

A Programming Model for Reduction of Import Dependence of Pakistan

by

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INTRODUCTION

The present study consists, in the main, of formulating, implementing and interpreting the results of a model which attempts to explore with the help of available data the possibilities of import substitution in Pakistan from 1963/64 to 1972/73. The model is designed to assess the realistic limits to the process of import substitution in the context of growth—taking into consideration such realistic factors as the growth of per capita consumption, export possibilities mobilization of domestic savings and the availability of foreign economic assistance. Although in a formal sense the model is deterministic, the variation of certain exogenous elements and parameters generates enough information on the behaviour of the model to indicate the extent of substitutability and trade-offs between strategic economic aggregates which is so crucial in evaluating the different policy objectives a planner may wish to pursue.

The focus of our model is on import substitution. This does not mean that other problems or objectives of economic development are unimportant. However, it is in the nature of model-building to isolate the chosen problem to the abstraction of others. The model does not take into account all the relevant scarcities, e.g., skilled labour, organizational ability, etc., nor does it seek to achieve all the complex — and sometimes conflicting — objectives the nation may set for itself, e.g., maximization of employment, reduction in income inequality, etc. Even in the sphere of foreign trade, the model is not as compre-

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hensive as it otherwise might be. Although the imports in the model are generated endogenously, exports are treated exogenously. This asymmetry, common to many similar models of growth and trade, is known to arise from the inelasticity of demand for exports which cannot be endogenously accommodated in a linear model. Although it was possible, in principle, to treat exports endogenously it would have added a completely new dimension to the problem and involved a separate empirical exercise far beyond the scope of the present study. We did, however, work with two estimates of export growth to see the effects of "export promotion" in our model.

One of the distinguishing features of our model is the imposition of realistic constraints in the model to ensure minimum levels of imports of each category. The model distinguishes three different kinds of imports of each category: *a*) investment imports related to the level of investment which in turn is related, *via* the accelerator, to changes in the level of output; *b*) interindustrial imports, related to the levels of output; *c*) consumption imports, determined as the excess of total imports over investment and interindustrial imports. The model requires that the demand for first two kinds of imports must *always* be met.

The theoretical basis for imposing the above kind of restrictions on imports is the phenomenon of a specific foreign-exchange bottleneck which lies behind the recent "two-gap" theories of economic growth. If certain essential imports are needed in fixed proportions for domestic production and if exports are exogenously given, there is bound to arise a situation where domestic savings cannot be transformed into productive investment due to lack of imported inputs. Empirical evidence suggests that "shortage of maintenance imports has resulted in unused capacity in both India and Pakistan" [13]. Of course, the problem of unused capacity is part of the wider problem of misallocation of resources — but neglect of the realistic constraints on import requirements contributes to such misallocation.

If the foreign-exchange constraint does inhibit growth in a country then it can be overcome by import substitution or increased foreign aid — assuming again that the maximum is already being done to increase exports. The supply of foreign aid, however, is not infinite and its availability to a particular country is certainly restricted. Import substitution is, thus, the most likely policy to adopt.

However, import substitution is a means to achieving economic growth and not an end in itself. The intensity of the import-substitution programme depends on the rate of growth which the economy aims to achieve. We have treated the total consumption expenditures as exogenous in our model and by varying the rate of growth of consumption one can see the effect of changes in

consumption growth on total import requirements and other macroeconomic variables.

The achievement of a given growth rate of consumption also gives rise to certain requirements of domestic savings; these requirements will depend not only on the growth-rate target but also on the intensity of the import-substitution programme. If all but the most essential imports are to be substituted by domestic production, the requirements of domestic savings are likely to be very high and possibly beyond the means of the developing country. Our model, therefore, ensures that the savings requirements of any import-substitution programme do not exceed those which the country is capable of mobilizing.

The basic model used in the study is a straightforward finite-horizon linear programming model. Models with similar structure have been used by Bruno [2], Chenery [3], Khan [9], Manne [11], Sandee [18] and Weisskopf [21] to solve a variety of planning problems. The use of a finite-horizon in a model of economic growth — which is supposedly a continuing process — is conceptually awkward. Ideally, a model involving growth and choices in the future should have infinity as its horizon. However, infinite-horizon programming — attractive though it is conceptually — is immensely difficult, computationally. Even if computational techniques were available, it would still be immensely difficult to specify the appropriate technology, welfare or cost function being optimized.

Most economy-wide planning models project the state of affairs from a given base-year to a given target-year. The intervening years are treated essentially as a single period. The problem of optimality or even consistency for the intervening years of the model is ignored. The comparison between the initial and target years is in the nature of comparative statics.

By conducting a sensitivity analysis on the various exogenous elements and parameters of the model, a variety of results is generated and analyzed. The essential elements of the sensitivity analysis are: a) consumption growth-rates varying from 4 per cent to 8 per cent; b) two different exports targets; c) alternative assumption about availability of foreign aid and domestic savings.

The paper is structured in the following way. Section A describes the formal structure of the model. Section B discusses data sources and empirical implementation of the model. Section C presents the results and discusses their economic significance.

A. ALGEBRAIC FORMULATION OF THE MODEL

In the following we shall give a mathematical formulation of our model. First, we formulate the basic model in which the availability of domestic savings and foreign aid is unrestricted. Subsequently, constraints on the availability of

each of these aggregate resources are introduced in two separate variants of the model.

The necessary notation is introduced in the following:

VARIABLES

(All variables are valued in constant rupees).

Endogenous

$$X_t = \{ {}_i X_t \} = n \times 1 \text{ vector of domestic outputs}$$

$$Z_t = \{ {}_i Z_t \} = r \times 1 \text{ vector of imports}$$

$$J_t = \{ {}_i J_t \} = n \times 1 \text{ vector of investment demands}$$

$$J_t^m = \{ {}_i J_t^m \} = r \times 1 \text{ vector of import component of investment demands}$$

$$\bar{Z}_t = \text{total imports}$$

$$\bar{J}_t = \text{total investments}$$

$$\bar{J}_t^m = \text{import component of total investment}$$

Exogenous

$$C_t = \{ {}_i C_t \} = n \times 1 \text{ vector of household and government demands}$$

$$E_t = \{ {}_i E_t \} = r \times 1 \text{ vector of exports}$$

$$\bar{C}_t = \text{total consumption expenditures}$$

$$\bar{E}_t = \text{total exports}$$

PARAMETERS

$$A = \{ a_{ij} \} = n \times n \text{ matrix of current input-output coefficients}$$

$$M = \{ m_{ij} \} = r \times n \text{ matrix of current import coefficients}$$

$$\bar{A} = \{ \bar{a}_{ij} \} = n \times n \text{ matrix of current domestic coefficients}$$

$$\hat{B} = \left\{ \hat{b}_{ij} \right\} = n \times n \text{ matrix of fixed capital coefficients}$$

$$B^m = \left\{ b_{ij}^m \right\} = r \times n \text{ matrix of imported capital coefficients}$$

$$\bar{N} = \left\{ \bar{n}_{ij} \right\} = n \times n \text{ diagonal matrix of inventory coefficients}$$

$$B = \left\{ b_{ij} \right\} = n \times n \text{ matrix of capital coefficients}$$

$$\beta = \left\{ \beta_j \right\} = 1 \times n \text{ vector of sectoral capital-output ratios}$$

$$\bar{B} = \left\{ \bar{b}_{ij} \right\} = n \times n \text{ matrix of domestic capital coefficients}$$

IDENTITIES

Some of the variables and parameters defined above are related with one another by means of identities. We list the identities in the following, briefly commenting on their significance where necessary.

(a) Endogenous Variables

$$(i) \bar{Z}_t \equiv \sum_{i=1}^r iZ_t$$

$$(ii) \bar{J}_t \equiv \sum_{i=1}^n iJ_t$$

$$(iii) \bar{J}_t^m \equiv \sum_{i=1}^r iJ_t^m$$

(b) Exogenous Variables

$$(i) \bar{C}_t \equiv \sum_{i=1}^n iC_t$$

$$(ii) \bar{E}_t \equiv \sum_{i=1}^r iE_t$$

Identities (a) and (b) merely show the way economy-wide aggregate are obtained from sectoral variables.

(c) Parameters

$$(i) \underset{n \times n}{A} \equiv \underset{n \times n}{\bar{A}} + \underset{n \times r}{I^*} \underset{r \times n}{M}$$

where I^* is an $n \times r$ matrix, whose first r rows form an $r \times r$ identity matrix and the remaining $(n-r)$ rows form a null matrix¹. In plain words, this identity means that for sectors 1 to r , an overall input-output coefficient a_{ij} is composed of a domestic input-output coefficient \bar{a}_{ij} , and an import coefficient, m_{ij} ; and for sectors $(r+1)$ to n , the overall input-output coefficient is identical with the domestic input-output coefficient.

In our model the role of imports as essential intermediate inputs for the domestic production processes is clearly brought out within the framework of interindustry analysis. The level of intermediate imports in each sector is determined not by an overall import coefficient for that sector but by a set of coefficients — one for each consuming sector.

Each of the overall input-output coefficients, the a_{ij} 's, is thus split into two components — a domestic and an import coefficient.

$$a_{ij} = \frac{X_{ij}}{X_j} = \frac{\bar{X}_{ij} + M_{ij}}{X_j} = \frac{\bar{X}_{ij}}{X_j} + \frac{M_{ij}}{X_j} = \bar{a}_{ij} + m_{ij}$$

The total interindustry flow of i -th commodity to j -th sector, X_{ij} , is decomposed into the domestically produced part, \bar{X}_{ij} , and the imported part, M_{ij} .

The assumption of separate, fixed coefficient for domestic and imported input is more restrictive than the original Leontief assumption of fixed coefficient for all inputs (regardless of origin). In the short run, this assumption does not do any serious violence to reality and, in fact, by relating interindustry imports to the level of production takes much of the guessing out of the total import requirements of the economy. However, the static framework is no longer adequate if the economy itself is undergoing structural change through investment and import substitution. Thus, the assumption of constancy of the input-output and import coefficients becomes invalid in the face of changes in technology (changes in a_{ij} 's) and in import-intensity (changes in m_{ij} 's) of production — which must take place in the long run. Insofar, however, as the constancy of the input-output coefficients is a plausible assumption, the double-celled method is conceptually and computationally a more efficient (in the sense of predictive accuracy) method than all the others. For purposes of development planning in underdeveloped countries with persistent balance-of-payments problems, this seems to be the most appropriate model to use.

$$(ii) B \equiv \hat{B} + \bar{N}$$

This identity merely says that the matrix of capital coefficients is formed by summing fixed capital coefficients and inventory coefficients.

¹The operator I^* has to be used in order to make it possible to sum matrices \bar{A} and M which do not have the same dimension.

$$(iii) \bar{B} \equiv B - I^* B^m$$

where I^* has already been defined (see footnote 1). This identity gives the relation between domestic, overall and imported capital coefficients.

$$and (iv) \beta_j \equiv \sum_{i=1}^n b_{ij} \text{ for } j = 1, 2, \dots, n$$

The economy is assumed to be divided into a total of n sectors. Sectors 1 to r are internationally trading and sectors $(r+1)$ to n derive their total supply solely from domestic sources of production.

The Basic or Unconstrained Model²

The basic model consists of two sets of constraints. The first set consists of the demand-and-supply balance equations based on the interindustry coefficients matrix, viz.,

$$(I-A) X_t + I^* Z_t = C_t + I^* E_t + J_t \quad (1) \star$$

The left-hand side of each equation in the above set represents the total supply of goods and services of each sector — either through domestic production or imports or both — and the right-hand side the final-demand uses to which they can be put.

To derive the increments³ from the base period, 0, to the terminal period, T , we put $t=T$ and $t=0$ in (1) and subtract to get:

$$(I-A) (X_T - X_0) + I^* (Z_T - Z_0) = (C_T - C_0) + I^* (E_T - E_0) + J_T - J_0 \quad (2)$$

putting

$$X_T - X_0 = x = \{ x_i \} \quad (3)$$

and

$$(C_T - C_0) + I^* (E_T - E_0) + I^* Z_0 - J_0 = y = (y_i) \quad (4)$$

we get

$$(I-A) x + I^* Z_T - J_T = y \quad (5)$$

²The model is basic in the sense that all the constraints used in this model also form part of the other models developed subsequently. It is unconstrained in that no restrictions are placed on the availability of the aggregative resources — foreign or domestic — for the financing of investment.

³The model is formulated in terms of increments in, rather than absolute values of, domestic production variables in order to avoid the all-or-nothing character of solutions yielded by linear models. The implicit assumption in such a formulation is that the supply of domestic factors of production in the initial year is inelastic and can be substituted by imports only at the margin.

* The Investment Assumption

In all projective models it is usual to determine the level of investment endogenously. In the interperiod model the rationale for investment in each year is provided by the simple unlagged accelerator, linking investment to incremental output levels through capital-output ratios. But in the terminal year of a finite-horizon model, whether single or interperiod, the accelerator is of little help — unless one is willing to extrapolate growth beyond the horizon on the basis of a balanced-growth assumption. We have, however, found it more satisfactory to adopt the assumption originally due to Manne [11] whereby investment in the terminal year is a fixed proportion of the total investment during the period. This proportion depends on the length of the horizon, the assumed rate of growth of investment and the gestation period of investments.

Let investment in the initial year be J_0 and let us assume that the rate of growth of investment is r so that the terminal-year investment, $J_T = e^{rT}J_0$, and total investment during the period,

$$\int_{t=0}^T J_t dt = \int_{t=0}^T e^{rt}J_0 dt = \frac{1}{r} (e^{rT} - 1) J_0$$

The stock-flow conversion factor (the ratio of terminal-year investment to total investment during the period), assuming no gestation lag, is given by

$$\frac{J_T}{\int_{t=0}^T J_t dt} = \frac{e^{rT} J_0}{\frac{1}{r} (e^{rT} - 1) J_0} = \frac{r}{1 - e^{-rT}}$$

(multiplying both numerator and denominator by re^{-rT}).

We now use an assumption regarding investment activity in the terminal year, T , discussed below. The assumption postulates that investment in capital good i in the T -th period is a fraction k (a scalar, the stock-flow conversion factor) of the total incremental capacity of that capital good during the period 0 to T , i.e.,

$$J_T = kB_x \quad (6)$$

Substituting (6) in (5), we get

$$(I - A - kB) x + I^* Z_T = y \quad (7)$$

We shall call (7) as the input-output or balance equations of our model.

Import Constraints

The second set of constraints in the model is designed to prevent imports from varying freely. It ensures that the domestic production process does not

encounter a bottleneck due to inadequate supply of essential imports of raw materials and investment goods.

In order to derive the essential or minimum import requirements of raw materials or intermediate goods we use the intermediate-import requirements matrix, M . For computing the investment requirements we make use of imported capital-output matrix B^m and make the following assumption for imported capital goods in the terminal year in the spirit of (6) above.

$$J_T^m = kB^m x \tag{6a}$$

Our import constraints for the terminal year, thus, become

$$\begin{aligned} Z_T &\geq M X_T + kB^m x \\ &\geq M (X_0 + x) + kB^m x \end{aligned}$$

or $-(M + kB^m) x + Z_T \geq MX_0$

putting $MX_0 = w = (w_i)$ we get

$$-(M + kB^m) x + Z_T \geq w \tag{8}^4$$

Combining the input-output balance equations (7) and the import constraints (8) our programming problem can be stated as⁵:

Objective Function 1: Minimize \bar{Z}_T (terminal-year imports)

Objective Function 2: Maximize $\bar{C}_T + \bar{E}_T + kB_x - \bar{Z}_T$
(terminal-year national income)

subject to: $(I - A - kB) x + I^* Z_T = y \tag{7}$

$$-(M + kB^m) x - Z_T \geq w \tag{8}$$

$$\begin{aligned} x_i, jZ_T &> 0 \quad \text{for } i = 1, 2, \dots, n \\ j &= 1, 2, \dots, r \end{aligned}$$

⁴ MX_0 or w is the vector of raw-material imports in the base year and thus consists of constant terms only.

⁵The constraints of the model may be written more compactly in matrix notation as follows:

$$\left[\begin{array}{c|c} (I - A - kB) & I^* \\ \hline -(M + kB^m) & 1 \end{array} \right] \left[\begin{array}{c} x \\ Z_T \end{array} \right] = \left[\begin{array}{c} y \\ w \end{array} \right]$$

* Choice Elements in the Model

In order to see clearly the choices provided by the model between different activities let us introduce an $r \times 1$ vector of non-negative variables $R = (R_i)$ to convert the r inequalities of (8) into strict equalities. We can, thus, rewrite our constraints in the following way:

$$\begin{aligned} (I - A - kB)x + I^* Z_T &= y & (7) \\ -(M + kB^m)x + Z_T - R &= w & (9)^6 \\ x, Z_T, R &> 0 \end{aligned}$$

This is a system of $(n + r)$ equations and $(n + 2r)$ variables, which can be identified as follows:

Equations	Name	Number
(5)	input-output	n
(6a)	import demand	r
Total		$(n + r)$

Variables

x	incremental domestic production	n
Z_T	terminal-year total imports	r
R	terminal-year non-essential imports	r
Total		$(n + 2r)$

In the optimal solution — assuming one exists for our problem — each of these variables will either assume a positive or a zero value. Also, the number of positive variables in the optimal solution will be equal to the number of independent equations in the system, barring degeneracy [7, pp. 97-98]. Thus, only $(n + r)$ variables will be at a positive level and the remaining r will be at a zero level in the optimal solution.

This property of the optimal solution can be used to illuminate the choice elements in the model. It is possible for us to tell on the basis of the structure of our model which of the endogenous variables of the model *will be* at a positive level. The effective choice of the model will then be restricted to the remaining variables — r of them will turn out to be zero in the optimal solution.

⁶ R can be interpreted as a vector of non-essential imports, since $R = Z_T - (M + kB^m)x + w = Z_T - (Mx + w) - (kB^m)x =$ total imports in year $T -$ (raw-material imports in year $T) -$ (import component of investment demands in year T).

In our model the last $(n - r)$ elements of the vector x represent the incremental domestic production levels of the noninternationally trading sectors. In order to satisfy the various final and intermediate demands for the output of these sectors the last $(n - r)$ variables in the vector x will have to be at a positive level — as long as final demands of some sectors increase between the base and terminal years — since domestic production is the only source of supply in these sectors⁷. The vector Z_T will be positive (*i.e.*, all its elements will be greater than zero) if we assume that w , the vector of raw-material imports in the initial year, is positive⁸. This condition is easily verified in the empirical implementation of our model. This leaves us only with the first r elements of vector x and the vector R (which also has r elements). The effective choice of the model is confined to these $2r$ variables — r of which will be at a positive and the other r will be at a zero level in the optimal solution. The model's choice mechanism is, thus, restricted to determining the extent to which incremental domestic output can replace non-essential imports in each sector.

In view of the above, the following mutually exclusive possibilities can arise in any internationally trading sector i ($i=1, 2, \dots, r$)

$$(a) \quad x_i > 0, \quad R_i = 0$$

$$(b) \quad x_i = 0, \quad R_i > 0$$

$$(c) \quad x_i > 0, \quad R_i > 0$$

$$(d) \quad x_i = 0, \quad R_i = 0$$

The possibility (d) can be ruled out on *a priori* grounds since the incremental final demands will have to be met either by increases in domestic output ($x_i > 0$) or by a level of imports beyond the barely essential level ($R_i > 0$). Possibility (a) implies that domestic production in the i -th sector will be able to meet all intermediate and final demand requirements, other than those of essential imported inputs. The second polar possibility is (b) in which all *incremental* intermediate and final demands are met entirely by imports, and incremental domestic production is zero. Possibility (c) takes a middle position between the two polar possibilities (a) and (b). However, in our basic or unconstrained model this possibility is ruled out because the number of degrees of freedom (*i.e.*, the number of x_i and R_i variables that can be positive) is just equal to the number of internationally trading sectors. If both x_i and R_i are positive in any sector it will imply that in some other sector j possibility (d) will hold, *i.e.*, both x and R_j will have to be zero. But as we have seen earlier, this possibility is

⁷Since ours is a projective model, the final demands of all sectors increase, though with varying intensity—depending on the postulated growth rate of total consumption and the sector's expenditure elasticity.

⁸This is a sufficient and not a necessary condition.

ruled out. The effective sectoral choice in our basic or unconstrained model, thus, reduces to (a) and (b) only⁹.

The Savings-Constrained Model

The unconstrained model formulated above implicitly assumes that domestic savings are available in elastic supply and whatever requirements of this aggregate resources the optimising solution may generate will be met by the economy. The model does not take into account the institutional limits to the mobilization of domestic savings and does not ensure that the import-substitution programme will optimize the use of this resource. In order to introduce this realistic element into our model we impose an additional constraint limiting the marginal savings rate in the economy.

The savings constraint which distinguishes this model from the basic model is of the form

$$S_T - S_0 \leq s(Y_T - Y_0) \quad (10)$$

where $s = \frac{S_t - S_{t-1}}{Y_t - Y_{t-1}}$ is the *maximum* marginal savings rate the economy is considered capable of¹⁰.

$$\text{and } S_t \equiv \bar{J}_t = \bar{Z}_t + \bar{E}_t \quad (11)$$

$$Y_t \equiv \bar{C}_t + \bar{E}_t + \bar{J}_t + \bar{Z}_t \quad (12)$$

$$\text{or } J_T - \bar{Z}_T + \bar{E}_T - S_0 \leq s(\bar{C}_T + \bar{E}_T + \bar{J}_T - \bar{Z}_T) - sY_0. \quad (13)$$

Rearranging the terms so that those involving endogenous variables (unknown) are on the left-hand side and those involving base-year and exogenous variables (constant) are on the right, we have:

$$(1-s)\bar{J}_T - (1-s)\bar{Z}_T \leq (S_0 - sY_0) + s\bar{C}_T - (1-s)\bar{E}_T \quad (14)$$

Substituting (6) into (a) (iii) on page 517 and the result into (14), we have

$$(1-s)k\bar{B}x - (1-s)\bar{Z}_T < (S_0 - sY_0) + s\bar{C}_T - (1-s)\bar{E}_T \quad (15)$$

since $0 \leq s \leq 1$, we can divide both sides by $(1-s)$ to get:

$$k\bar{B}x - \bar{Z}_T < \left\{ (S_0 - sY_0) + s\bar{C}_T \right\} - \bar{E}_T$$

$$\text{or } \frac{-k}{1 \times 1} \bar{B} \frac{x}{(n \times 1)} + \frac{\bar{Z}_T}{(1 \times 1)} \geq \left\{ (sY_0 - S_0) - s\bar{C}_T \right\} + \bar{E}_T \quad (16)$$

⁹In the savings-constrained and foreign-aid-availability models described below the degree of freedom is increased by one if the new constraint is effective and possibility becomes admissible.

¹⁰The value of s and all the base-year variables, *i.e.*, variables with time subscript 0, is assumed to be known.

which is the final form in which the savings constraint is imposed. The other constraints of the model and the objective functions are the same as those in the basic (unconstrained) model, *i.e.*, (7) and (8) and objective functions 1 (minimization of terminal-year imports) and 2 (maximization of terminal-year national income).

Foreign Aid Availability Model

Another variant of our basic model is based on the assumption that a certain proportion of the total resources needed to undertake the development programme can be met by reliance on foreign assistance¹¹. Again, this also corresponds to the facts of life and the developing country can take advantage of the availability of such assistance, even though its long-term goal may be to reduce dependence upon such aid.

We can also look upon the foreign-aid constraint as resulting from the decisions of aid-giving countries. In that event, however, it would be more logical in the framework of our model to maximize national income rather than to minimize total imports. Maximizing national income in our model is equivalent to maximizing domestic savings¹². In the empirical implementation of our models we have tried both objective functions and found that although in the unconstrained case the solutions to both objective functions are the same, in the case of foreign-aid-availability cases the solutions are different.

Let the postulated minimum ratio between foreign and total investment be λ , then our foreign-aid constraint takes the form:

$$\bar{Z}_T - \bar{E}_T \geq \lambda \bar{J}_T \quad (17)$$

Again substituting the value of \bar{J}_T from (1) and (2) transposing the constant terms to the right-hand side we get:

$$-(k.\lambda) \begin{matrix} B \\ 1 \times 1 \end{matrix} \begin{matrix} x \\ (1 \times n) \end{matrix} + \bar{Z}_T \geq \bar{E}_T \quad (18)$$

The other constraints of the model are (7) and (8) the same as in the basic (unconstrained) model. The objective functions of this model are also the same as those of the basic (unconstrained) model.

¹¹The ratio of foreign aid to total investment has often been suggested as a criterion by aid-givers to determine the performance and, hence, the future needs of the recipient country. This ratio, on which the recipient may have little influence, is likely to depend on the size, stage of development and geographic and political importance of the country.

¹²This follows from the fact that consumption is an exogenous variable in our model and that the maximum value of two linear functions is the same if they differ only by a constant term (exogenously given terminal-year consumption, in our case).

The Dual Problem

In this subsection we shall discuss the dual problem with respect to the single-period programming model minimizing total imports (objective-function 1) in the terminal year. We shall try to give an economic interpretation to the dual variables or "shadow prices" of the problem.

Corresponding to our primal problem there is the dual problem:

$$\text{Minimize } \sum_{i=1}^n p_i^y y_i + \sum_{i=n+1}^{n+r} p_i^z w$$

subject to

$$\left[\begin{array}{ccc|c} I & -A & -.15B & I^* \\ \hline & -M & +.15B^m & I \end{array} \right] \begin{bmatrix} p^x \\ p^z \end{bmatrix} \leq \begin{bmatrix} (0) \\ (1) \end{bmatrix} \quad (19)$$

where p^x is an $n \times 1$ vector of dual variables associated with the n input-output balance equations and p^z is an $r \times 1$ vector of dual variables associated with the r import equations and (0) is an $n \times 1$ vector consisting of zero elements (being the coefficients corresponding to domestic production activities in the objective function) and (1) is a $r \times 1$ vector consisting of unit elements (being the coefficients corresponding to importing activities in the primal objective function).

In the previous section we elaborated the choice mechanism through which the model decides to include or exclude domestic production and importing activities. The inclusion or exclusion of a particular activity in the optimal solution is also indicated by the "shadow prices" or the dual variables.

Each activity in a linear programme can be represented by a column vector of coefficients showing the per unit costs of (or revenues from) running the activity at a unit level. If the activity is included in the optimal solution (*i.e.*, if the corresponding variable is at a positive level) then the cumulative product of its coefficients and the corresponding shadow prices should be exactly equal to its coefficient in the cost function. Let α_{ij} be the per unit requirement of resource i for running the j -th activity, then if activity j is to be included in the optimal solution we must have

$$\sum_{i=1}^n \alpha_{ij} \pi_i = c_j \text{ where } \pi_i \text{ is the dual variable associated with the } i\text{-th}$$

resources and c_j is the revenue obtained by operating the activity at a unit level and $\sum_{i=1}^n \alpha_{ij} \pi_i \leq c_j$ if the j -th activity is not included in the optimal solution [6].

The economic meaning of this is that an activity is profitable (and hence included in the optimal solution) only if its net marginal revenue is equal to its marginal cost and is not profitable (and hence excluded from the optimal solution) if its net marginal revenue falls short of its marginal cost.

The shadow prices associated with each constraint represent the marginal reduction in the value of the minimand which can be achieved by relaxing the *i*-th constraint by 1 unit. Thus, if the final demand of sector *i* is reduced by 1 unit the value of total imports is reduced by p_i^x , the price corresponding to the input-output balance equation of the *i*-th sector. The prices corresponding to the second group of constraints, *i.e.*, the import constraints measure the costs of having to import certain fraction of intermediate and investment goods rather than produce them domestically; they can be interpreted as the prices of specific factors whose scarcity prevents complete import substitution. The constraints on domestic savings and foreign aid can also be similarly interpreted. The shadow prices of the domestic-savings constraint reflect the scarcity of domestic savings which prevents complete import substitution. The shadow price of the foreign-aid constraint, on the other hand, reflects the scarcity of foreign aid which prevents further substitution of domestic savings by foreign aid and determines the lower limit of the import-substitution process.

The theorem that if a particular activity is included in the optimal solution its marginal cost must equal its marginal revenue, enables us to analyse the cost structure of each domestic production activity, in terms of the shadow prices of the model.

If the incremental domestic production activity to produce the *j*-th sector's output is included in the optimal solution then the following should hold¹³.

$$p_j^x = \sum_{i=1}^n (a_{ij} + .15b_{ij}) p_i^x - \sum_{i=1}^r (m_{ij} + .15b_{ij}^m) p_i^z = 0 \tag{20}$$

But since all the import activities are likely to be at a positive level, we have from (19)

$$\left(\frac{I^*}{I}\right)' p = (1), \text{ where } p = \begin{pmatrix} p^x \\ p^z \end{pmatrix}$$

$$\text{or } p_{i+n}^z + p_i^x = 1 \tag{21}$$

$$\text{for } i = 1, 2, \dots, r$$

and hence:

$$p_j^x - \sum_{i=1}^n (a_{ij} + .15b_{ij}) p_i^x - \sum_{i=1}^r (m_{ij} + .15b_{ij}^m) (1 - p_i^x) = 0$$

¹³In the following the value of the stock-flow conversion factor, *k*, is assumed to be .15.

$$\text{or } p_j^x - \sum_{i=1}^n [(a_{ij} - m_{ij}) + .15(b_{ij} - b_{ij}^m)] p_i^x - \sum_{i=1}^r (m_{ij} + .15b_{ij}^m) = 0$$

$$\text{or } p_j^x - \sum_{i=1}^n [\bar{a}_{ij} + .15(\bar{b}_{ij})] p_i^x + \sum_{i=1}^r (m_{ij} + .15 b_{ij}^m)$$

$$\text{or } p_j^x = \sum_{i=1}^n [\bar{a}_{ij} + .15\bar{b}_{ij}] p_i^x + \sum_{i=1}^r (m_{ij} + .15 b_{ij}^m) \quad (22)$$

The left-hand side represents the net marginal revenue of the j -th domestic product activity and the right-hand side represents its marginal cost. The cost structure of the domestic production activity can be analysed in the following way:

COST STRUCTURE OF A DOMESTIC PRODUCTION ACTIVITY

Category		Capital	Total
Origin	Current		
Domestic	$\sum_{i=1}^n \bar{a}_{ij} p_i^x$	$\sum_{i=1}^n .15 \bar{b}_{ij} p_i^x$	$\sum_{i=1}^n (\bar{a}_{ij} p_i^x + .15 \bar{b}_{ij} p_i^x)$
Foreign	$\sum_{i=1}^r m_{ij}$	$\sum_{i=1}^r .15 b_{ij}^m$	$\sum_{i=1}^r (m_{ij} + .15 b_{ij}^m)$
Total	$\sum_{i=1}^n \bar{a}_{ij} p_i^x + \sum_{i=1}^r m_{ij}$	$\sum_{i=1}^n .15 \bar{b}_{ij} p_i^x + \sum_{i=1}^r .15 b_{ij}^m$	$\sum_{i=1}^n (\bar{a}_{ij} p_i^x + .15 \bar{b}_{ij} p_i^x) + \sum_{i=1}^r (m_{ij} + .15 b_{ij}^m)$

Since from (21) $p_i^x < 1$, it follows that the use of imported, rather than domestic, current inputs requires the payment of a premium $= (1 - p_i^x)$. The pricing of capital and the interpretation of the stock-flow conversion factor also emerges very clearly from the above table. The expression

$$\sum_{i=1}^n b_{ij} p_i^x \quad \text{and} \quad \sum_{i=1}^r b_{ij}^m p_i^x$$

represent the total value of the (incremental) capital stock according to the implicit shadow prices of the model. In effect, this capital is rented out for use

by activity j at a rate given by the stock-flow conversion factor of .15, which can be interpreted as the implicit gross rate of interest in the model.

B. EMPIRICAL IMPLEMENTATION

The empirical implementation of an economy-wide model such as ours requires the estimation of a large number of parameters and exogenous variables involved in the various equations and constraints of the model. Since the estimation of each of these hundreds of parameters and exogenous variables is itself a task involving considerable time and labour we had to rely a great deal on available data in the field¹⁴.

In this section, we outline the sources of data and methodology employed for the implementation of our models. Part I discusses the degree of aggregation of sectors and the aggregated input-output table on which most of the computations are based. Part II describes the estimation of the other parameters of the model. Part III is devoted to the discussion of the estimates of the exogenous variables of the model, *e.g.*, consumption and exports. All statistical tables are attached in Appendix A.

I. Sectoral Aggregation and the Input-Output Matrix

The degree of sectoral aggregation adopted in our empirical implementation of the single-period models was influenced by our desire to keep the computational burden within manageable limits and by the degree of aggregation of the available input-output table on which our programming model crucially depended. Although a larger number of sectors could be distinguished, we decided to divide the economy into twelve sectors by the aggregation of some groups of the thirty sectors into which the economy is divided by Conrad, Stern and Tims in their interindustry relations table for 1960/61. Our sectoral classifications and that of the original table are compared in Table I. The first eight sectors of our model derive their total supplies from both domestic production and imports, while the total supplies of the last four are made up entirely of domestic production.

The 12×12 current input-output coefficients (matrix A, Appendix Table A-1) was obtained by the aggregation of the corresponding 30×30 table prepared by Conrad *et. al* [5]. The aggregation was performed, with the base-year values of gross output of the sectors being aggregated as weights¹⁵.

¹⁴To a large extent, the degree of the sophistication of our model was conditioned by the availability of data on the economy of Pakistan.

¹⁵The aggregation procedure is the same as given in [4, p. 36, equation 2.14].

TABLE I

**COMPARISON OF SECTORAL CLASSIFICATION
WITH THE ORIGINAL TABLE***

Our sector number	Name of the sector	Corresponding sectors/sectors in Conrad <i>et. al</i>
International		
I	Agriculture	1, 19
II	Mining	2
III	Food processing	3
IV	Textiles	4, 7, 8
V	Wood and paper manufactures	5, 6
VI	Chemicals	9, 10, 11
VII	Machinery	12, 13, 14, 15
VIII	Transport equipment	16
National		
IX	Construction	18
X	Small-scale manufactures	20, 21
XI	Electricity and gas	22
XII	Services (including government services)	23-30

*The original input-output tables also had a "Miscellaneous" manufacturing-industries sector (sector 17 in the Conrad table). Such sectors, as is well known, are always a problem in interindustry studies as they represent a conglomerate mass whose functional characteristics cannot be identified with any of the constituent sectors, nor with any other sector in the table. In order to overcome this "nuisance" aspect of the miscellaneous industries sector we distributed the total value of gross output of this sector among the other manufacturing sectors (sectors 2-16) and combined the coefficients of this sector with those of each of the latter in proportion to the value of gross output of the "miscellaneous" sector falling in a sector's share and its own value of gross output, respectively.

II. Other Parameters of the Model

(a) *The Current Import Coefficients Matrix M*: A fundamental contribution of the Conrad-Stern-Tims study [5] was to separate imports from interindustry flows matrix for 1960/61 and construct a 16×30 interindustry import coefficients matrix. The 8×12 import coefficients matrix in Appendix Table A-2 has been obtained by aggregating the Conrad *et. al* import coefficients matrix.

The aggregation procedure and the sector correspondence between our import matrix and the Conrad matrix is the same as that for the overall input-output coefficients matrix A, outlined earlier¹⁶.

(b) *The Matrix of Capital Coefficients B*: The 3×12 matrix of fixed capital coefficients (Appendix Table A-3)¹⁷ and the inventory coefficients in Appendix Table A-3a are derived not from Pakistan sources — to the best of our knowledge, no such table based entirely on Pakistani resources was available at the time the present study was carried out — but on the basis of two such matrices prepared for India at the Indian Statistical Institute [12;18]. The three sectors producing capital goods in our classification are Sector VII (machinery), VIII (transport and communication equipment) and IX (construction). Suitable adjustments were made in adapting the two Indian tables to our sectoral classification scheme. The total capital-output ratio for each sector was obtained as a sum of fixed and inventory. These coefficients are given in Appendix Table A-3b.

(c) *The Matrix of Capital Import Coefficients B^m*: The 2×12 matrix of capital import coefficients is derived from the matrix of fixed capital coefficients in the following way: Sector IX (construction) is excluded as it is a purely domestic sector. The coefficients of Sectors VII (machinery) and VIII (transport equipment) are multiplied by 0.8. The latter is done on the assumption that 80 per cent of each of the capital stocks of Sectors VII and VIII used in each sector are of imported origin. While the basis of this assumption on the aggregate level is empirically sound, the assumption that each stock has the same import intensity in each sector is rather arbitrary¹⁸. However, in the absence of detailed information on the composition of different capital stocks it is hard to remove this arbitrariness.

(d) *The Stock-Flow Conversion Factor*: In the previous section, we discussed the significance of the stock-flow conversion factor in order to derive the level of investment activity in the terminal year. To recapitulate, the stock-flow conversion factor is the ratio of terminal-year investment in capital good *i* to the total increment in the stock of capital goods during the period. If we further assume that this conversion factor is the same for all capital goods then the stock-flow conversion factor is also the ratio between terminal-year total investment and the sum of investments during the period.

¹⁶The matrix was used by Conrad *et. al* to project the raw-material import requirements per unit final demand for each sector and is given by $M(I-A)^{-1}$, where M is the import coefficients matrix, $(I-A)^{-1}$ is the Leontief inverse.

¹⁷Appendix Table A-3 can be considered as a 12×12 matrix with all but three rows having zero elements.

¹⁸For instance, *the Third Five-Year Plan* [17] gives the following figures: output of investment goods in 1965, 340 million rupees [17, Table 2, p. 20]; imports of capital goods in 1965, 2,015 million rupees [17, Table 6, p. 24], giving us a ratio of nearly 0.8.

Since our planning period is ten years of duration, $T = 10$. We have seen that the value of k depends on the assumed rate of increase of investment. We have assumed the value of $k = .15$ which corresponds to the rate of growth of investment of about 9 per cent per annum. Since the stock-flow conversion factor can also be interpreted as the "shadow rate of return on capital" our assumption implies that this rate will be 15 per cent for the Pakistani economy during the period. On both accounts the assumption of the 15-per-cent figure does not seem unreasonable¹⁹.

(e) *The Planning Period and the Base Year*: The model formulated in the last section is stated in terms of increments over the planning period. We, therefore, had to choose a base year and the length of the planning period in order to get the terminal year. We chose the year 1962/63 as the base year for the empirical implementation of our model, since that was the most recent year for which most of the data we needed were available. The choice of ten years as the length of the planning period is also the time-worn practice in all finite-horizon planning.

For the base year, 1962/63, we generated the values of the endogenous elements (e.g., J_0, MX_0) internally rather than taking their actual values in that year. The logic behind this kind of procedure in the implementation of a theoretical model is the desire to get an internally consistent set of variables. The input-output table from which most of the parameters of our model were derived related to the year 1960/61 and the first year of our model was 1962/63. We, therefore, needed to extend the input-output projections to 1962/63, in order to get consistent estimates of MX_0 and J_0 vectors²⁰.

The extension was performed by assuming a modest rate of growth of 5 per cent per annum in all the variables in the initial year. Although theoretically a little awkward and difficult to justify the assumption is computationally very convenient²¹.

By making this assumption we ensure that the available data for the base year 1962/63 are consistent with our model and, when the actual values for the exogenous variables are inserted, the model exactly reproduces the 1962/63 input-output table. For any future year, the model will produce a consistent

¹⁹Manne [11] has used a value of .15 for the conversion factor in his study on Mexico and a value of .17 for the consistency model on India. Weisskopf [21] has used two conversion factors, .14 and .18, for his import-substitution model on India. (All the models had 10-year time horizon). Weisskopf has noted the relative insensitivity of the qualitative nature of his results to changes in the value of the conversion factor.

²⁰J. Sandee, [18, p. 15] adopts a similar, though different in detail, approach to extrapolate the "assumed state of affairs" in 1960—the first year of his model—from the input-output table relating to 1953/54.

²¹It is easy to show that with the stated assumption $X_0 = (I - A - .05I)^{-1} (C_0 + E_0 - Z_0)$ and $J_0 = .05BX_0$. The estimation of C_0, E_0 and Z_0 is described below.

set of sectoral accounts once a set of values for its exogenous variables is inserted. However, the model cannot be used to provide consistent "back-casts" for years prior to 1962/63 as the model is not designed to explain Pakistan's economic history.

The choice of the 5-per-cent growth rate for the base year — not in itself implausible — was influenced by our desire to conduct a sensitivity analysis on rates of growth between 4 per cent to 8 per cent per annum. Neither the choice of the pattern nor that of the rate of growth in the initial year is likely to be of great consequence to the terminal-year solution since the planning horizon is sufficiently long

III. The Estimation of Exogenous Elements in the Model

Besides the fixed coefficients of the model, we also had to estimate the exogenous variables of the model in order to get a determinate solution for the unknowns of the problem, *viz.*, the domestic production increases and import levels in the terminal year. Since the purpose of our model was not to give a unique and deterministic solution of the unknowns but to provide a milieu of choices we also conducted a sensitivity analysis on the model by changing the final demand vectors of the problem. Thus, the solutions obtained are in the nature of conditional predictions: given certain assumptions about the growth of final demand elements, what will be the effect on the choices between domestic production and imports.

Thus in order to compute the 12×1 vector y , (Eq. (4)) we need to have an estimate of consumption expenditures, exports, imports, investment and total output of each sector in the initial year. In addition, we have to estimate consumption expenditures and exports for the terminal year. Imports, investment and total outputs in the terminal year are endogenously determined. We shall detail below how each of the seven exogenous vectors was estimated.

(a) *C₀, Sectoral Consumption Expenditures in the Initial Year:* The sectoral distribution of domestic consumption expenditures has been estimated from a variety of sources. Although some unofficial estimates of gross national expenditure have been made in the aggregate, sectoral breakdowns of such expenditures are not available.

In arriving at the sectoral breakdowns, we have relied mainly on production and foreign-trade statistics, deriving consumption expenditures as a residual between the total availability of a commodity (domestic production plus imports) and exports, investment changes in stocks and interindustrial uses.

For many commodities and sectors we have used the per capita "availability" figures in East and West Pakistan for 1959 worked out at the Pakistan Institute of Development Economics [16] and adjusted the figures for price

changes from 1959/60 to 1962/63. Multiplying the regional per capita figures thus obtained by regional population figures for 1962/63, and adding we have obtained the national consumption figures for Sectors I to VIII. For Sectors IX to XII we have relied mainly on the national income figures for these sectors. Column (4) of Appendix Table A-4 gives the estimate of sectoral consumption expenditures in the initial year.

(b) Z_o, E_o , *Sectoral Distribution of Imports and Exports in the Initial Year*: These are derived from the published sources on foreign-trade statistics [15]. The only manipulation involved was to reclassify the figures into the mould of our sectoral classification. Columns (2) and (5) of Appendix Table A-4 give the sectoral distribution of imports and exports, respectively, in 1962/63.

(c) C_t , *The Vector of Consumption Expenditures in the Terminal Year*: The projections of consumption expenditures in 1972/73 were based on the rates of growth of total consumption and expenditure elasticities of the goods and services of different sectors. Instead of assuming a single rate of growth, we conducted a sensitivity analysis on the model with growth rates ranging from 4 per cent to 8 per cent at 0.5 per cent intervals. This was done to add richness to the variety of results, since the rate of growth of consumption is often an instrument the planners have to choose and work with.

The formula used for projecting consumption expenditures of each category was derived from the assumption of a logarithmic relationship between per capita consumption of a particular category and per capita total expenditure. *i.e.*,

$$\left(\frac{{}_iC_t}{P_t} \right) = \frac{\left(\sum_{i=1}^n {}_iC_t \right) \eta_i}{P_t}$$

where ${}_iC_t$ is the value of consumption expenditure on the goods and services of the i -th sector, P_t is the population at time t , η_i is the total expenditure elasticity of sector i 's goods and services. Taking logs and differentiating both sides of the above expression with respect to time we have:

$$r_{ci} = (1 - \eta_i) \rho + \eta_i r_c$$

where r_{ci} is the annual rate of growth of consumption expenditures on sector i 's goods and services, ρ , is the annual rate of growth of population and r_c is the postulated annual rate of increase of total consumption expenditures²².

A wide variety of empirical estimates are available on total expenditure or income elasticities of individual commodities in underdeveloped countries.

²²The value of ρ , the annual growth rate of population, used in the present estimate is 2.6 per cent — the same used by the Pakistan Planning Commission.

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where r_{ci} is the annual rate of growth of consumption expenditures on sector i 's goods and services, ρ , is the annual rate of growth of population and r_c is the postulated annual rate of increase of total consumption expenditures²².

A wide variety of empirical estimates are available on total expenditure or income elasticities of individual commodities in underdeveloped countries.

²²The value of ρ , the annual growth rate of population, used in the present estimate is 2.6 per cent — the same used by the Pakistan Planning Commission.

Such estimates have also become available for Pakistan recently. However, our task was more than that of choosing the estimates of individual elasticities, we also had to obtain the aggregate elasticity of each sector as a weighted average of the elasticities of individual commodities produced by it. This was by no means an easy task and we had to rely considerably on our personal judgment in combining the available estimates to obtain the sectoral elasticities. The elasticities finally chosen for use in the implementation of the model are given in Column (1) of Appendix Table A-6.

Having obtained r_{ci} we applied it to the initial levels of sectoral consumption expenditures given in Appendix Table A-4 to obtain the ten-year consumption increases, *i.e.*,

$$({}_i C_{10} - {}_i C_0) = \left\{ (1 + r_{ci})^{10} - 1 \right\} {}_i C_0$$

These are given in Appendix Table A-5.

(d) E_{10} , *The Vector of Exports in the Terminal Year*: The estimates for exports in the final year were derived by applying the rates of growth of sectoral exports implicit in the estimates given in [8] to the sectoral distribution of exports in the initial year (Appendix Table A-4). The estimates of exports in the final year are given in Column (1) of Appendix Table A-7. A more optimistic forecast of exports is obtained by raising all sectoral export-targets in the terminal year by 10 per cent and is given in Column (2) of Appendix Table A-7.

The right-hand side vector of our programming problem y changes with each growth rate in the first 12 of its elements and remains invariant in the last eight, (consisting of interindustrial import requirements in the initial year, w). Appendix Table A-8 gives the value of the y vector for different rates of growth²³. As already indicated, w vector is given in Appendix Table A-4.

C. COMPUTATIONAL RESULTS OF THE SINGLE-PERIOD MODEL

In this section we discuss the computational results obtained by the various runs of the single-period model. The section is divided into four parts. In Part I we describe the various runs of the model according to the changes in the different essential parameters, and outline the relationships between the sectoral and macroeconomic results of the model. Part II discusses the general characteristics of the results in summary form. Part III analyses in detail the macroeconomic results and the relationships revealed by them between the two major resource scarcities, *viz.*, domestic savings and foreign aid. Part IV discussing the microeconomic or sectoral results in detail and with reference to the pricing and the dual problem.

²³The y vector is based on export targets given in Column (1) of Appendix Table A-9.

The large variety of optimal solutions obtained by the present exercise can be considered to represent alternative strategies of import substitution. Our model should, thus, be considered as a convenient device for generating alternative solutions and not as a deterministic model pointing toward a single optimal strategy. The result can be used by a planner to choose from among the many alternative import-substitution strategies they represent, enabling him to explore in great quantitative detail the implication of his assumptions or targets.

I. Classification of Results

Table II below gives a summary classification of the various runs of the single-period linear programming model which differ in the specification of the constraints, sensitivity to the choice of objective function, exogenous variables (such as the consumption and export targets, changes in coefficients, savings and foreign aid ratios). All four groups have at least 20 constraints: 12 input-output balance equations and 8 import constraints. Group II has an additional constraint on the maximum marginal rate of savings — which is postulated at 20 per cent and 15 per cent respectively, in cases A and B. Groups III and IV have an additional constraints on the minimum foreign-aid to total-investment ratio. All models were run with two objective functions: *i*) minimization of total terminal-year imports; and *ii*) maximization of terminal-year national income. In all models, other than those where a minimum foreign-aid to total-investment ratio was specified, the solutions to the two objective functions were identical. Group III and Group IV results differ when only the objective function is changed, other things remaining the same, and are hence presented separately. We have tried two estimates of terminal-year exports in a limited number of cases to see the effect of "export expansion" in the model. Two cases (cases C and D of Group I) in which import coefficients have been lowered in the terminal year have also been tried for the unconstrained model.

A selection of the results of these runs is presented in Appendix B in four sets of tables for each case²⁴. (The case and group numbers are indicated at the head of each table.) The first set, Appendix Table B-1 for each case, summarizes the macroeconomic aggregates based on the optimal solution to each individual run. The second set of tables, Appendix Table B-2, gives the optimal solution consisting of the incremental output of each sector. The third set, Appendix Table B-3, gives the value of terminal-year total imports in each sector and the fourth set, Appendix Table B-4, gives the value of non-essential imports in each sector (given by the value of the 'surplus' variables in the optimal solution).

²⁴In Appendix B, solutions to only four formulations of the model (*viz.*, Group I, case A; Group II, case A; Group III, case B; and Group IV, case A) are given. Other solutions can be obtained on request from the author.

TABLE II
CLASSIFICATION OF SINGLE-PERIOD MODEL RUNS

Name	Additional constraints	Objective function	Export performance	Import coefficients	Consumption growth rates (%)
Group I					
(Basic or unconstrained)					
Case A	<i>nil</i>	1,2	$\bar{E}_{10} = 4679.6$	No change	4, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0
Case B	<i>nil</i>	1,2	$\bar{E}_{10} = 5147.6$	No change	4, 6, 8
Case C	<i>nil</i>	1,2	$\bar{E}_{10} = 4679.6$	All import coefficients 50% of the initial level	4, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0
Case D	<i>nil</i>	1,2	$\bar{E}_{10} = 4679.6$	Import coefficients of capital-goods producing sectors reduced to zero	4, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0
Group II					
(Savings-constrained)					
Case A	Max. marginal rate of savings = 20%	1,2	$\bar{E}_{10} = 4679.6$	No change	4, 4.5, 5.0, 5.5 6.0, 6.5, 7.0, 7.5, 8.0
Case B	Max. Marginal rate of savings = 15%	1,2	$\bar{E}_{10} = 4679.6$	No change	4, 5, 6, 7, 8

(Contd.)

TABLE II — (Contd)

Name	Additional constraints	Objective function	Export performance	Import coefficients	Consumption growth rates (%)
Group III					
Foreign aid availability I					
A	Min. For. Aid to total investment ratio = 30%	1	$\bar{E}_{10} = 4679.6$	No change	4, 4.5, 5.0, 5.5 6.0, 6.5, 7.0, 7.5, 8.0
B	Min. For. Aid to total investment ratio = 40%	1	$\bar{E}_{10} = 4679.6$	No change	4, 5, 6, 7, and 8
B1	Same as above	1	$\bar{E}_{10} = 5147.6$	No change	4, 6, and 8
Group IV					
Foreign aid availability II					
A	Min. For. Aid to total investment ratio = 30%	2	$\bar{E}_{10} = 4679.6$	No change	4, 4.5, 5.0, 5.5, 6.0 6.5, 7.0, 7.5, and 8.0
B	Min. For. Aid to total investment ratio = 40%	2	$\bar{E}_{10} = 4679.6$	No change	4, 5, 6, 7 and 8
B1	Same as above	2	$\bar{E}_{10} = 5147.6$	No change	4, 6, and 8

Objective Function 1 = Minimization of total terminal-year imports.

Objective Function 2 = Maximization of national income in the terminal year.

Derivation of Macroeconomic Aggregates: The optimal solution to each of the several linear programmes listed in Table II gives two sets of values of the variables from which, in the main, our macroeconomic aggregates are constructed. These are: *a*) the values of the incremental outputs of each of the 12 sectors given in Appendix Table B-2; and *b*) the values of terminal-year imports of the eight different categories given in Appendix Table B-3. Since consumption and exports are exogenously given, imports and investment are the only other variables needed to derive the various macroeconomic aggregates, such as foreign aid, domestic savings and national income.

Total Investment: This is obtained in the model by making use of the relationship between terminal-year investment level and the incremental productive capacity assumed in Section A and the value of stock-flow conversion factor, $k (= .15)$.

Total investment in the terminal year is, thus, determined by multiplying the ten-year incremental output of each sector by .15 of the aggregate capital-output ratio of that sector and summing the result over the 12 sectors. Thus

$$\bar{J}_{10} = .15 \sum_{j=1}^{12} \beta_j x_j$$

where \bar{J}_{10} is total investment in the terminal year and β_j is the aggregate capital-output ratio of sector j .

Total value of imports in the terminal year is merely the sum of the value of endogenously determined imports of each of the eight categories.

$$\bar{Z}_{10} = \sum_{i=1}^8 i Z_{10}$$

The value of consumption expenditures in each run depends directly on the rate of growth of consumption postulated in it.

$$\bar{C}_{10} = \bar{C}_0 (1+r_c)^{10}$$

where \bar{C}_{10} and \bar{C}_0 are total consumption expenditures in terminal and initial periods, respectively, and r_c is the yearly rate of consumption growth (*see* Section B, for the derivation of \bar{C}_0 and r_c).

The total value of exports in the terminal year, whose sectoral breakdown is exogenously given, is merely the sum of the sectoral export levels.

$$\bar{E}_{10} = \sum_{i=1}^8 i E_{10}$$

Foreign aid in the terminal year is defined as the difference between terminal-year imports and exports.

$$F_{10} = \bar{Z}_{10} - \bar{E}_{10}$$

Domestic savings in each run are given as the difference between total investment and foreign aid.

$$S_{10} = \bar{J}_{10} - F_{10}$$

National income in the terminal year is obtained as the sum of exogenously given terminal-year consumption and endogenously determined domestic savings

$$Y_{10} = \bar{C}_{10} + S_{10}$$

II. Salient Features of Different Results

The large variety of results obtained (a selection of which is presented here) are intended to derive economically meaningful generalizations about the effect of making the different changes on the exogenous variables and parameters — which can to some degree be influenced by public policy — on the final outcome in the terminal year.

The differences among Groups I to III for identical assumptions about exports, import coefficients and consumption growth, arise from the effect of imposing an additional constraint regarding the availability of foreign aid or domestic savings. The primary effect when an additional constraint becomes binding in a linear programme is, of course, to increase the value of the minimand or decrease the value of the maximand. That this, in fact, is the case can easily be verified by comparing the value of imports in comparable cases of each of the group for any rate of growth of consumption. A few illustrative examples are given below.

TABLE III
VALUE OF IMPORTS IN THE OPTIMAL SOLUTIONS

Group	Case	r = .04	r = .05	r = .06	r = .07	r = .08
I	A	4407.4	5389.5	6502.3	7761.3	9149.5
II	A	4832.6	5690.2	6711.6	7814.0	9150.5
III	A	6199.4	6808.0	7552.4	8390.6	9417.2
IV	A	6540.8	7173.3	7877.0	8636.7	9516.2

The maximum degree of import substitution is possible when there are no supply limits on the availability of either kind of resources. If there are limits on the degree to which domestic savings can be mobilized, maximum possible import substitution can not be undertaken as additional foreign resources are needed to supplement domestic savings. If the high proportion of

total investment can be financed by foreign aid, the need for domestic savings is reduced and import substitution need not be carried to its limit. By the same token, the value of imports is higher in Case B than in Case A of Group II since the maximum marginal rate of savings in the former is lower (15 per cent) than in the latter (20 per cent). For the same reason Case B of Group III is always (for identical assumptions regarding export levels, import coefficients and consumption growth) more import-oriented than Case A. In general, Case B of Group III, *cet. par.*, always gives the most import-oriented solution among all other cases in Groups I-III. Case A of Group I, on the other hand, gives the least import-oriented or most favourable to domestic-production solutions. The two can, thus, be regarded as polar cases for each set of other assumptions, *i.e.*, relating to export levels, import substitution and consumption growth.

In Table IV we contrast the level and structure of investment and imports of these two polar cases for a number of consumption growth rates. The corresponding values for the intermediate cases falls, naturally, between those of the two polar solutions²⁵.

The table points up the interesting, though obvious, relationship between import substitution and industrialization. Total industrial investment — defined as investment in Sectors II-VII— is higher in both absolute and relative terms in Group I, where import substitution is greatest. Industrial investment in Group I accounts for about a third of all investment and this ratio is relatively independent of the rate of growth of consumption. On the other hand, for Group III this ratio, which is only about 17 per cent at the 4-per-cent growth rate rises to about 30 per cent when the growth rate is raised to 8 per cent. Thus, higher consumption growth rates, which raise the demand for the output of industrial sectors, through the operation of Engel's law, call for a high ratio of industrial investment, which in turn promotes import substitution even when the condition for the supply of foreign aid is very favourable. In the unconstrained model the ratio of industrial investment is high even at low growth rates as import substitution is being pushed to its maximum possible limit.

Table IV also shows the difference between Group I and Group III solutions with respect to the level and composition of imports in the terminal year. Not only is the level of imports in Group III solutions uniformly higher than those of Group I, for all growth rates, the composition of the imports in the two groups also points out the degree of austerity of the polar regimes they represent. Whereas Group I imports consist only of the essential investment

²⁵The relative position of the different import-minimizing solutions is fairly fixed. The following is their ranking in decreasing order of import-orientation: unconstrained case, savings-constrained case with maximum marginal rate of saving = 20 per cent, savings-constrained case with maximum M.R.S. = 15 per cent.

TABLE IV
COMPARISON OF POLAR CASES

Identification		Investment		Imports			Foreign aid	Domestic savings
Group case	Rate of growth of consumption	Total	Industrial II-VII	Total	Material	Capital		
I A	.04	6984.9	2190.0 (31.4)	4407.4	3016.9	1390.5 (31.5)	0	7257.2
III B	.04	4848.2	750.6 (15.5)	6617.6	1845.8	887.8 (27.9)	3884.0 (13.4)	2909.6
I A	.05	8965.3	2848.3 (31.8)	5389.5	3606.1	1783.4 (33.1)	0	8225.4
III B	.05	6657.2	1235.1 (18.6)	7342.5	2253.0	1299.2 (30.7)	3790.3 (17.7)	3994.2
I A	.06	1116.3	3617.7 (32.4)	6502.3	4282.9	2219.4 (34.2)	0	9338.9
III B	.06	8767.0	1916.4 (21.9)	8183.0	2955.3	1737.7 (36.1)	3490.0 (21.2)	5263.4
I A	.07	13596.0	4502.2 (33.1)	7761.3	5061.0	2700.3 (34.8)	0	10514.3
III B	.07	11302.9	2882.5 (25.5)	9196.3	3894.1	2265.7 (42.3)	3036.5 (24.6)	6786.2
I A	.08	16256.4	5325.1 (33.5)	9149.5	6160.0	2989.5 (32.7)	0	11786.4
III B	.08	14263.1	4152.7 (29.1)	10379.2	5109.8	2871.9 (49.2)	2397.5 (27.7)	8563.1

Note: Figures in parentheses are percentages of total. Absolute values are in millions of rupees.

and intermediate-goods imports and no other imports, as much as half of Group III imports are devoted to consumer goods²⁶.

III. The Substitutability between Domestic and Foreign Resources

The macroeconomic results derived for the different cases are presented in Appendix Table B-1 for each case. Apart from providing a consistent picture of the macroeconomic aggregates, these tables can be used to see the trade-offs between domestic and foreign resources for achieving a given targeted growth rate of consumption. For this purpose we plot in Figures 1—3 the foreign resources, represented by the value of the foreign-trade deficit, on the vertical axis and domestic savings, average and marginal saving rates, respectively, representing the domestic effort on the horizontal axis for each of the cases in Groups I-III as listed in Table III. By joining the different points corresponding to each growth rate of consumption we obtain a family of iso-consumption curves. These contours show the different minimal combinations of foreign aid and domestic savings which result in the same consumption growth rate²⁷. These curves have downward slopes like ordinary production isoquants and their slope measures the trade-off between the two kinds of resources at a given point.

These contours show that domestic savings are easily substitutable for foreign aid when the ratio between the former and the latter is low. For any ratio of foreign aid to domestic savings, the substitutability of domestic savings, as represented by the slope of these contours diminishes as the rate of growth of consumption increases. This reflects the fact that as consumption targets are raised the requirements of intermediary and investment-goods imports rise and with fixed export targets the foreign-exchange gap becomes more and more important to the economy relative to domestic savings.

The extreme right-hand point of each of these contours represents the unconstrained case, where the import requirements are lowest and domestic savings requirements highest. Beyond this point, domestic savings are not substitutable for foreign aid as the implied import requirements are at the minimum level needed to sustain the given rate of consumption growth. Any attempt to step up domestic savings effort at the expense of foreign aid beyond

²⁶These are principally the imports of consumer durables, though in solutions of Group III for low growth rates, other consumer-good imports are also indicated.

²⁷In drawing these curves we have left out runs of Group IV in which the objective function was the maximization of national income rather than the minimization of imports (and, hence, of foreign aid). The resultant foreign aid and savings combination in these cases is not minimal. As can be seen by a comparison with a case where, *cet. par.*, the objective function is minimization of imports, the imports in these cases are higher. Also, since exports and consumption are exogenous, maximization of national income is equivalent to the maximization of domestic savings. The point corresponding to such a case will always lie above the minimal foreign aid-domestic savings contour corresponding to a given consumption growth rate. Also excluded are runs with higher export targets of \$147.6 and changes in import coefficients.

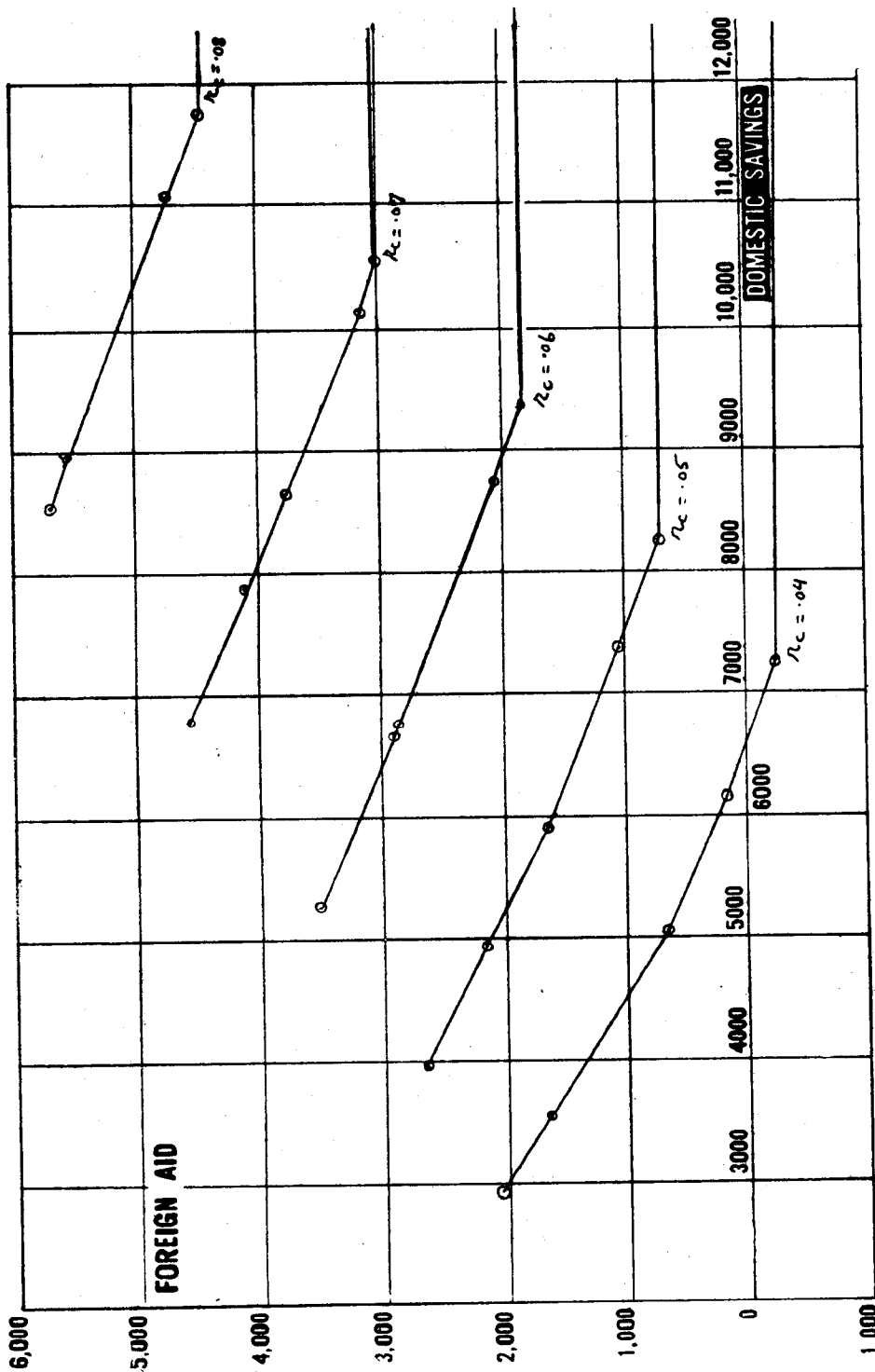


Figure 1. Trade-Off between Foreign Aid and Domestic Savings

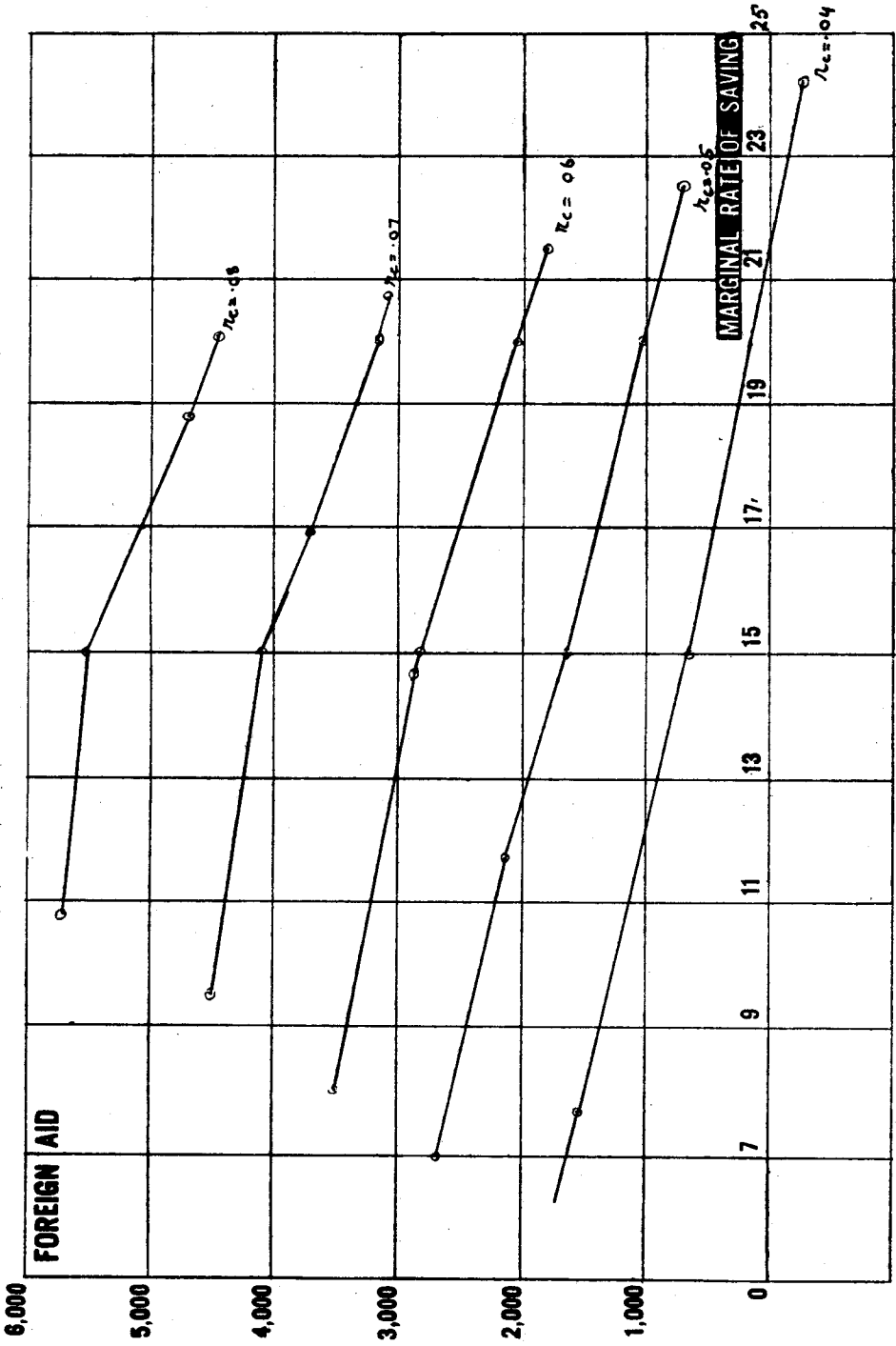


Figure 2. Relationship between Foreign Aid and Marginal Rate of Savings

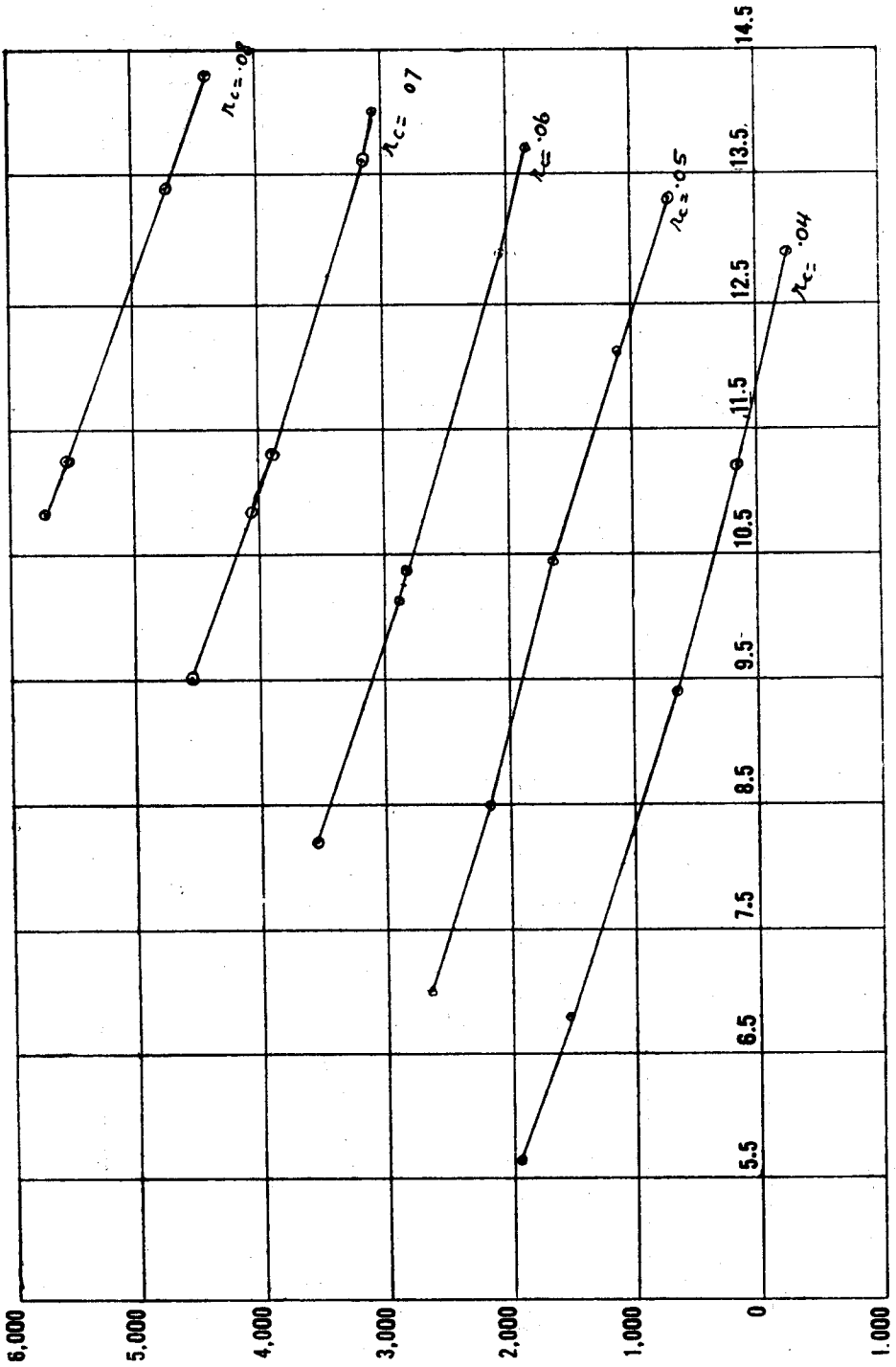


Figure 3. Relationship between Foreign Aid and Average Rate of Savings

this point can be successful only if a lower growth rate of consumption is acceptable. Otherwise domestic savings will be dissipated due to lack of complementary imported inputs needed to transform them into productive investment. Thus, a given level of foreign aid can support a unique maximum growth rate of consumption regardless of the complementary domestic savings resource. However, a given level of domestic savings can sustain a growth rate of consumption only if the necessary amount of complementary foreign aid can be obtained.

The extreme left-hand point on each contour is represented by the case in which the foreign-aid to total-investment ratio is equal to 40 per cent. It is, of course, possible to extend these contours leftwards by postulating a higher value for this ratio. This could have also been done by postulating a value for the maximum marginal rate of saving lower than 15 per cent²⁸. The length of the contours in Figures 1—3 gets smaller and smaller as the consumption growth rate increases, reflecting the fact that the variation in the value of the two variables plotted gets smaller and smaller as the consumption growth rate increases. As the targeted rate of consumption is increased the requirements of domestic savings and foreign aid both increase and the differences between the two extremal cases progressively narrow down. For instance, the 5-per-cent target of consumption growth can be sustained, on the one hand, by a combination of foreign-aid to total-investment ratio of 40 per cent and an average savings ratio of 7 per cent or, on the other hand, by a combination of 2.5 per cent and 13.5 per cent respectively, of the two ratios. The raising of the target rate to 8 per cent requires 40 per cent and 10.8 per cent respectively, in Group III Case B and 27.5 per cent and 14.3 per cent of the two ratios in Group I Case A.

The slopes of the iso-consumption curves in Figures 1—3 also reflect the shadow price of foreign exchange. For the ratio of the increment in domestic savings to the decrease in foreign aid, holding consumption and exports constant, can be regarded as the rate of transformation of rupees into foreign exchange, through the process of import substitution. Our empirical results show that to replace a million rupee worth of foreign exchange the Pakistani economy needs to generate 1.5 to 3.0 million rupee worth of domestic savings. This rate varies according to the consumption target postulated and the ratio of foreign aid to domestic savings available as discussed earlier. In terms of savings rates, a billion rupee worth of foreign aid can enable the economy to reduce the average savings rate by 3 to 4 per cent and the marginal savings rate by 6 to 9 per cent²⁹.

We may now address ourselves to the problem of assessing, in the light of our results, the relative importance of the two aggregate resource scarcities in

²⁸All cases of foreign-aid to total-investment ratio of 40 per cent yield marginal savings rates less than 15 per cent.

²⁹Bergsman and Manne in [1] calculate this rate for India at "roughly two rupees" during 1965/66 through 1975/76. Weisskopf's study for India [21] also gives similar results.

the economy — domestic savings and foreign aid. In an optimizing model, such as ours, the two resource “gaps” are the same and the necessary import substitution to equalise the two gaps has already taken place through the process of import substitution. However, the different runs of our model give us the requirements for the two resources and we can use our judgment as to whether they are realistic in the Pakistani context.

We have seen earlier that, for a given level of foreign aid, there is a maximum rate of saving which can be translated into productive investment, and hence also a maximum target of consumption growth in the terminal year. Conversely, for a given target of consumption growth there is a minimum requirement of foreign aid. These maximum (realizable) savings and minimum foreign-aid combinations are given by the solution of the unconstrained cases. Figure 1 shows that the maximum rate of growth of consumption achievable with no foreign aid is a little over 4 per cent. The average and marginal saving rates corresponding to this situation are 13 per cent and 24 per cent respectively. For achieving a 6-per-cent consumption growth rate target the minimum foreign-aid requirement is roughly the same as in the initial year although the average savings rate needed to achieve this target is 13.7 per cent or more than double than in the initial year. For the 8-per-cent consumption growth target the minimum foreign-aid requirement is about 2.5 times the initial level and the average domestic savings rate needed is 14.26 per cent.

We, thus, see that if the foreign-aid requirements are to be kept at a minimum, the economy needs to generate marginal saving rates ranging between 20 per cent to 24 per cent and average saving rates ranging from 13 per cent to 14.25 per cent. Whether these maximum realizable rates of savings suggest that there exists in the Pakistani context the possibility of a loss of potential savings depends directly on what the order of magnitudes of these rates in fact are and whether the country is, in fact, capable of achieving them. No definitive estimates of these rates are available although the Planning Commission and other related sources have provided some educated guesses about them which put the marginal saving rates in the 20—24 per cent and the average saving rates in the 12—14 per cent range. To the extent that the economy can generate rates higher than these, it is possible for the phenomenon of foreign-exchange bottleneck to arise in the Pakistani situation. A comparison with similarly situated countries like India and Burma shows that the Pakistani economy is capable of generating higher saving rates.

On the other hand, however, it must be pointed out that the maximum realizable saving rates described above are yielded by the model when it is assumed that the import coefficients in the terminal year remain unchanged. As has been pointed out earlier this assumption limits the possibilities of import substitution in the model, essentially, to that of consumption goods. The two

cases in which import coefficients were lowered — Group I, Cases C and D — show that as high as 17—22 per cent average savings rate (corresponding to 30—35 per cent marginal saving rates) may be needed in order to undertake any serious import-substitution programme. To that extent it can be argued that the effective constraint in the Pakistani context is that of domestic savings and not of foreign exchange.

IV. Sectoral Outputs and Imports in the Terminal Year

In this subsection we analyse the sectoral composition of total outputs and imports in the terminal year given by the different runs of our model. Appendix Table B-2 for each case shows the levels of incremental outputs in each sector in the terminal year. Appendix Table B-3 shows the sectoral levels of imports in the terminal year and Appendix Table B-4 for each case shows the level of nonessential imports in the terminal year³⁰.

Due to the rather high degree of aggregation of our sectors it is not possible to attach very great operational significance to our sectoral results. Even though the more damaging consequences of the all-or-nothing character of sectoral solutions are avoided by formulating the model in terms of increments over the ten years, the high degree of aggregation makes the assumption of linearity a little distant approximation to reality. Also, from the point of view of the planner desirous of finding out which of the thousands of projects he should choose in order to pursue the import-substitution strategy efficiently the sectoral results of this highly aggregative nature are not likely to be of immediate help. Nevertheless, the present results provide a valuable insight into the nature of choices between domestic production and imports available to the economy.

As pointed out in subsections B and C of this section, the unconstrained model solutions, Group I, provide the greatest opportunity for import substitution. The level of incremental domestic production is the highest among all groups, for each growth rate, and the level of imports of each category is the lowest. Also, only the most essential imports of intermediate and investment goods are undertaken and there is no import of consumption goods.

The fact that all incremental domestic outputs are at a positive level is itself significant. It implies that, in the absence of any constraints on the availability of domestic savings and foreign aid, all domestic production activities are feasible and more economical than (nonessential) imports. Although this result is in conformity with similar results obtained by Manne for Mexico [11] and Weisskopf for India [21], the contrary possibility cannot be ruled out

³⁰In the jargon of linear programming, Appendix Tables B-2 and B-3 represent the value of the non-slack variables and Appendix Table B-4 that of slack variables in the import constraints. The slack variables associated with the foreign aid or savings constraint are always zero since this constraint is always found to be effective in all runs of our model.

either on theoretical or empirical grounds. Theoretically, there is nothing in the model to prevent it from discarding some of the incremental domestic production activities in favour of imports. Empirically, it has been conjectured by Soligo and Stern [19] that in the Pakistan context, there may be several industries whose establishment has led to negative value added — in that the direct and indirect per unit costs of such industries, may be higher than the domestic price of the commodities produced, net of the implicit subsidy afforded through protection³¹.

The unconstrained model provides only one pattern of domestic production and imports for all growth rates; *i.e.*, the optimal basis rates remains the same, regardless of the growth rate of consumption and, hence the final demand vector. This invariance of the optimal basis is due to the applicability, in the unconstrained model, of the well-known substitution theorem on open Leontief input-output systems [10]. The theorem can be summarized to state that the optimal choice of activities in an open Leontief system is independent of the levels of final demand if there is only a single basic resource scarcity. In the unconstrained model the only basic resource scarcity is foreign exchange and, hence, the optimal basis does not change with changes in the final demand levels.

The savings-constrained and foreign-aid availability models, on the other hand, present more interesting optimal solutions from the point of view of choice between imports and domestic production. Incremental domestic production in some sectors is unprofitable and it is cheaper to satisfy all incremental demands for the output of that sector by direct imports. As a general rule, the lower the rate of growth the larger is the number of sectors where incremental domestic production levels are zero or where nonessential imports are at a positive level, and the larger the ratio of nonessential to total imports. This implies that at higher consumption growth rates a larger degree of import substitution is called for in the face of a limit on the availability of savings of foreign aid. The explanation for this in the savings-constrained model (Group II solutions) is that low rates of consumption growth, in general, lead to low growth rates of national income and, since savings are related to incremental national income, to low levels of savings, which constrains the economy from carrying import substitution farther. For Group III solutions the explanation is that the maximum available foreign aid, which is related to total investment, permits a relaxation of domestic savings effort at low growth rates and reduces the need for import substitution.

For the same reason the Group II solutions with a lower marginal savings rate and Group III and Group IV solutions with a higher foreign-aid to investment ratio have a larger number of sectors where incremental domestic outputs

³¹Since our model does not take tariff protection into account, any such domestic production activities will be "priced out" and their activity level in the optimal solution will be zero.

are at a zero level or nonessential imports at a positive level and the ratio of nonessential to total imports is higher. Thus, it can be seen that in Group II solutions with 15-per-cent maximum marginal rate of savings (Case B) there are two domestic sectors whose incremental outputs are at a zero level and three sectors where nonessential imports are at a positive level at 4-per-cent growth rate of consumption; for higher growth rates only one sector where domestic incremental output is at a zero level and two sectors where nonessential imports are at a positive level. For, the higher marginal savings rate of 20 per cent domestic incremental production of only one sector is at a zero level—that too only at the low growth rates of 4 and 4.5 per cent. At these low growth rates two sectors have nonessential imports, while at higher growth rates all incremental domestic production activities are at a positive level and there is only one nonessential importing sector. In fact, for the 8-per-cent growth rate the solution is almost identical to that of Group I, Case 1 for the same growth rate.

In Group III, Case B (foreign-aid to total-investment ratio 40 per cent) solution for 4-per-cent rate of growth as many as 4 sectors have incremental domestic production at zero level and five sectors have nonessential imports at a positive level. At 5—6 per cent growth rate the number of such sectors are two and three respectively. At 7—8 per cent growth rate only one sector has incremental domestic production at a zero level and only two have nonessential imports at a positive level. For this group, Case A (foreign-aid to total-investment ratio 30 per cent) solutions are qualitatively the same for growth rates up to 5.5 per cent as those for Case B but for higher growth rates fewer incremental domestic production levels are zero and nonessential import levels positive. At 7.5—8 per cent growth rates in the solutions for this case all domestic production sectors are at a positive level and nonessential imports are positive for only one sector.

It is interesting to note which of the sectors show incremental domestic production at a zero level and nonessential imports at a positive level in the various solutions³². The most sensitive sector in this respect is Sector VIII (the transport equipment sector). Out of the total 28 runs in Group II and Group III all show positive nonessential imports for this sector. In more than half of the runs incremental domestic output in this sector is at a zero level. The next least viable incremental domestic production activity is that for producing the output of Sector VII (the machinery sector). In seven out of 28 runs incremental domestic production in this sector was at a zero level and in 19 runs the imports of this sector's output were above the essential level. The other three sectors susceptible to savings or foreign-aid constraints are Sectors V (wood

³²Group IV solutions show incremental outputs in all sectors at all growth rates are at a positive level. The only nonessential imports occur in the agriculture sector. One is tempted to call this solution as the "PL 480" solution. Foreign aid is utilized to import foodgrains and domestic resources diverted to increase investment.

products) or VI (chemicals) and IV (textiles). Table V gives in summary from the data described above.

TABLE V

NUMBER OF RUNS IN WHICH SECTORS DOMESTIC OUTPUT IS AT A ZERO LEVEL (OR NONESSENTIAL IMPORTS AT A POSITIVE LEVEL)

Sector	Group II		Group III		Total
	Case A	Case B	Case A	Case B	All cases
VIII Transport equipment	2 (9)	5 (5)	7 (9)	5 (5)	19 (28)
VII Machinery	0 (2)	1 (5)	3 (7)	3 (5)	7 (19)
V Wood products	0 (0)	0 (0)	1 (1)	2 (3)	3 (4)
VI Chemicals	0 (0)	0 (0)	1 (3)	1 (2)	2 (5)
IV Textiles	0 (0)	0 (0)	0 (1)	0 (1)	0 (2)
No. of runs	9	5	9	5	28

Table V also reflects on the relative profitability of the various domestic production activities. The interpretation of the results summarized above is that Sectors VIII and VII are not profitable when there is a scarcity of savings or foreign aid. The reason for this is obviously the high capital cost of these activities — their capital coefficient is the highest of the eight internationally trading sectors.

A satisfactory discussion of the relative profitabilities of different domestic activities can be given only in terms of the dual variables generated by the different optimal solutions of the primal problem. As pointed out earlier in this Section, the solution to Group I Cases A and B runs all produce a single optimal basis. Group II solutions produce two optimal bases for each of Case A (marginal rate of saving=20 per cent) and Case B (marginal saving=15 per cent). Group III solutions have four distinct optimal bases for each of Case A (foreign-aid to total-investment ratio = 30 per cent) and Case B (foreign-aid to total-investment ratio = 40 per cent). The values of the dual variables resulting from 10 of these optimal bases are given in Appendix Table B-5³³.

When the objective function is the minimization of imports the shadow prices corresponding to each input-output balance equation reflect the minimal

³³Their applicability to different runs is shown in Table VI.

TABLE VI
RANKING OF SECTORS ACCORDING TO SHADOW PRICE

Optimal basis	Group	Case	Rate of growth	Sectors in increasing order of values of p*
1	I	A,B	4.0-8.0	I, IV, III, II, V, VI, VIII, VII
2	II	A	5.0-8.0	I, III, IV, II, V, VI, VII, VIII
3	II	A	4.0-5.0	I, III, IV, II, V, VI, (VII, VIII)*
4	II	B	4.0	I, III, IV, II, (V, VI, VII, VIII)*
5	II	B	5.0-8.0	I, III, IV, II, V, VI, (VII, VIII)*
6	III	A	7.5-8.0	I, III, IV, II, V, VI, VII, VIII
7	III	A	5.5-7.0	I, III, IV, II, V, VI, (VII, VIII)*
8	III	A	4.5-5.0	I, III, IV, II, V, (VI, VII, VIII)*
9	III	B	7.0-8.0	I, III, IV, II, V, VI, VII, VIII
10	III	B	6.0	I, III, IV, II, V, (VI, VII, VIII)*

*These sectors have equal prices.

foreign-exchange content of a unit output from the corresponding domestic-output activity. The corresponding price of import activity in each case is unity—equal to its coefficient in the objective function. The ratio of the shadow price in each sector to the alternative import price (equal to unity), thus, represents the relative foreign-exchange content of domestic production. The higher this ratio, the lower the net saving of the foreign exchange afforded by import substitution. A ranking of the domestic sectors in ascending order of the value of their shadow prices, thus, affords us a way of determining the comparative advantage (in descending order) of the different sectors.

In Table VI the ranking of the eight sectors where an alternative import choice is permissible in ascending order of the values of dual variables (and, hence, in descending order of static comparative advantage) resulting from the different optimal bases is given. This ranking of the different sectors also reflects in decreasing order their comparative advantages, based on this relative price criterion.

The relative position in the ranking of Sectors I (agriculture) and VII (machinery) and VIII (transport and communications equipment) does not

change in any of the bases. There are some minor shifts in the ranking position of other sectors in some of the bases. The only significant difference between the ranking in the unconstrained group bases and others is that the machinery sector in the former has a higher relative price than the transport equipment sector. This is explained by the fact that the capital coefficient in transport equipment sector is higher than in the machinery sector and as our analysis of the dual problem at the end of Section A above shows the shadow prices in the savings-constrained and foreign-aid availability models have an additional cost element involving the capital coefficient.

The majority of the bases have the following ranking:

Rank

1.	Sector I	Agriculture
2.	Sector III	Food processing industries
3.	Sector IV	Textile industries
4.	Sector II	Mining
5.	Sector V	Wood products
6.	Sector VI	Chemicals
7.	Sector VII	Machinery
8.	Sector VIII	Transport equipment

The above ranking conforms to the commonly held view about the pattern of comparative advantage in an underdeveloped economy. However, it cannot be too strongly emphasized that this ranking is based on the current value of the structural parameters of the economy. In particular, the import coefficients used are those existing at the initial year. During the process of growth, import substitution will become a necessity in order to close the savings and foreign-exchange gaps and many of the presently fixed import coefficients will have to be lowered. It is quite likely that the first sectors to be affected by an import-substitution strategy would be those very sectors where comparative advantage, at present, is very low.

In addition, two more qualifications to the above results must be pointed out. First, the model does not take into account either external economies or economies of scale — both of which are known to be highly important factors in determining the pattern of comparative advantage in a developing country. Secondly, the length of planning horizon could also be a significant factor: *e.g.*, domestic production in the machinery sector may be profitable with a 20-year horizon but not with a 10-year horizon.

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Appendix A

TABLE A-1
TWELVE-SECTOR CURRENT INPUT-OUTPUT COEFFICIENTS (MATRIX A) FOR PAKISTAN: 1959/60

Consuming sectors →	Producing sectors ↓											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
I	.04520	.00067	.41693	.28078	.04996	.02687	.00179	—	—	.76130	—	.00096
II	—	—	.00240	.00638	.00065	.15002	.00290	.00107	—	.00052	.08132	—
III	.00231	—	.03249	.00072	—	.04089	—	—	—	—	—	.00076
IV	—	—	.00447	.06750	.00412	.02669	.00772	.02233	—	—	—	.00529
V	—	—	.01452	.01103	.20392	.04372	.00394	.00238	.02033	—	—	.00875
VI	.00252	.00863	.00931	.02903	.15654	.09527	.02080	.00480	.11467	—	.04699	.01918
VII	—	—	.00702	.01469	.03102	.01196	.29534	.06929	.10469	.00138	—	.00046
VIII	.00066	—	—	—	—	—	—	.11105	.00130	—	—	.00213
IX	.00466	—	—	—	—	—	—	—	—	—	—	—
X	—	—	—	—	—	—	—	—	.06608	.06487	—	—
XI	.00014	.01853	.01048	.01361	.00284	.00745	.00456	.00286	.00048	—	—	.00132
XII	.02209	.10362	.23361	.21353	.36679	.26211	.26414	.56225	.13033	.07308	.11858	.10238

Source: [5]

TABLE A-2
EIGHT-SECTOR CURRENT IMPORT COEFFICIENTS MATRIX (MATRIX M) FOR PAKISTAN: 1959/60

Producing sectors ↓	Consuming sectors →											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
I	.00004	.00067	.04520	.01388	.00802	.01339	.00079	—	—	—	—	—
II	—	—	—	—	—	.07192	.00056	—	—	.00024	.04392	—
III	—	—	—	—	—	.02562	—	—	—	—	—	—
IV	—	—	.00089	.02142	.00193	.00417	.00695	.01876	—	—	—	.00196
V	—	—	.00227	.00558	.04993	.00404	.00003	.00238	.01015	—	—	—
VI	.00220	.00695	.00640	.02328	.06246	.06873	.00411	.00428	.02710	—	.02415	.00984
VII	—	—	.00230	.00886	.00809	.00156	.29534	.02609	.9341	.00109	—	.00038
VIII	.00066	—	—	—	—	—	—	.09522	.00130	—	—	.00213

Sources: [5]

TABLE A-3
THREE-SECTOR FIXED CAPITAL COEFFICIENTS MATRIX (MATRIX B) FOR PAKISTAN: 1962/63

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Consuming sectors →	0.210	1.070	0.250	0.260	0.300	0.490	0.490	0.400	0.100	0.240	2.950	0.150
Producing sectors ↓	—	0.450	0.150	0.100	0.050	0.100	0.150	0.050	0.050	—	0.150	0.400
VII	1.270	1.140	0.200	0.500	0.500	0.460	1.600	2.000	0.640	0.240	3.200	1.000
VIII												
IX												

TABLE A-3a
INVENTORY COEFFICIENTS (N_{ij})

0.150	0.500	0.200	0.306	0.230	0.340	0.520	0.840	—	0.150	—
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TABLE A-3b
SECTORAL CAPITAL COEFFICIENTS (B_i)

1.630	3.160	0.800	1.166	1.080	1.390	2.560	3.290	0.790	0.630	6.300	1.550
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TABLE A-5
 INCREMENTAL CONSUMPTION VECTORS, $C_{10}-C_0$, CORRESPONDING TO DIFFERENT
 RATES OF GROWTH OF OVERALL CONSUMPTION

Sectors ↓	(million rupees)										
	Annual growth rate of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08	
I	7,292.6	8,281.0	9,234.6	10,249.5	11,279.5	12,359.3	13,472.3	14,635.8	15,831.4		
II	32.1	34.0	35.9	37.7	39.7	41.6	43.5	45.5	47.6		
III	882.8	1,051.9	1,231.6	1,421.7	1,624.3	1,838.0	2,067.3	2,304.5	2,559.0		
IV	1,407.8	1,677.3	1,963.9	2,267.0	2,590.2	2,930.9	3,296.6	3,674.8	4,080.6		
V	127.3	151.7	177.6	205.0	234.2	265.0	298.1	332.3	369.0		
VI	359.9	447.0	541.9	644.9	756.7	879.0	1,011.1	1,153.6	1,309.1		
VII	606.1	767.7	952.4	1,153.5	1,375.3	1,616.6	1,888.7	2,181.7	2,510.3		
VIII	289.3	379.4	481.2	596.4	726.4	872.9	1,040.3	1,223.2	1,434.5		
IX	536.4	666.3	807.7	961.2	1,127.9	1,310.1	1,507.0	1,719.4	1,951.2		
X	778.8	927.7	1,086.4	1,254.1	1,432.8	1,621.3	1,823.6	2,032.8	2,257.3		
XI	113.3	135.0	158.1	182.4	208.4	235.9	265.3	295.7	328.4		
XII	3,334.4	3,635.3	3,978.6	4,273.8	4,577.1	4,858.5	5,037.6	5,238.6	5,377.4		

TABLE A-6

**SECTORAL EXPENDITURE
ELASTICITIES**

Sector	Elasticity
I Agriculture	0.8
II Mining	0.3
III Food processing	1.4
IV Textiles	1.4
V Woodwork and paper	1.4
VI Chemicals	2.0
VII Machinery	2.4
VIII Transport equipment	3.0
IX Construction	2.0
X Cottage industries	1.4
XI Electric, power and gas	1.4
XII Services	1.0

TABLE A-7

**EXPORT ESTIMATES
FOR 1972/73**

Sector	Estimate I	Estimate II
I	2,926.0	3,218.6
II	65.0	71.5
III	80.0	88.0
IV	1,155.0	1,270.5
V	242.0	266.2
VI	100.0	110.0
VII	100.0	110.0
VIII	11.6	12.8
Total	4,679.6	5,147.6

TABLE A-8
 'Y' VECTORS CORRESPONDING TO DIFFERENT OVERALL RATES OF
 GROWTH OF CONSUMPTION

Sector ↓	Annual growth rate of total consumption ↑											
	.04	.045	.05	.055	.06	.065	.07	.075	.08	(million rupees)		
I	9,007.8	9,996.2	10,949.2	11,964.7	12,994.7	14,074.5	15,187.0	16,351.0	17,546.6			
II	154.5	156.5	158.4	160.2	162.2	164.1	166.0	168.0	170.1			
III	1,204.5	1,374.0	1,553.7	1,743.8	1,946.4	2,160.1	2,389.4	2,626.6	2,881.1			
IV	2,218.7	2,488.2	2,774.8	3,077.9	3,401.1	3,741.8	4,107.5	4,485.7	4,891.5			
V	409.1	433.5	459.4	486.8	516.0	546.8	579.9	514.1	550.8			
VI	1,257.2	1,344.3	1,439.2	1,542.2	1,654.0	1,776.3	1,908.4	2,050.9	2,206.4			
VII	1,732.9	1,894.5	2,079.2	2,280.3	2,502.1	2,743.1	3,015.5	3,308.5	3,637.1			
VIII	335.3	425.4	527.2	642.4	772.4	918.9	1,086.3	1,269.2	1,480.5			
IX	-1,791.9	-1,662.0	-1,520.6	-1,367.1	-1,200.4	-1,018.2	-821.3	-608.9	-377.1			
X	778.8	927.9	1,086.4	1,254.1	1,432.8	1,621.3	1,823.6	2,032.8	2,257.3			
XI	113.3	135.0	158.1	182.4	208.4	235.9	265.3	295.7	328.4			
XII	3,334.3	3,635.3	3,978.6	4,237.8	4,577.1	4,858.5	5,037.6	5,238.6	5,377.4			

Appendix B

A NOTE ON APPENDIX B

The tables in this appendix are based on the optimal solutions of the different runs of our single-period programming model, obtained by changing different parameters and exogenous variables. The identification of different runs with reference to these parametric changes is given in Table II (p. 537).

Appendix Table B-1 provides for each case the different macroeconomic aggregates for 1972/73, the terminal year of our planning period. The derivation of these aggregates is discussed and the notation described on page 539.

Appendix Table B-2 gives for each case the value of increases in domestic production for each sector, between 1962/63 and 1972/73.

Appendix Tables B-3 and B-4, presented together for each case, give, respectively, the values of total sectoral imports in the terminal year, 1972/73. (Z_1 , for example, represents total imports of agricultural products in 1972/73) and the nonessential imports of each sector in 1972/73.

All values in Appendix Tables B-1 through B-4 are in million rupees.

Whereas Appendix Tables B-2 through B-4 represent the primal solution of each run, Appendix Table B-5 represents the dual solution. As many of the primal solutions have the same dual solution the number of the latter is necessarily smaller than that of the former.

TABLE B-1
 MACROECONOMIC AGGREGATES
 (Group III — Case B)

Rate of growth of consumption →	.04	.05	.06	.07	.08
Variable ↓					
\bar{Z}_{10}	6617.6	7342.5	8183.0	9196.3	10379.2
F_{10}	1938.6	2662.9	3503.4	4516.7	5700.0
\bar{J}_{10}	4848.2	6657.2	8767.0	11302.9	14263.1
S_{10}	2909.6	3994.3	5263.6	6786.2	8563.1
\bar{C}_{10}	48595.8	53584.9	58807.5	64586.4	70890.8
Y_{10}	51505.4	57579.2	64071.1	71372.6	79453.9
$\frac{S_{10}}{Y_{10}}$	5.65%	6.94%	8.22%	9.51%	10.78%

TABLE B-1
MACROECONOMIC AGGREGATES
(Group IV — Case A)

Variable ↓	Rate of growth of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
\bar{Z}_{10}		6,540.8	6,848.3	7,173.3	7,511.1	7,877.0	8,260.0	8,656.7	9,066.6	9,516.2
F_{10}		1,861.2	2,168.7	2,493.7	2,831.5	3,197.4	3,580.4	3,957.1	4,387.0	4,836.6
\bar{J}_{10}		6,203.9	7,229.0	8,312.3	9,438.2	10,658.0	11,934.2	13,190.0	14,623.4	16,122.1
S_{10}		4,342.7	5,060.3	5,818.6	6,606.7	7,460.6	8,354.2	9,233.2	10,236.4	11,285.5
\bar{C}_{10}		48,595.8	50,989.5	53,484.9	56,082.2	58,807.5	61,664.1	64,586.4	67,672.9	70,890.8
Y_{10}		52,938.5	56,049.8	59,303.5	62,688.9	66,268.1	70,018.3	73,819.6	77,909.3	82,176.3
S_{10}/Y_{10}		8.20%	9.03%	9.81%	10.54%	11.26%	11.93%	12.51%	13.14%	13.73%
$\frac{S_{10}-S_0}{Y_{10}-Y_0}$		11.85%	13.51%	14.83%	15.86%	16.78%	17.53%	18.08%	18.79%	19.23%

TABLE B-2
VALUES OF INCREASES IN DOMESTIC PRODUCTION
(Group I — Case A)

	Annual growth rate of total consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
	Incremental domestic outputs ↓									
x ₁		11677.4	13087.4	14480.8	15962.0	17485.5	19085.1	20750.6	22480.7	24288.0
x ₂		249.9	272.8	297.1	322.6	350.4	379.8	411.0	442.7	478.4
x ₃		1325.4	1513.6	1713.1	1924.0	2148.8	2385.8	2639.9	2902.7	3184.8
x ₄		2396.5	2702.7	3028.5	3372.7	3740.2	4127.5	4542.6	4971.5	5432.5
x ₅		620.6	684.3	752.8	824.0	901.2	982.0	1067.0	1031.0	1125.9
x ₆		1355.7	1544.6	1747.1	1963.5	2197.9	2448.9	2715.6	2985.3	3293.1
x ₇		1667.8	1886.5	2132.9	2399.5	2692.8	3010.0	3364.5	3739.6	4163.7
x ₈		433.9	556.5	694.5	847.7	1020.6	1212.9	1429.4	1664.0	1933.8
x ₉		2860.0	3630.0	4441.1	5290.4	6207.0	7172.3	8182.9	9230.0	10371.3
x ₁₀		1060.4	1279.6	1512.0	1757.2	2019.4	2295.8	2590.7	2895.7	3224.3
x ₁₁		197.2	230.1	265.3	302.2	341.8	383.7	428.4	474.3	524.3
x ₁₂		6885.4	7736.5	8672.9	9552.5	10571.4	11569.8	12508.0	13452.9	14460.7

TABLE B-2
VALUES OF INCREASES IN DOMESTIC PRODUCTION
(Group II — Case A)

Rate of growth of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
Incremental domestic outputs ↓									
x ₁	11,648.4	13,061.5	14,457.8	15,942.6	17,469.5	19,072.9	20,739.6	22,476.7	24,287.8
x ₂	242.9	267.2	292.8	319.0	347.4	377.5	408.4	441.9	478.4
x ₃	1,323.9	1,512.4	1,712.0	1,923.1	2,148.0	2,385.3	2,639.4	2,902.5	3,184.8
x ₄	2,390.9	2,697.2	3,023.0	3,368.1	3,736.4	4,124.6	4,540.1	4,970.5	5,432.5
x ₅	603.7	670.3	741.2	814.2	893.1	975.8	1,061.0	1,029.0	1,125.9
x ₆	1,299.2	1,497.7	1,709.3	1,931.0	2,171.1	2,428.5	2,692.5	2,978.6	3,293.1
x ₇	1,079.5	1,587.2	2,083.5	2,357.3	2,658.4	2,983.8	3,272.5	3,730.9	4,163.7
x ₈	0	0	36.4	291.6	562.7	863.1	1,169.7	1,548.6	1,933.8
x ₉	2,453.4	3,265.2	4,114.6	5,014.5	5,979.8	6,998.7	8,028.5	9,172.7	10,371.3
x ₁₀	1,031.0	1,253.1	1,488.3	1,737.2	2,002.9	2,283.3	2,579.5	2,891.6	3,224.3
x ₁₁	191.7	225.7	261.8	299.3	339.5	381.9	426.6	473.7	524.3
x ₁₂	6,351.8	7,221.7	8,177.8	9,134.1	10,226.9	11,306.6	12,284.2	13,366.0	14,460.7

TABLE B-2

VALUES OF INCREASES IN DOMESTIC PRODUCTION

(Group III—Case B)

Rate of growth of consumption →	.04	.05	.06	.07	.08
Incremental domestic outputs ↓					
x ₁	11306.5	14321.0	17355.8	20647.5	24199.4
x ₂	123.0	180.8	318.3	385.2	459.3
x ₃	1300.2	1690.2	2142.5	2634.8	3180.5
x ₄	1572.7	2983.1	3718.8	4522.9	5413.4
x ₅	0	0	375.9	1007.6	1078.3
x ₆	0	469.4	1930.4	2508.9	3132.7
x ₇	0	0	0	1125.1	3771.8
x ₈	0	0	0	0	0
x ₉	1617.7	3073.7	4710.4	6739.1	9120.3
x ₁₀	970.5	1413.0	1911.0	2486.1	3133.8
x ₁₁	159.6	235.6	318.9	408.5	509.3
x ₁₂	5054.6	6684.7	8604.2	10636.7	12690.2

TABLE B-2
VALUES OF INCREASES IN DOMESTIC PRODUCTION

(Group IV — Case A)

	.04	.045	.05	.055	.06	.065	.07	.075	.08
Rate of growth of consumption →									
Incremental domestic outputs ↓									
x ₁	9,116.3	10,725.0	12,339.4	14,053.7	15,835.2	17,709.9	19,637.0	21,693.6	23,847.7
x ₂	244.5	267.8	292.5	318.6	346.9	376.8	407.5	441.0	477.5
x ₃	1,317.9	1,506.7	1,706.8	1,918.4	2,143.9	2,381.8	2,636.5	2,900.4	3,183.5
x ₄	2,394.2	2,700.5	3,026.5	3,371.0	3,738.7	4,126.2	4,541.0	4,970.8	5,432.1
x ₅	607.2	672.3	741.9	814.3	892.8	975.0	1,059.4	1,027.1	1,123.7
x ₆	1,293.6	1,487.3	1,695.8	1,917.2	2,157.8	2,415.6	2,678.8	2,966.2	3,282.4
x ₇	1,636.7	1,857.8	2,106.8	2,376.3	2,672.7	2,993.3	3,276.3	3,730.0	4,158.4
x ₈	429.8	552.8	691.1	844.6	1,018.0	1,210.7	1,394.2	1,662.8	1,933.1
x ₉	2,255.5	3,072.4	3,935.7	4,840.6	5,817.5	6,847.7	7,878.7	9,044.2	10,267.4
x ₁₀	1,016.7	1,239.2	1,475.4	1,724.6	1,991.2	2,272.3	2,568.6	2,882.3	3,216.8
x ₁₁	195.5	228.5	263.8	300.8	340.7	382.8	427.0	473.8	524.0
x ₁₂	6,692.5	7,558.4	8,511.6	9,408.8	10,447.1	11,466.2	12,370.9	13,396.6	14,427.5

TABLE B-3
 VALUE OF TOTAL SECTORAL IMPORTS IN TERMINAL YEAR, 1972/73
 (Group I — Case A)

Annual growth rate of total consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
Total imports ↓									
Z ₁	242.6	258.7	275.8	293.8	313.2	333.6	355.5	377.0	401.5
Z ₂	167.5	182.7	199.0	216.4	235.2	255.3	276.7	298.4	323.1
Z ₃	51.3	56.2	61.4	66.9	72.9	79.3	86.2	93.1	101.0
Z ₄	197.6	210.7	225.0	240.0	256.5	274.0	292.7	312.1	333.7
Z ₅	167.7	181.9	197.0	212.8	229.9	248.0	266.9	280.4	301.9
Z ₆	807.1	867.2	931.2	997.6	1069.8	1145.5	1224.4	1297.7	1386.5
Z ₇	2156.5	2429.1	2721.6	3030.9	3366.2	3722.0	4102.7	4494.0	4931.0
Z ₈	616.9	694.0	778.5	863.1	958.6	1056.7	1156.2	1258.3	1371.0

TABLE B-3

VALUE OF TOTAL SECTORAL IMPORTS IN TERMINAL YEAR, 1972/73
(Group II—Case A)

Total Imports ↓	Rate of growth of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
Z ₁		241.1	257.6	275.0	293.2	312.6	333.2	355.0	376.9	401.5
Z ₂		162.8	178.9	196.1	213.9	233.1	253.8	274.9	297.9	323.1
Z ₃		49.9	55.0	60.4	66.1	72.2	78.8	85.6	92.9	101.0
Z ₄		183.9	196.8	211.0	228.2	246.8	266.6	286.6	309.6	333.7
Z ₅		161.5	176.0	191.4	208.1	226.0	244.9	264.4	279.4	301.9
Z ₆		781.3	844.2	910.8	980.3	1,055.6	1,134.6	1,214.4	1,294.1	1,386.5
Z ₇		2,358.9	2,460.3	2,605.7	2,932.9	3,285.6	3,660.4	4,027.2	4,473.7	4,931.0
Z ₈		893.1	1,071.3	1,239.9	1,252.9	1,279.6	1,301.9	1,306.9	1,339.2	1,371.0

TABLE B-4

NONESENTIAL IMPORTS, 1972/73

(Group II — Case A)

Nonessential imports ↓	Rate of growth of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
R ₁		0	0	0	0	0	0	0	0	0
R ₂		0	0	0	0	0	0	0	0	0
R ₃		0	0	0	0	0	0	0	0	0
R ₄		0	0	0	0	0	0	0	0	0
R ₅		0	0	0	0	0	0	0	0	0
R ₆		0	0	0	0	0	0	0	0	0
R ₇		504.5	233.8	0	0	0	0	0	0	0
R ₈		361.8	468.5	557.0	470.7	337.5	296.1	191.5	97.7	1.0

TABLE B-3

VALUE OF TOTAL SECTORAL IMPORTS IN TERMINAL YEAR, 1972/73

(Group III — Case B)

Rate of growth of consumption →					
Total imports ↓	.04	.05	.06	.07	.08
Z ₁	205.5	249.2	302.6	349.9	397.7
Z ₂	67.3	104.5	213.4	259.7	310.3
Z ₃	16.6	28.6	66.1	80.9	96.9
Z ₄	856.9	185.5	212.2	245.3	285.8
Z ₅	522.4	658.3	557.3	244.9	281.4
Z ₆	1625.9	1657.6	941.7	1132.2	1307.6
Z ₇	2595.9	3385.6	4388.4	4858.5	5015.4
Z ₈	727.0	1073.2	1501.4	2024.9	2684.1

TABLE B-4

NONESSENTIAL IMPORTS

(Group III — Case B)

Rate of growth of consumption →					
Nonessential imports ↓	.04	.05	.06	.07	.08
R ₁	0	0	0	0	0
R ₂	0	0	0	0	0
R ₃	0	0	0	0	0
R ₄	707.1	0	0	0	0
R ₅	409.5	520.0	372.5	0	0
R ₆	1033.0	932.8	0	0	0
R ₇	1411.5	1796.7	2304.4	1873.5	768.8
R ₈	322.9	540.8	813.1	1163.0	1628.5

TABLE B-3
 VALUE OF TOTAL SECTORAL IMPORTS IN TERMINAL YEAR, 1972/73
 (Group IV—Case A)

Total imports ↓	Rate of growth of consumption →	.04	.045	.05	.055	.06	.065	.07	.075	.08
Z ₁		2,588.1	2,422.1	2,236.9	2,041.4	1,824.2	1,593.0	1,372.2	1,097.8	804.6
Z ₂		162.9	178.4	195.2	212.9	232.2	252.9	274.0	297.0	322.3
Z ₃		49.7	54.7	60.0	65.7	71.9	78.5	85.2	92.6	100.7
Z ₄		196.6	209.8	224.1	239.3	255.9	273.4	291.0	311.8	333.5
Z ₅		160.7	175.1	191.1	207.6	225.4	244.1	263.2	278.3	300.6
Z ₆		777.9	840.1	906.7	975.7	1,050.9	1,129.7	1,208.7	1,288.7	1,381.4
Z ₇		2,006.1	2,290.3	2,595.9	2,918.8	3,269.3	3,641.2	4,001.8	4,447.8	4,905.2
Z ₈		598.9	677.4	763.4	849.6	946.9	1,047.0	1,140.5	1,252.8	1,367.9

TABLE B-5
VALUES OF DUAL VARIABLES IN DIFFERENT OPTIMAL BASES

Optimal bases →	1	2	3	4	5	6	7	8	9	10
Dual variables ↓										
P_1^*	.0910	.1794	.1946	.1996	.2266	.0968	.0973	.0972	.1184	.1221
P_2^*	.3078	.3770	.3874	.3874	.4041	.2035	.1735	.1936	.2356	.1588
P_3^*	.2180	.2801	.2899	.2899	.3084	.1512	.1324	.1449	.1763	.1656
P_4^*	.2175	.2874	.2987	.2987	.3258	.1551	.1398	.1493	.1817	.1747
P_5^*	.3823	.4167	.4217	.4217	.4252	.2250	.1826	.2107	.2565	.2273
P_6^*	.4179	.4311	.4324	.4324	.4296	.2328	.1846	.2162	.2630	.2253
P_7^*	.5442	.5676	.5707	.5707	.4296	.3064	.1846	.2852	.3471	.2273
P_8^*	.4600	.6340	.5707	.5707	.4296	.3423	.1846	.2852	.3471	.2273
P_9^*	.2518	.2588	.2595	.2595	.2578	.1398	.1109	.1299	.1577	.1377
P_{10}^*	.1355	.2409	.2589	.2589	.2960	.1301	.1272	.1295	.1576	.1594
P_{11}^*	.6305	.8317	.8656	.8656	.9260	.4487	.3975	.4325	.5265	.4957
P_{12}^*	.1506	.2174	.2276	.2276	.2489	.1174	.1070	.1139	.1385	.1338

(Continued)

TABLE B-5 (Contd.)

Dual variables ↓	Optimal bases ↑									
	1	2	3	4	5	6	7	8	9	10
P_1^*	.9090	.4550	.3761	.3761	.2030	.2455	.0873	.1880	.2288	.1052
P_2^*	.6922	.2570	.1833	.1833	.0255	.1388	.0111	.0916	.1115	.0685
P_3^*	.7820	.3540	.2807	.2807	.1212	.1911	.0522	.1403	.1708	.0618
P_4^*	.7825	.3466	.2719	.2719	.1038	.1871	.0447	.1359	.1654	.0526
P_5^*	.6177	.2173	.1490	.1490	.0045	.1173	.0020	.0744	.0906	.0000
P_6^*	.5821	.2029	.1382	.1382	.0000	.1095	.0000	.0690	.0841	.0020
P_7^*	.4558	.0664	.0000	.0000	.0000	.0359	.0000	.0000	.0000	.0000
P_8^*	.5400	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
P_9, P_{10}^*	.3650	.4287	.4287	.4287	.5704	.6577	.8159	.7148	.6529	.7727

*These are the dual variables corresponding to the savings constraint (16) in bases 2-5 and to the foreign-aid constraint(18) in bases 6-10.