Choice of Technique and the Volume of Saving

A RE-EXAMINATION OF THE COBB-DOUGLAS MODEL

bу

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INTRODUCTION

The purpose of this paper is to re-examine the relationship between the degree of aggregate labour-intensity and the aggregate volume of saving in an economy where a Cobb-Douglas production function in its traditional form can be assumed to give a good approximation to reality. The relationship in question has an obviously important bearing on economic development policy in the area of choice of labour intensity. To the extent that and in the range where an increase in labour intensity would adversely affect the volume of savings, a conflict arises between two important social objectives, *i.e.*, higher rate of capital formation on the one hand and greater employment and distributive equity on the other. If relative resource endowments in the economy are such that such a "competitive" range of labour-intensity falls within the nation's attainable range of choice, development planners will have to arrive at a compromise between these two social goals.

An analysis of this question was undertaken by Qayum [2] where he used a production function of the traditional Cobb-Douglas variety and deduced that there is essentially no conflict between the two social objectives in question. In the present paper, we re-examine the question within the framework of the same model used by Qayum, and discover that the conclusion reached by Qayum was rather premature: a "competitive" range of labour-intensity does exist and significantly so.

We first present an outline of Qayum's analysis in Section II. This is followed by a re-examination in Section III of the model bringing out in analyti-

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cal and numerical terms the nature and extent of the conflict between increasing the degree of labour intensity and the volume of aggregate savings. Two important limitations of analysis of this sort are noted in the concluding section to stimulate further research.

II. OUTLINE OF QAYUM'S ANALYSIS

Qayum assumes a Cobb-Douglas production function for the economy as $P=L^{\alpha}\,K^{1-\alpha}$. In period 0, L_{o} and K_{o} amounts of labour and capital are available, and \bar{L}_{o} (= $\gamma\,L_{o}$) and \bar{K}_{o} (=K_{o}) amounts are utilized. The wage rate (w̄) and rental (r̄) for labour and capital respectively in period 0 are given by respective marginal productivities, so that

$$\begin{split} &\bar{w} = \alpha \left(\frac{\bar{K}_o}{\bar{L}_o} \right)^{1-\alpha} \\ &\bar{r} = \left(1-\alpha \right) \left(\frac{\bar{K}_o}{\bar{L}_o} \right)^{-\alpha} \end{split}$$

Labour force grows at a rate λ per period, and capital stock at a rate λ — δ , where λ is the ratio of 'fluid' capital K_1^* in period 1 to capital stock K_0 in period 0, and δ the rate of depreciation of capital stock, and also gives the rate at which labour force is released as a result of capital depreciation.

Choice of technique in period 1 is restricted to combination of 'fluid' labour with 'fluid' capital only. Capital being scarce, fluid capital will be fully used. With this, according to Qayum's policy, that amount of labour L_1^{\bullet} will be used which makes the labour-capital ratio in this 'fluid' sector equal the ratio of total availabilities in period 1, so that $L_1^{*} = K_1^{*}$. L_1/K_1 , where L_1 and K_1 represent total availabilities in period 1.

For institutional and other reasons wage rate and rental in period 1, however, remains the same as in period 0, underwritten by the state which would finance any overall deficit (excess of factor payments over production) by deficit financing [2, Chapter VIII, Section 8-3f and Chapter IX, Section 9.9].

In general, the technique thus chosen in the 'fluid' sector will be more labour-intensive than the technique, represented by the ratio \bar{L}_0/\bar{K}_0 , used 'traditionally'. The excess of total saving in period 1 resulting from Qayum's policy over what would have been if the 'traditional technique' (henceforth called the T-technique) were continued for the fluid resources in this period, is given by $ES_1 = EP_1 - \emptyset EW_1$, where EP_1 is the corresponding excess of production, EW_1 the corresponding excess of wage payments (income of capital-owners remains unchanged), and \emptyset the ratio of change in consumption of the

labour force to change in the total wage-bill. ES₁, the excess of saving in question, turns out to be

$$ES_{1} = \lambda \bar{P}_{0} \left[\left\{ \frac{1+\lambda}{\gamma (1+\lambda-\delta)} \right\}^{\alpha} - 1 - \emptyset \alpha \left\{ \frac{1+\lambda}{\gamma (1+\lambda-\delta)} - 1 \right\} \right]$$

Qayum computes various values of ES₁ as a proportion of total product \bar{P}_0 of period 0, corresponding to various alternative combinations of values of the parameters α , λ , δ , λ , γ and \emptyset , keeping these latter values within bounds of realism. From the results Qayum concludes 'confidently' that the volume of saving would not be adversely affected by his policy and, moreover, would be greater the greater a) the degree of unemployment $(1-\gamma)$ in period 0, and/or b) the rate of growth of labour force λ between periods 0 and 1, and/or c) the rate of depreciation δ of capital.

A similar analysis shows that the inflationary effect of deficit financing to finance the difference between total factor payments and total production resulting from Qayum's policy would be negligible, so that the above may be taken to hold for the volume of *real* saving as well as for saving in terms of current prices.

In essence, Qayum's analysis seems to suggest that the more labour-intensive technique — which yields greater direct productivity (DP) if it does not require same or more capital for the same output than a less capital-intensive technique and if all capital is utilized — also offers the higher 'marginal re-investment coefficient' (MRC) as defined by Galenson and Leibenstein [3]. For, with a given volume of 'fluid' capital for period 1, a greater volume of saving generated with a constant-return technology as the Cobb-Douglas function represents means a greater MRC in the technique concerned. Thus, as long as capital is relatively scarce and labour is surplus, adoption of more labour-intensive techniques should increase output both in the short and in the long run, apart from increasing employment and giving a better distribution of the national product. There should then be no conflict between the two social objectives referred to earlier which are often regarded as conflicting.

III. A RE-EXAMINATION OF THE MODEL

Qayum's rather strong conclusions are not fully warranted under his assumptions, and are in fact the results of stopping short of a fuller analysis of the question. An examination of the expression ES_1 will show that it is a non-linear function of the parameters mentioned in a), b), and c) above, so that all the conclusions need to be modified. It can be verified that this holds even for the range of values for the respective parameters that Qayum has

considered himself: for example, ES₁ attains a maximum¹, with respect to γ , at

$$\gamma = \varnothing^{-1/(1-\alpha)} - \frac{1+\lambda}{1+\lambda-\delta}$$
, which is approximately .80 for $\lambda=.03$, $\lambda-\delta=.05$,

 $\emptyset=.95$, and $\alpha=.75$, whereas the range of values Qayum considers for λ , λ — δ , \emptyset and γ respectively are .01 to .03, .05 to .15, .70 to 1.00, and .70 to .90 with $\alpha=.75$. (In Qayum's as well as in the present analysis, α is taken as .75, and in subsequent enquiries this will be assumed without further notice.)

Instead of pursuing this point further, it seems more interesting to examine the responsiveness of ES_1 to change in the amount of labour to be combined with the amount of fluid capital K_1^* in period 1 — in other words, to enquire about the effect on the volume of saving of choice of technique itself, rather than of changes in certain parameters with choice of technique given by Qayum's policy as $L_1^* = K_1^* \cdot L_1/K_1$ (henceforth referred to as the Q-technique or simply Q).

Expressing ES₁ as a function of the amount of labour (call it $L_1^{*\prime}$) to be combined with the amount K_1^* of fluid capital in period 1, we find that

$$ES_{i}=(EP_{i})-E_{C_{i}}=\left\{ \lfloor \frac{1}{2} + \frac{1$$

where \bar{L}_1 (= λ \bar{L}_o) is the amount of labour that would be combined with K_1^{\bullet} (= λ \bar{K}_o) amount of capital if the traditional technique were continued.

$$\frac{1}{d\text{ES}_{1}} = \lambda \left(\frac{1+\lambda}{1+\lambda-\delta} \right)^{\alpha} - (\alpha) \cdot \frac{1}{\gamma^{\alpha+1}} - \lambda \cdot \frac{1+\lambda}{1+\lambda-\delta} \cdot \emptyset \cdot \alpha \cdot (-1) \cdot \frac{1}{\gamma^{2}}$$

$$= \frac{\alpha\lambda}{\gamma} \left\{ \frac{1+\lambda}{\gamma(1+\lambda-\delta)} \right\}^{\alpha} \left[\emptyset \left\{ \frac{1+\lambda}{\gamma(1+\lambda-\delta)} \right\}^{1-\alpha} - 1 \right]$$
For maximization,
$$\emptyset \left\{ \frac{1+\lambda}{\gamma(1+\lambda-\delta)} \right\}^{1-\alpha} = 1$$
whence
$$\gamma = \emptyset^{1/(1-\alpha)} \frac{1+\lambda}{1+\lambda-\delta}$$

It can be verified that $\frac{d ES_1^2}{d^2 \gamma}$ < 0 at the values postulated for the parameters

This shows that the smaller the \emptyset , the greater is ES₁ for any given technique, other things remaining the same. If we call $(1-\emptyset)$ the 'marginal rate of saving' of the labour force, then we see that the volume of saving will be greater, the greater is the marginal rate of saving of the labour force.

We can see also that ES₁ is a non-linear function of
$$L_1^{*'}$$
, and
$$\frac{dES_1}{dL_1^{*'}} = \alpha \left(\frac{K_1^*}{L_1^{*'}}\right)^{1-\alpha} - \varnothing \bar{w}$$
 yielding maximum ES₁ at
$$\frac{K_1^*}{L_1^{*'}} = \left(\frac{\varnothing \bar{w}}{\alpha}\right)^{1/(1-\alpha)}$$
 or, with $K_1^* = \lambda \bar{K}_0$, at $L_1^{*'} = \left(\frac{\alpha}{\varnothing \bar{w}}\right)^{1/(1-\alpha)} \lambda \bar{K}_0 = \left(\frac{\alpha}{\varnothing \alpha}\right)^{1/(1-\alpha)} \lambda \bar{K}_0$
$$= \bar{L}_0 \quad \frac{\lambda}{\varnothing^{1/(1-\alpha)}} = \bar{L}_0 \frac{\lambda}{\varnothing^4}$$

This shows that a) the saving-maximizing technique, identified by the ratio $\frac{K_1^*}{L_1^{*'}}$ is independent of the volume of fluid capital (and hence the same in all periods), and b) for any given K_1^* , it is more labour-intensive the greater is the 'marginal rate of saving' of the labour force.

It is interesting to compare this saving-maximizing value of $L_1^{*'}$ with L_1^{*} of the Q-technique. Define saving-maximizing $L_1^{*'}$ as L_1^{**} . Then $L_1^{**} - L_1^{*} =$

$$\lambda \bar{L}_{0} \left[\frac{1}{\varnothing^{4}} - \frac{1+\lambda}{\gamma(1+\lambda-\delta)} \right] \stackrel{\geq}{\sim} 0 \text{ if}$$

$$\gamma(1+\lambda-\delta) \stackrel{\geq}{\sim} \varnothing^{4}(1+\lambda)$$

Table I shows the respective values of \varnothing that make $L_1^{**} = L_1^*$ for different combinations of a set of realistic values of the parameters γ , $\lambda - \delta$ and λ . It shows, in other words, that if \varnothing exceeds the relevant values shown in the table, then $L_1^{**} < L_1^*$, *i.e.*, the saving-maximizing technique (henceforth called the S-technique is less labour-intensive than the Q-technique; and viceversa if $L_1^{**} > L_1^*$. For example, the S-technique is less (more) labour-intensive than the Q-technique if \varnothing is more (less) than .95, with a 5-per-cent

rate of growth of capital stock $(\lambda - \delta)$, 2.0-per-cent rate of growth of labour force (λ) , and 20-per-cent initial unemployment $(1-\gamma)$. To put it another way, the S-technique is more (less) labour-intensive than the Q-technique if the 'marginal rate of saving' of the labour force is greater (less) than 4.7 per cent under such conditions.

A look at Table I will show that realistic values of the relevant parameters, including \emptyset , can be conceived both for L_1^{**} to be less than L_1^* as well as for L_1^{**} to exceed L_1^{*2} . In the situation where $L_1^{**} > L_1^*$, there would be a range of more labour-intensive techniques offering higher volumes of saving, i.e., higher MRC as well as higher DP, than the Q-technique. In the other situation where $L_1^{**} < L_1^*$, there would be a range of more capital-intensive techniques than Q, offering higher MRC as against smaller DP. In the former situation, if more labour-intensive techniques are in fact available, the adoption of the Q-technique would miss possibilities of improving both short-term and long-term efficiency, and in the latter situation possibility of improving long-term efficiency might be left unexplored.

The main reason why Qayum advocates the choice of technique for the fluid resources in period 1 to be given precisely by $L_1^* = K_1^* \cdot L_1/K_1$, and similarly for successive periods, seems to be a desire to see a gradual convergence of the difference between the market prices and marginal productivities, or what Qayum calls the 'accounting prices', of the two factors of production³. But convergence for its own sake is irrelevant in a search for long-term efficiency of choice of technique. Qayum does not suggest any serious economic cost of a lack of this property; even if there were some economic costs of a lack of convergence — perhaps in terms of administering a tax-cum-subsidy policy — it would still be necessary to consider the benefits it might have in terms of yielding a higher DP and/or a higher MRC. For, we have already seen that more capital-intensive techniques than Q might up to an extent offer higher MRC.

The following analysis brings out the extent of unemployment of labour that would remain after the adoption of the Q-technique, in order to show the range of more labour-intensive techniques, offering at least higher DP, that might conceivably exist.

²The three alternative values of the rate of growth of capital stock considered by Qayum in his tables are .05, .10, .15. For underdeveloped economies the last two are not conceivable. With unchanged techniques, the rate of growth of output will equal the rate of growth of capital stock; with a population growth rate of about 2 per cent per period and with a 5-per-cent growth rate for capital stock, per capita output would then grow at about 3 per cent per period. Many underdeveloped economies are not faring even as well.

³Apart from the *negative* reason that the short-run disadvantage of this technique, due to loss of extra output offered by more labour-intensive techniques, will not persist in the long run, *see* [2, pp. 78-80].

Since total fluid labour force in period 1 equals

$$L_1 - \bar{L}_0 (1-\delta) = L_1 - L_1 \cdot \frac{\gamma}{1+\lambda} (1-\delta)$$

and the Q-technique absorbs
$$L_1^* = \lambda \bar{L}_0 \frac{1+\lambda}{\gamma (1+\lambda-\delta)} = L_1 \frac{\lambda}{1+\lambda-\delta}$$

unemployment with the Q-technique is given by

$$L_{1}\left(1-\frac{\gamma(1-\delta)}{1+\lambda}-\frac{\lambda}{1+\lambda-\delta}\right) = L_{1}\left(\frac{1-\delta}{1+\lambda-\delta}-\frac{\gamma(1-\delta)}{1+\lambda}\right)$$

This may be compared with unemployment with the T-technique, which is given by $L_1 - \bar{L}_0$ $(1-\delta) - \bar{\bar{L}}_1$

$$= \ L_1 \ \left[1 - \frac{\gamma(1-\delta)}{1+\lambda} - \frac{\lambda\gamma}{1+\lambda}\right] = \ L_1 \left[1 - \frac{\gamma(1+\lambda-\delta)}{1+\lambda}\right]$$

Tables II and III show unemployment as a proportion of total labour force available in period 1, corresponding to the T-technique and the Q-technique respectively for a set of alternative combinations of values of the relevant parameters. It is seen that realistic values of the parameters can be conceived for which substantial unemployment remains even after the adoption of the Q-technique which would in such cases absorb only a small portion of the labour force that would be unemployed if the T-technique were continued. For example, with $\gamma=.80,\ \delta=.10,\ \lambda=.02$ and $\lambda=.15$, the Q-technique would leave a 15-per-cent unemployment as compared with a 17.7-per-cent unemployment that would be given by the traditional technique.

This suggests that a sufficient range of more labour-intensive techniques than Q can be conceived to warrant keeping open the choice-of-technique question, to be settled by more specific considerations of long-term perspectives the society concerned may have. The same presumption would be suggested if we think in terms of the S-technique instead of the Q-technique, since the two can conceivably be close enough to one another, and the former may even be less labour-intensive than the latter.

This calls for a re-examination also of the inflationary effect of a policy of deficit financing to finance the excess of factor payments over production resulting from adoption of more labour-intensive techniques without changing factor prices. While inflation corresponding to the Q and, perhaps, to the S-techniques may not be significant, it may be so at or close to full employment of labour force, as demonstrated below.

Output at full employment in period 1, say P_1^f is given by output flowing from previous investments plus output resulting from combining the remaining labour force with fluid capital that is available. In other words,

$$\begin{split} P_1^f &= \overline{P}_o \left(1 - \delta \right) + \left[\overline{L}_o \left\{ \frac{1 + \lambda}{\gamma} - (1 - \delta) \right\} \right]^{\alpha} - (\lambda \overline{K}_o)^{1 - \alpha} \\ &= \overline{P}_o \left[(1 - \delta) + \lambda \cdot \left\{ \frac{(1 + \lambda) - \gamma(1 - \delta)}{\lambda \gamma} \right\}^{\alpha} \right] \end{split}$$

Full employment factor payments, say F₁, is given by

$$F_{1}^{f} = \overline{w} \cdot \overline{L}_{o} \cdot (\frac{1+\lambda}{\gamma}) + \overline{\gamma} \cdot \overline{K}_{o} (1+\lambda-\delta)$$

$$= \overline{P}_{o} \left[\alpha \cdot \frac{1+\lambda}{\gamma} + (1-\alpha)(1+\lambda-\delta) \right]$$

The index of output-price in period 1 is then given by

$$\frac{\mathbf{F_{1}^{f}}}{\mathbf{P_{1}^{f}}} = \frac{.75\left(\frac{1+\lambda}{\gamma}\right) + .25\left(1+\lambda-\delta\right)}{(1-\delta)+\lambda \cdot \left\{\frac{(1+\lambda)-\gamma(1-\delta)}{\lambda\gamma}\right\}^{.75}}$$

Table IV, giving values of F_1^f/P_1^f for different alternative combinations of the relevant parameters, shows that a significant rise in the price of output at full employment and unchanged factor prices cannot be ruled out.

In practice some fractional unemployment will always remain; moreover, 'efficient' alternative techniques (in the sense of not requiring, in comparison with other available techniques, same or more of one factor with no less of the other per unit of output) may not exist enough to justify employment approximating full use of available labour force. To assume, however, such constraints on the production function to ensure that the choice of technique would not extend as much as to bring about a significant price rise would be arbitrary. With the door opened, as we have done, to more labour-intensive techniques than Q for consideration, it would therefore be necessary either a) to include a 'price-parameter' in the derivation of (real) volume of saving, recognizing at the same time that to continue giving the same money wage rate to labourers and the same money rental to capitalists while adopting more labour-intensive techniques would tend to reduce real unit-earnings of both, and also constitute a transfer of real purchasing power from the latter to the former; or b) to do away with deficit financing and taxing directly the capitalists to finance an unchanged real wage rate for labourers so as to have stable prices; or c) perhaps a

combination of the two. This will take care of the case when inflation resulting from a policy of deficit financing or otherwise might be significant, and also the case when it might not.

An explicit analysis of the effect of choice of techniques on the volume of real saving is not presented in any details as it would not add any further insight concerning the basic choice-of-technique question raised in this paper. The essential proposition that the volume of saving would be a non-linear function of employment remains valid.

TABLE I
$$\text{Values of } \varnothing \ = \ \left[\begin{array}{c} \gamma \ (1+\lambda-\delta) \\ 1+\lambda \end{array} \right]^{\frac{1}{4}}$$

(Requirement for Q to be the saving-maximizing technique)

ı	γ_ →	.75			.80			.85		
	λ—8 →	.03	.05	.07	.03	.05	.07	.03	.05	.07
.02	(1)	0.933	0.937	0.942	0.948	0.953	0.957	0.963	0.967	0.974
.03	(2)	0.931	0.935	0.941	0.946	0.950	0.956	0.960	0.965	0.972

TABLE II

$$1 - \frac{\gamma (1+\lambda-\delta)}{1+\lambda}$$

(Unemployment as a percentage of labour force with the traditional technique)

J.	γ _→	.75		.80		.85	
	λ—8 →	.03	.05	.03	.05	.03	.05
.02	(1)	0.243	0.228	0.192	0.177	0.142	0.125
.03	(2)	0.250	0.235	0.200	0.185	0.150	0.134

TABLE III

$$\frac{1-\delta}{1+\lambda-\delta} - \frac{\gamma(1-\delta)}{1+\lambda}$$

(Unemployment as a percentage of labour force with Qayum's technique)

	γ _→		.75	.8	0	.85		
Y	<u>λ</u> —8	.03	.05	.03	.05	.03	.05	
	δ →	.05 .10	.05 .10	.05 .10	.05 .10	.05 .10	.05 .10	
.02	(1)	0.224 0.212	0.206 0.195	0.178 0.169	0.160 0.151	0.131 0.124	0.113 0.107	
.03	(2)	0.232 0.214	0.213 0.202	0.184 0.175	0.167 0.158	0.138 0.131	0.121 0.114	

TABLE IV

$$\frac{.75\left(\frac{1+\lambda}{\gamma}\right) + .25\left(1+\lambda-\delta\right)}{(1-\delta)+\lambda\cdot\left\{\frac{(1+\lambda)-\gamma\left(1-\delta\right)}{\lambda\cdot\gamma}\right\}^{.75}}$$

(Index of output price at full employment)

λ.	γ →	.75	.80	.85
.02	(1)	1.028	1.017	1.009
.03	(2)	1.030	1.019	1.010

IV. SUMMARY AND CONCLUDING COMMENTS

To sum up, the results of our re-examination of the choice-of-technique implications of the Cobb-Douglas model, a move towards more labour-intensive techniques than the 'traditional' one, may give a higher volume of saving (equivalently, higher MRC) for some range, but there will be a saving-maximizing technique somewhere, and beyond this the volume of saving will start falling. The particular technique that Qayum suggests will be more, or less, labour-intensive than the saving-maximizing technique according as the marginal rate of saving of the labour force in the neighbourhood is smaller, or greater, than some critical value depending on other parameters of the model. With the assumptions of the Cobb-Douglas model the relevance of Qayum's technique in a search for efficiency does not appear to be very convincing, and the basic choice-of-technique question remains to be settled only by specific operational identification of social objectives.

The saving-maximizing technique is superior to more capital-intensive techniques in the sense of offering both higher direct productivity and higher volume of saving by way of higher marginal reinvestment coefficient. The search for efficiency may, therefore, be confined to techniques that are more labour-intensive than the saving-maximizing technique. This enables a preliminary screening to dispense with a range of techniques as inefficient irrespective of the objective function. The more difficult question of choice, involving a conflict between short- and long-term efficiency and also between alternative long-term objective functions themselves conceivably remains over a significant range where techniques with higher direct productivity offer lower marginal reinvestment coefficients and vice versa.

In conclusion, it may be worthwhile for the sake of completeness to repeat two important limitations already pointed out by the present author elsewhere [3; 4] of models of the type examined in this paper. 1) The use of a production function of the traditional Cobb-Douglas type ignores the possibility that the efficiency of labour, and hence its contribution to the product it helps produce, is something more than a matter of hours of work only. To the extent that this efficiency is a function of the reward itself that labour obtains for its work, the traditional Cobb-Douglas production function misses a "factor of production" whose contribution to the product, and hence to aggregate savings, is yet to be studied thoroughly. A preliminary analysis of this question will be presented separately.

2) The savings function implicit in the above analysis ignores the interdependence of consumer (hence savings) behaviour to which Duesenbury draws attention in [1], and to which Sen has more recently alluded to in his theory of the "isolation paradox" [5]. The validity of the Duesenbury hypothesis is hardly in question, and it is conjectured that incorporation of this hypothesis in the analysis may materially alter the character of the conclusion reached in this paper. Analytically, the task of formally examining the effect of interdependence of consumer behaviour on aggregate savings remains difficult because of having the income-distribution parameter as a variable in such a problem instead of being a constant as in Duesenbury's original study. The importance of the question requires, however, that vigorous attention be given to find ways of making it analytically tractable.

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