. It relies on the accuracy of the data already collected and the assumptions made. The PGE represents the best effort so far undertaken to collect new and accurate information on Pakistan's vital rates. The results may be too high or too low. So may the estimates obtained through quasi-stable analytical approaches. But, we have narrowed the range of doubt and are after all close to agreement. The general conclusion is clear; a birth rate of around 50, a death rate of around 20 and a growth rate of approximately 3 per cent per year.

TABLE I

PERCENT DISTRIBUTION OF POPULATION BY DETAILED AGE GROUPS, SEX AND PROVINCE, 1961 CENSUS OF PAKISTAN AND 1962 PRELIMINARY AND FINAL PGE ESTIMATES*

Age	Final 1962 PGE				1961 Census				Preliminary 1962 PGE			
	East Pakistan		West Pakistan		East Pakistan		West Pakistan		East Pakistan		West Pakistan	
	M	F	M	F	M	F	M	F	M	F	M	F
All ages	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Under 5	15.0	16.5	14.6	15.9	17.4	19.1	15.6	17.2	14.9	17.6	15.6	16.9
5 - 9	17.6	18.1	15.6	16.3	18.6	19.0	16.3	16.5	16.6	17.3	15.6	15.3
10 - 14	12.8	10.8	11.4	10.5	9.9	8.3	9.9	9.4	12.8	11.3	11.8	10.5
15 - 19	8.0	8.1	9.5	8.8	7.3	8.1	9.1	8.9	8.3	7.5	8.8	8.1
20 - 24	5.9	8.1	7.2	8.3	6.9	8.1	7.7	7.9	6.5	7.4	7.0	8.2
25 - 29	7.6	8.4	7.9	9.1	7.6	8.2	7.5	7.9	7.4	8.3	8.5	9.3
30 - 34	6.7	6.3	6.3	6.6	6.4	6.3	6.3	6.6	6.7	6.0	6.6	6.9
35 - 39	5.9	5.1	5.4	5.1	5.9	5.1	5.3	5.2	5.9	5.5	5.0	5.5
40 - 44	5.1	4.8	4.9	4.6	4.8	4.5	4.9	4.8	5.1	4.6	4.6	4.6
45 - 49	3.7	3.5	3.9	3.3	3.9	3.3	4.0	3.7	4.1	3.8	4.0	3.6
50 - 54	3.9	3.5	3.9	3.6	3.6	3.3	3.9	3.5	3.8	3.4	3.5	3.1
55 - 59	2.3	1.7	2.1	1.6	2.3	1.8	2.1	2.0	2.3	1.7	1.9	1.8
60 - 69	3.6	3.2	4.3	3.5	3.4	3.0	4.2	3.6	3.6	3.4	4.2	3.5
70 & over	3.1	1.9	3.0	2.7	2.1	1.9	3.2	2.9	2.1	2.1	2.9	2.7

M=male; F=female

Source: [10; 18].

* All distributions exclude Chittagong Hill Tracts in East Pakistan and Frontier Regions and Quetta and Kalat Divisions in West Pakistan. PGE Estimates (preliminary and final) have been adjusted to allow for aging process between time of reporting and mid-year time reference.

TABLE II

PERCENT DISTRIBUTION OF POPULATION BY CUMULATIVE AGE GROUP, SEX AND PROVINCE, 1961 CENSUS OF PAKISTAN AND 1962 PRELIMINARY AND FINAL PGE ESTIMATES*

Cumulative age group	Final 1962 PGE				1961 Census				Preliminary 1962 PGE			
	East Pakistan		West Pakistan		East Pakistan		West Pakistan		East Pakistan		West Pakistan	
	M	F	M	F	M	F	M	F	M	F	M	F
Under 5	15.0	16.5	14.6	15.9	17.4	19.1	15.6	17.2	14.9	17.6	15.6	16.9
Under 10	32.6	34.6	30.2	32.2	36.0	38.1	31.9	33.7	31.5	34.9	31.2	32.2
Under 15	45.4	45.4	41.6	42.7	45.9	46.4	41.8	43.1	44.3	46.2	43.0	42.7
Under 20	53.4	53.5	51.1	51.5	53.2	54.5	50.8	52.0	52.6	53.7	51.8	50.8
Under 25	59.3	61.6	58.3	59.8	60.1	62.6	58.6	59.9	59.1	61.1	58.8	59.0
Under 30	66.9	70.0	66.2	68.9	67.7	70.8	66.1	67.8	66.5	69.4	67.3	68.3
Under 35	73.6	76.3	72.5	75.5	74.1	77.1	72.4	74.4	73.2	75.4	73.9	75.2

M=male; F=female
* For qualifications, see Table I.

Source: Table I

TABLE III

QUASI-STABLE ESTIMATES OF VITAL RATES FOR PAKISTAN BY SEX AND PROVINCE, ESTIMATED USING THE 1951-1961 INTERCENSAL GROWTH RATE AND FINAL 1962 PGE CUMULATIVE AGEGROUPS

Cumulative age group used	All Pakistan[1]	East Pakistan			West Pakistan		
		Both sexes	Male	Female	Both sexes	Male	Female
Under 10 Years							
Birth rate	57.6	65.6[2]	63.1	68.5	48.2[2]	45.2	51.7
Death rate	33.8	44.2[3]	43.4	45.2	21.5[3]	19.0	21.5
Under 30 Years							
Birth rate	56.7	60.4[2]	55.5	66.5	52.5[2]	49.1	56.5
Death rate	32.5	40.1[3]	35.6	45.6	23.2[3]	19.8	27.1

Source: See Appendix A.

[1] Simple weighted average of Province rates using the following weights:
East Pakistan .54 West Pakistan .46

[2] Computed from the formula $\frac{(1+u)(b_m\ b_f)}{u(b_f)+b_m}$

Where b_m = male birth rate
b_f = female birth rate
u = sex ratio at birth, which was assumed to be 1.05

[3] Simple weighted average of male and female death rates using weights derived from the sex ratios as follows:

	Male	Female
East Pakistan	51.9	48.1
West Pakistan	53.7	46.3

TABLE IV

ZELNIK-RAHMAN KHAN BIRTH RATE ESTIMATES FOR CUMULATIVE AGE GROUPS UNDER 5 TO UNDER 30, BY SEX AND PROVINCE, PAKISTAN: 1962

Cumulative age group	East Pakistan		West Pakistan	
	Male	Female	Male	Female
Under 5	38.0	50.0	37.0	42.0
Under 10	55.0	68.0	47.0	49.0
Under 15	62.0	65.0	49.0	46.0
Under 20	55.0	56.0	46.0	43.0
Under 25	50.0	54.0	44.0	44.0
Under 30	51.0	60.0	48.0	51.0

Source: [19, Figure 1].

TABLE V

SELECTED VALUES FROM VARIOUS LIFE TABLES FOR SPECIFIED PORTIONS OF INDO-PAKISTAN SUBCONTINENT: 1931 to 1962

Area and year	Male		Female	
	e_o	l_5	e_o	l_5
A. Undivided India (1931-1941)	32.09	78250	31.37	79610
B. Indo-Pakistan subcontinent	33.5	66600*	34.7	58500*
C. Punjab (1950-52)	33.04	78185	34.64	80364
D. India & Pakistan (1961)	44.2	77400*	45.7	80300*
E. Pakistan (1962) **				
East	48.61	81307	44.98	81477
West	49.46	80350	47.45	79641

Sources: A: [5].
B: [4].
C: [12].
D: [4].
E: [9].

e_o = life expectancy at birth.

l_5 = number surviving to exact age 5 of 100,000 persons born.
*these are estimates which assume very high levels of infant mortality.
**Provisional

TABLE VI

QUASI-STABLE ESTIMATES OF VITAL RATES FOR PAKISTAN BY SEX AND PROVINCE, ESTIMATED USING FINAL 1962 PGE CUMULATIVE AGE GROUP UNDER 30 AND OFFICIAL ESTIMATE OF CURRENT GROWTH RATES [1]

Type of rate	All Pakistan [2]	East Pakistan			West Pakistan		
		Both sexes	Male	Female	Both sexes	Male	Female
Birth rate	52.6	53.1[3]	49.0	58.2	52.1[3]	49.1	55.7
Death rate	20.9	20.1[4]	16.4	24.1	21.8[4]	19.7	24.3

Source: See Appendix A.

[1] Growth rates used:

	Male	Female
East Pakistan	2.6	2.8
West Pakistan	2.4	2.6

[2] *See*, Table III, footnote 1 for method of combining province rates.

[3] *See*, Table III, footnote 2 for method of combining male and female birth rates.

[4] *See*, Table III, footnote 3 for method of combining male and female death rates.

TABLE VII

QUASI-STABLE ESTIMATES OF VITAL RATES FOR PAKISTAN BY SEX AND PROVINCE, ESTIMATED USING FINAL 1962 PGE CUMULATIVE AGE GROUP UNDER 30, OFFICIAL ESTIMATE OF CURRENT GROWTH RATES[1] AND A TEN-YEAR MORTALITY DECLINE[2]

Type of rate	All Pakistan[3]	East Pakistan			West Pakistan		
		Both sexes	Male	Female	Both sexes	Male	Female
Birth rate	53.5	52.9[4]	48.8	58.0	54.2[4]	48.9	55.4
Death rate	19.8	19.1[5]	15.5	22.9	20.7[5]	18.7	23.1

Source: See Appendix A.

[1] For growth rates used see Table VI, footnote 1.

[2] Mortality decline over a ten-year period beginning in 1952.

[3] *See*, Table III, footnote 1 for method of combining province rates.

[4] *See*, Table III, footnote 2 for method of combining male and female birth rates.

[5] *See*, Table III, footnote 3 for method of combining male and female death rates.

Appendix A

ON THE COMPUTATION OF STABLE AND QUASI-STABLE ESTIMATES OF THE VITAL RATES IN PAKISTAN

The logic of the quasi-stable model is explained in the text of this note and also in the Zelnik-Rahman Khan article. The detailed procedures used to apply this model to Pakistan are not given in the article. As these procedures are fairly intricate, this appendix will show, as an illustration, how one specific set of estimates of the vital rates were derived. Specifically, we will derive quasi-stable vital rates for the East Pakistan female population using the 1951-1961 intercensal growth rate (2.1 per cent) and the proportion of the population under age 10 (34.6 per cent) taken from the final 1962 PGE tabulations.

The process of arriving at the final estimates may be seen as involving essentially two steps. First vital rates are estimated assuming the population under study to be stable. The published tables of stable population values associated with various population parameters prepared by Coale and Demeny were used for this purpose [3]. Referring to Appendix Table A-1, Column (1) of this table contains the assumed parameter values; Columns (2) and (3) show the values from the Coale-Demeny tables such as to fall above and below the assumed growth rate and such that the indicated proportions under 10 are all below the assumed proportion. Interpolating between Columns (2) and (3) gives the values shown in Column (4) corresponding to the assumed growth rate for a proportion under 10 which is just below the assumed proportion aged under 10. Columns (5) and (6) repeat this process for the assumed growth rate when the proportions aged 0-9 are above the assumed proportion. Column (7) represents a second interpolation between Columns (5) and (6). Columns (4) and (7), thus, are sets of values representing the vital rates corresponding to two stable populations each with the assumed growth rate and with proportions under age 10 just above and just below the assumed proportions. Column (8) is then a final interpolation between Columns (4) and (7) such that the stable population has both the assumed growth rate and the assumed proportion aged 0-9. These estimates are the ones corresponding to stable conditions.

The second step is to adjust these stable estimates for the effect of the assumed change in the mortality conditions which is believed to have occurred. The Zelnik-Rahman Khan article assumed a twenty-year decline in mortality such that life expectation at birth (e_o) had increased by one year per year for twenty years, fertility all the while remaining constant. The stable estimate is an equilibrium rate and in such a process of rapid transition will differ from the quasi-stable estimates and the rate actually to be found if one could compute the current rate [1, p. 186; 4, Figure 2]. The exact nature of the relationship

among these three sets of rates (stable, the quasi-stable and actual) will depend on the parameters employed in the original estimations, the level of mortality before destabilization occurred, the duration and speed of the mortality decline and other factors. For our purposes we have made use of the set of correction factors worked out by Demeny for illustrative purposes [6, Tables A-1 to A-5]. These were developed using the East not the West model life tables but Demeny suggests they will at least be indicative of the direction and general magnitude of the correct adjustment for other populations.

Demeny's factors show the quasi-stable estimates as percentages of the actual rates. He also presents the corresponding intrinsic (stable) rates and the quasi-stable can thus be expressed as percentages of these stable estimates. Column (9) shows these correction factors with the quasi-stable rate expressed as a per cent of the stable rate. Column (10) is then our final quasi-stable estimate of the actual rates prevailing at the present moment in time (1962).

In summary:

—Column (1) gives the assumed input parameters.

—Columns (2), (3), (5) and (6) come from Coale and Demeny's prepared tables [3] for West Model Population, mortality level as indicated.

—Columns (4) and (7) are interpolated based on Columns (2) and (3), and (5) and (6), respectively.

—Column (8) is an interpolation based on Columns (4) and (7).

—Column (9) shows the correction factors derived from Demeny, [6, Appendix Tables A-1 to A-5] to allow for changes in mortality such that e_o has been increasing by one year per year for 20 years.

—Column (10) is Column (8) divided by Column (9).

Subsequent to the preparation of this note one of the present authors received in a personal communication from Demeny, a new suggested technique for the derivation of correction factors. The application of these correction factors however, yield somewhat different quasi-stable estimates than those found by Zelnik and Rahman Khan (our Table IV) and for this reason we have continued to employ the correction factors based on Demeny's paper [6] in our own estimates (Table III).

In closing this methodological appendix, we feel compelled to comment on the lack of a comprehensive and explicit exposition of the quasi-stable population techniques. The theory underlying the quasi-stable approach is undoubtedly complex and its implications in practice are still being explored and refined by Coale, Demeny and others. The actual use-value of the technique would be considerably enhanced if basic methodological developments were reported on, in print, at more frequent intervals. This is particularly important because many students of the quasi-stable technique are cut off from later developments at the very time that they are applying these techniques in various countries around the world. In short, it is time that the scribes be called out and a fine oral tradition be turned into an excellent written one.

TABLE A-1

ILLUSTRATION OF DERIVATION OF STABLE AND QUASI-STABLE ESTIMATES OF VITAL RATES FOR EAST PAKISTAN FEMALES

	Assumed parameters (1)	"West Model" Mortality Level 4			"West Model" Mortality level 3			Stable estimates (8)	Correction factors (9)	Quasi-stable estimates (10)
		(2)	(3)	(4)	(5)	(6)	(7)			
Growth rate	2.1	2.0	2.5	2.1	2.0	2.5	2.1	2.1		
Percent under 10 years	34.6	33.51	36.28	34.06	34.52	37.28	35.07	34.60		
Birth rate		59.21	65.69	60.51	64.33	71.06	65.68	63.28	92.4	68.48
Death rate		39.31	40.69	39.59	44.33	46.06	44.68	42.29	93.6	45.18
Gross reproduction rate		3.788	4.302	3.891	4.129	4.688	4.241	4.076		
Expectation of life at birth		27.500	27.500	27.500	25.000	25.000	25.000	26.175		

Source: See text of Appendix A.

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