

A Simple Optimisation Model for Cotton Processing Activities in Pakistan

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Introduction

The importance of cotton and its products for the economy of Pakistan is obvious enough. Cotton is the main cash crop of Pakistan. In fact, it is one-fourth of the value of the major crops of Pakistan and provides income to about 5 million people. It is a raw material for the textile industry which is the single largest industrial activity accounting for 48% of the value-added of the large-scale manufacturing and 27% of its employment. The value of the installed textile machinery complex is around 15% of the total industrial investment in the country. Exports of cotton and its products constitute about a half of the total export proceeds of Pakistan. The final products of cotton, cloth and garments also play an important part in planning for a better standard of living in Pakistan. After food and shelter,¹ this is the most essential item of consumption and hence adequate domestic availability of cloth assumes an important role in public policy. Cotton and its products have also been a major

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¹An important by-product of cotton, cotton seed, is also used in manufacture of edible oils. However, our model does not cover its production as the basic exogenous variable of our model is ginned cotton.

source of government revenues derived by imposing excise duties and export taxes.

The growth of textile industry since independence has been spectacular. The spinning industry which had only 78,000 spindles at the time of independence now comprises of 3.6 million spindles. Similarly, capacity of the large-scale, textile mills increased from 3,000 looms in 1947 to 30,000 looms by 1971-1972 but has remained more or less the same till now. In fact in 1947 textile production was hardly sufficient to provide one yard of cloth per capita per annum but now the same has increased twenty-fold for a much larger population. The exemption of capacity tax on 4 or less powerlooms in 1967-68 led to a mushroom growth of units containing four or less powerlooms. According to the official estimates the number of powerlooms in this sector are around 50,000, producing 1250 million yards of cloth.

The monetary crises in developed countries leading to a fall in international prices and the high and fluctuating export duties on yarn and cloth created serious problems for the textile industry. Since this industry is export-oriented, it exports for its survival and viability about 33% of yarn produced and 60% of cloth. At the time of low international prices huge stocks of yarn and cloth piled up and the large-scale mills were the victims of an acute liquidity crises. The powerloom units were also badly hit leading to the closure of 50% of powerlooms in Punjab only. All this make it imperative to take a closer look at this industry because the future projections depend very much on its present state.

The export composition of cotton and its products show that we export cotton more in raw than in processed form (i.e. yarn, cloth etc.). In 1960-1961 the exports of raw cotton and raw cotton equivalent of yarn and cloth were 59.24%, 28.27% and 12.48% respectively.² This composition has remained more or less the same with slight variations. The FAO report [3] comparing the per pound value realised from exports of raw cotton, cotton yarn and cotton cloth show that the latter two, i.e. yarn and cloth, have higher relative export values as compared to raw cotton. However, they point out that the export price of yarn and cloth did not cover their respective costs of production³ to a larger extent in the case of yarn—implying that as we move horizontally towards finishing activities this difference decline. It is interesting to note, however, that the difference between the per unit price of cloth realized and the cost of conversion of cotton into both yarn and cloth has been declining over time.

The sort of export composition mentioned earlier may be a reflection of the inherent imbalance found in our textile industry where "spinning capacity exceeds weaving capacity while the bleaching, dyeing and finishing capacity covers the need of only 30 to 40% of the weaving industry" [3].

The questions of optimal product mix, exports, employment and investment in the textile industry have therefore important policy implications. In this paper an attempt will be made to answer these important questions through the implementation of a linear programming model, with multiple objective

²FAO report [3]. However, the product and export mix of the industry has progressively moved in favour of coarser varieties, which is reflected in increasing output of yarn and cloth per spindle and loom, respectively (See Table 14, FAO Report).

³The exporters of yarn and cloth were, however, compensated by the Export Bonus Scheme.

functions. The paper is structured in the following way. Section I describes the model structure. Section II discusses its empirical implementation. Section III describes the characteristics of the different results. Section IV compares the results with the Planning Commission's estimates and explores the possibility of finding an acceptable solution. In the concluding Section V, suggestions for possible improvements and extensions of the model are outlined.

I. The Model

The basic model consists of 20 (non-slack) activities and 12 constraints. In the following we shall describe the model in terms of these activities and constraints. The algebraic formulation of the model and the values of parameters are given in the Appendix.

Production Activities

$X_1,$	X_4	4 Yarn
$X_5,$	X_8	4 Cloth

Export Activities

$X_9,$	X_{15}	7 Export activities (1 raw cotton 3 yarn, 3 cloth).
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Capacity Expansion Activities

$X_{16},$	X_{20}	5 (2 spinning, 3 weaving).
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The first four activities consist of producing three varieties of yarn (coarse, medium and fine). The coarse variety of yarn can be produced in two different ways, i.e., through spinning by spindles or by the 'open-end' spinning method employing rotors. The two activities differ in terms of capacity and investment requirements and employment absorption. The three activities producing different varieties of yarn through spindles also differ among themselves in terms of capacity requirements for spindles and employment per unit of output since less of the finer varieties are produced per unit of time than the coarser variety.

The next four activities are for producing three varieties of cloth (coarse, medium and fine). It is assumed that the large-scale mills can produce all three varieties of cloth but the small-scale units can produce only coarse and medium varieties—in certain fixed proportion of their total output. (Three different sets of proportion have been experimented with: all coarse cloth, 50% coarse cloth and 30% coarse cloth).⁴

Seven different export activities (or export products) have been postulated in the model. These are: the exportation of raw cotton, the three different varieties of cotton yarn and the three different varieties of cloth. Although the production of all the six processed commodities is endogenous to the model, cotton production is assumed to be exogenously given.

⁴In this paper only the results based on the assumption that 50 percent of the output of small-scale industries is of the coarse variety are reported.

The last five activities are capacity expansion activities for (1) spindles, (2) rotors, (3) imported looms, (4) domestically produced looms in large-scale mills and (5) domestically produced looms employed in small-scale mills. It is assumed that large-scale mills can expand their capacity by installing both imported and domestically produced looms, whereas the small-scale industries install only domestically-produced looms.

Basic Constraints

A total of 12 basic constraints are imposed in the model. The first seven constraints are in the nature of balance equations for exogenously-given amounts of raw cotton and endogenously-determined amounts of the different varieties of yarn and cotton cloth. The next four basic constraints relate to the capacities of spindles, rotors, imported and domestically-produced looms respectively. The outputs of the different end-products are restricted in such a way that they do not exceed initial capacities and the increments in them during the five-year period. The last constraint relates to the production capacity of the domestic looms industry during the five-year period.

Additional Resource Constraints

These 12 basic constraints are not sufficient to ensure that a realistic solution in terms of aggregate resources (e.g. investment, export) would be obtained with any meaningful objective function. As stated earlier, the three objective functions in terms of aggregate resources tried in the model were the minimization of investment and maximization of exports and employment. If any of these objective functions were used with the 12 basic constraints mentioned above, they would produce implausible results in respect of the other two. Thus if investment were minimised, exports would most likely be zero. On the other hand, if exports or employment were to be maximized they would yield very high investment requirements. Thus in order to yield realistic results, certain resource constraints have to be applied. The specific nature of these constraints and the considerations behind them will be discussed in the next section.

The purpose of the model is to get some idea of the optimal pattern of domestic production and exports for the cotton textile industry in Pakistan. The choice elements in the model mainly consist of the following:

- (a) What is the optimum composition of outputs and exports of different varieties of yarn and cloth?
- (b) What is the optimum stage of processing for exports?
- (c) What is the pattern of investment required to meet a given final demand?
- (d) What is the choice of techniques between spindles and rotors in spinning?
- (e) What is the role of the small-scale units in the industry, i.e., how much and which varieties of cloth they are better suited to produce?

Each set of solution gives a definite answer to these elements of choice. The answer of the model to these micro-level questions, is generally different for the three objective functions and for the different aggregate constraints imposed.

This enables us to determine the trade-offs between the different aggregate resources for achieving a given level of final demand.

It is well-known that programming models like the present one produce rather rigid all-or-nothing type of results. To overcome this and yet retain the optimal characteristic of the solutions we have conducted sensitivity analysis on two aspects of the model viz. export prices and the output mix of the small-scale industries. There are a number of other directions in which similar sensitivity analysis could be conducted and these are pointed out in the last section but we were prevented from doing so because of the need to save computer time.

Objective Functions

Although the purpose of the three objective functions may seem obvious, a few words in justification and elaboration of them are perhaps, in order. The three objective functions reflect the most important policy concerns relating to the textile industry in Pakistan, since it constitutes such a large share in industrial employment and investment and in total exports. Another objective function that could also have been chosen is the maximization of value added. However, in a previous effort to implement this model, the authors found it extremely difficult to estimate the value added coefficients for different stages and different varieties of products. This effort had therefore, to be given up. However, the results of this objective function are unlikely to be very different from those for the employment objective function. An important aspect of policy-making in regard to the textile industry that our model is unable to capture is in regard to the geographical distribution of textile industries. For this, however, the model has to be reformulated not only with respect to the objective functions, but also with respect to variables and constraints so as to include transport costs of shipment of raw cotton and end-products.

II. Empirical Implementation

The model is used to find optimal levels of production and exports in 1980-81 and investment in spinning and weaving during the period 1975-76 and 1980-81. The model takes as given the output availability of cotton in the terminal year net of exogenous demand for cotton for non-mill use.⁵ Yarn for ancillary industries (hosiery etc.) was assumed to be 115 m. pounds, of which 90 m. pounds was assumed to be of medium variety and the rest of coarse variety. Domestic demand for the three varieties of cloth was estimated for the base year 1975-76 and projected for 1980-81 with the help of expenditure elasticities.⁶ Initial capacity estimates for spindles, rotors, looms in large-scale industry and power looms were obtained from the Planning Commission.

⁵The amount of cotton in 1980-81 is estimated at 6.1 m. bales of which 0.775 million bales is assumed to be devoted for exogenous, non-mill use.

⁶The total domestic availability of cloth for 1975-76 was obtained from the Planning Commission [10] as 1.112 million yards. This was disaggregated into the three varieties of cloth according to the percentage distribution 55, 40 and 5 for coarse, medium and fine cloth, respectively as given in [11]. These estimates were then projected for a population growth rate of 3 percent and expenditure elasticities of 0.9, 1.0 and 1.3 for coarse, medium and fine cloth respectively. In the absence of availability of direct estimates, the expenditure elasticities of textiles for rural and urban households obtained from [5] for coarse and fine cloth, respectively and the weighted average of the two for medium cloth.

The input-output coefficients for yarn and three varieties of cloth have been taken from a study by one of the present authors [1]. The finer is quality of cloth the lesser is the amount of yarn per yard consumed. A pound of yarn (of the appropriate quality) produces 3.25 yards of coarse, 4.1 yards of medium and 6.0 yards of fine cloth.

The output mix of textile industry is extremely heterogenous. Even if we confine our attention to the two main products, cotton yarn and cotton cloth, a very large number of varieties are produced. The fineness of yarn is determined by the thickness of the thread and in technical language is identified with the number of counts. The higher the count, the finer is the yarn. Yarn of counts 20 and lower is classified as "coarse" and the bulk of cotton presently being produced in and exported by Pakistan is of the coarse variety. Coarse yarn fetches lower prices than finer yarn, but per spindle more of it can be produced than the latter. The amount of yarn produced by spindle per hour is inversely related to yarn counts.⁷ We have distinguished between three varieties of yarn in the model and have assumed the following rates of output per spindle per hour.⁸

Coarse	.35 lbs. per shift (of 8 hours)
Medium	.22 lbs. per shift (of 8 hours)
Fine	.15 lbs. per shift (of 8 hours)

Assuming that there will be an average of 1000 shifts per year, the corresponding output per spindle per annum is arrived at the coefficients ay_1k , ay_2m and ay_3f are obtained by finding out the requirements of spindles per 1000 lbs. of yarn of each variety. These are: 2.85, 4.55 and 6.67, respectively. It is assumed that rotors produce thrice as much as spindles and since they are further assumed to produce only coarse yarn, $r = .95$.

The varieties of cloth are almost infinite: they differ not only in the counts of yarn used, construction structure (no. of warps and wefts), width but also in colour and design. It would be impossible for any model, however complicated, to take account of all these differences. For our simple model, we have distinguished only three varieties parallel with the varieties distinguished for yarn. The basis for the distinction has been the no. of picks per inch and the average count of yarn used in its construction.

Variety	No. of Picks/in.	Average count
Coarse	48	16
Medium	64	30
Fine	80	48

Coarse quality cloth is, generally, cheaper and heavier than finer quality cloth but more of it is produced per loom than finer quality cloth during a given period of time. The amount of cloth produced per hour is inversely related to

⁷The amount also depends, directly, on spindle speed which is a function of the modernity of equipment.

⁸We are grateful to Mr. M. Y. Siddiqui of Planning Commission and Mr. Cyril Halstead UNIDO Advisor to the Cotton Textile Industry Research and Development Centre, Karachi, for assistance in estimating the parameters.

number of picks per inch. We have assumed the following rates of output per year per loom for the different varieties.

	<i>Per year</i>
Coarse	47,619 yds.
Medium	27,027 yds.
Fine	20,000 yds.

For small-scale (or power looms) which are assumed to produce coarse and medium cloth an annual rate of output of 25,000 yds is assumed allowing for a smaller number of shifts during the year. From the above estimates the following capacity coefficients⁹ are derived:

$$l_k = .021, l_m = 0.37, l_f = .05, l_s = .040$$

Employment Coefficients

The employment coefficients for each of the production activities (4 for producing yarn and for producing cloth) were derived in the following way. It was assumed that one production worker could handle 400 spindles and that three shifts operated in a day. Thus 400 spindles working for 1000 shifts per year provided employment to three persons. The labour per spindle ratio is then multiplied by the spindle-output ratio derived earlier to get the labour-output coefficient, i.e.

$$\frac{L}{X} = \frac{L}{S} \times \frac{S}{X}$$

These labour-output ratios are inflated further to take account of peripheral workers in each activity (by 1.7, in case of coarse yarn and by 1.45 for medium and fine yarn). The labour-output ratio for coarse yarn produced by rotors is assumed to be half that of coarse yarn produced by spindles.¹⁰ A similar procedure is adopted for deriving the labour-output coefficients for cloth production. It is assumed that each production worker can handle 6 looms. The ratios thus obtained are raised by a third to take account of peripheral workers. The labour-output ratio for small-scale or power loom sector, which is assumed to produce only coarse and medium cloth, is taken to be close to that of medium cloth production in large-scale sector on the assumption of its being more labour-intensive than large-scale coarse production.

The values of labour coefficients are:

$$Ly_1^s = 35.71, Ly_1^f = 18.0, Ly_2 = 56.8, Ly_3 = 83.34,$$

$$L_k = 13.89, L_m = 25.0, L_f = 33.33, L_s = 20.0$$

Investment Coefficients

The model being an optimising projection model, requires final demand at the end of five years to be met through augmentation of existing capacity.

⁹For the definition of these and other coefficients discussed in this section refer to the list of variables and coefficients in Appendix I.

¹⁰Based on [3].

Yarn making capacity can be augmented through investment in either spindles or rotors (for coarse yarn only). Cloth making capacity can be increased through investment in imported looms (by large-scale sector only) and locally manufactured looms (whose capacity is limited). The per unit cost of investment (including non-plant investment) is derived from the Planning Commission based on feasibility studies [4]. These are:

$$k_s = 3.0, \quad k_r = 8.5, \quad k_m = 65.0, \quad k_l = 20.0$$

Export Prices

The data on export prices has been obtained from the Export Promotion Bureau. Export prices of cotton and its products fluctuate widely. We have constructed four hypothetical price vectors from the data provided by them. The "high" price vectors assume the raw cotton price to be \$235.2 and in the low price vectors the cotton price is assumed to be 164.0. Two sets of prices of yarn and cloth, by varieties, have been combined with these assumed prices of raw cotton. The price vectors are given in Table 1 below.

Table 1

Export Price Vectors								(Price in US \$)
Price Vector	Price of Cotton (per bale)	Price of Coarse Yarn (per lb.)	Price of Medium Yarn (per lb.)	Price of Fine Yarn (per lb.)	Price of Coarse Cloth (per yd.)	Price of Medium Cloth (per yd.)	Price of Fine Cloth (per yd.)	
P ₁	164.0	.63	.69	.90	.20	.24	.29	
P ₂	164.0	.50	.55	.86	.20	.20	.29	
P ₃	235.2	.63	.69	.90	.20	.24	.29	
P ₄	235.2	.73	.79	.96	.35	.40	.45	

III. Results

The three objective functions used in the model, in general, yield different results. In order to make the results of different objective functions comparable we have to hold certain strategic variables at a given level. This enables us also to observe the trade-offs between the three aggregate resources, viz. employment, exports and investment, which the model seeks to optimize. The

comparisons between the results of the three pairs of comparison are arranged and specified as follows:

Group	Objective Functions	Additional Constraints
A	Max. Employment and Min. Investment.	$X_0 = 2.5, (1.5, 1.0)$ $\bar{X} = 1120$
B	Max. Employment and Max. Exports.	$\bar{I} = 10,000 (8,000, 12,000)$
C	Max. Exports and Min. Investment.	$\bar{E} = 180,000 (150,000)$

The rationale for additional constraints has been explained in general terms in the above. For group A solutions, comparing the objective of maximising employment and minimising investment, the two additional constraints are intended to keep total exports equal to the level targeted by the Planning Commission (US\$ 1120 m) and to prevent excessive changes in the processing of raw cotton. If these constraints were removed, the solutions to the investment objective function would tend to yield rather low exports and, for any given level of export target, would tend to give a high figure for cotton exports. On the other hand, the solutions to the employment (maximizing) objective function, would yield very high total export figures. For Group B, the additional constraint is a ceiling on investment. For Group C, the additional constraint is on employment.

For the sake of simplicity, results relating to only one assumption about the product composition of the small-scale industry, viz the proportion of coarse cloth to total output in small-scale industries is assumed to be 0.5 although we have obtained results for other assumption as well (viz., $\theta=0.3$ and 1.0).

In each Table four solutions for each of the objective functions is obtained.¹¹ These correspond to a different set of export prices for the seven major export commodities given in Table 1.

Group A Solutions

For both sets of solutions in Table 1, the highest investment is obtained for P_2 price vector, i.e., when both raw cotton and yarn and cloth prices are relatively low. When raw cotton prices rise the investment requirement in both solutions fall—almost precipitously, in the investment minimising solution. In terms of product mix, export composition and investment pattern the two sets of solutions show marked differences. In the employment objective function all coarse yarn production is spindle based, whereas in the investment minimising functions spindles are used to produce coarse yarn only when cotton prices are high (and hence relatively low export of yarn and cloth are required to meet the export target). In the latter case expansion in yarn production capacity is achieved mainly through increase in rotors, while in the

¹¹The results are insensitive to changes in export prices in a number of cases, i.e., different price vectors yield the same optimal solution. This is generally so when the investment or employment constraint is used.

investment minimisation case capacity expansion is never recommended for small scale industries. In the employment maximization case capacity expansion is recommended for small-scale industries in most solutions. Small-scale industry output and investment rises to the highest feasible level, i.e., 1750, in most cases, implying that, in addition to full utilization of existing capacity, the entire output of the domestic looms producing industry is used to increase the capacity in the powerlooms sector.

Pattern of Export

The pattern of export varies from solution to solution. But most solutions indicate export of fine cloth. In all solutions with low price of cotton (\$164.0 per bale) exports of fine cloth are indicated. They contribute more than 50% of total export receipts in case of investment minimizing functions. In case of employment maximizing function both medium and fine cloth exports are recommended at the two price vectors with low raw cotton price. For the most favourable price vectors, the investment objective function solution switches from exports of fine cloth to that of coarse cloth, while the employment solutions substitute the export of medium cloth by fine yarn. In the other high price vector (P_3) the employment maximizing solution shows medium and coarse cloth exports. The amount of export of cotton manufactures all as the price situation becomes more favourable (the export target being fixed in value terms). In the investment minimizing solutions, exports of coarse yarn are shown at a positive level in all cases, while in the employment maximizing solutions only one solution shows exports of yarn and that of the highest quality. Thus, in general, the employment maximising solutions show the highest degree of processing.

Investment Pattern

The pattern of investment in various solutions is closely related to their output mix and export composition. Since investment in rotors costs less than in spindles (per unit of output of yarn) and since rotors are specific to coarse yarn production, the investment minimizing solutions require the installation of rotors and some export of coarse yarn. Spindleage capacity is installed only when the export of cloth has to be at a substantially high level.

For Group A, we have obtained solutions for three levels of exports of raw cotton (2.5, 1.5 and 1.0 m. bales). These are presented in Tables 1-3 respectively. The result of these solutions, not unexpectedly, show that as the level of exports of raw cotton is progressively raised, investment requirements in both sets of solutions fall. Thus, in the investment minimizing solutions the investment requirements for the most favourable export price case (P_1) fall from Rs. 13,366.57 m. when only 1m. bales of raw cotton is required to be exported, to Rs. 9,354.64 when raw cotton exports are 2.5 m. bale drop in exports of raw cotton necessitate an increase of Rs. 4,000 in worth of investment. A similar drop in the amount of cotton exported, requires an increase of Rs. 8,000 m. worth of investment in the employment maximizing case.

However, the output, export and investment mix of Group A solution does not change much with the change in the level of raw cotton exports. Due to a fixed limit on total exports (which acts as a ceiling for the employment

maximizing and as a floor for the investment minimizing solutions), lower cotton exports imply a lower degree of processing of exports.

The imposition of an overall export constraint simultaneously with a constraint on the amount of raw cotton exports (when the constraints act alternatively as a ceiling and as floor in each of the objective functions) gives a certain degree of rigidity and artificiality to the results. We therefore, decided to have solutions which would, allow only one of the two export constraints to be binding. The results of this exercise for (minimum) export of raw cotton at 2.5, 1.5 and 1.0 and overall export of 1120 m. \$ are given in Tables 4-6. For investment minimising solutions, cotton exports are always above the required minimum (they are 2.98 m. bales when cotton prices are low and 3.44 m. bales when cotton prices are low and 3.44 m. bales when cotton prices are high) while total exports are just equal to the required minimum (\$ 1120 m.). For employment maximising solutions the reverse situation holds (the total exports range from \$ 1303.6 to 1974.7).

What is more interesting, however, is that with this change in the form of constraints both objective functions yield fine cloth as the only (or the most preferred) processed export. In the investment minimising solutions some export of fine yarn is indicated at the more favourable sets of export prices. However, if the overall export target were raised, the solution would again favour fine cloth exclusively.

Group B Solutions

In this Group (See Tables 7-9) solutions have been obtained by fixing investment at a given level. Exports and employment have been allowed to vary freely.¹² The results of the solution are not very different. Since there is no export constraint, the employment solution are not affected by differences in export prices. The export objective function, does however, depend on prices. But all the four sets of prices produce identical solutions, for any given level of investment.

There is only one significant difference between the solutions to the export and employment objective functions. This relates to the production of coarse yarn. Although both sets of solutions give identical amount of production of coarse yarn, the export solutions show that coarse yarn is produced only by rotor (the open end method) whereas in the employment solution it is produced by rotors only to the extent of existing rotors capacity—the remainder being produced by the traditional method of spinning by spindles. Since spindles cost more than rotors to produce the same amount of output, in the employment solutions less of the fixed investment sum is left for investment in looms. As a result, the output and exports of cloth in this set of solutions are lower than those for export maximising solutions. The employment solutions also process .01 million bales of cotton less than the export solutions and to that extent their export of raw cotton is higher. The export maximising solutions, on the other hand, show larger exports of fine cotton cloth and more investment in looms as compared to the employment solutions.

¹²However, in Group B and C solutions of exports of cotton have been constrained, to be, at least, one million bales. This constraint was binding in only one case.

In other respects the two sets of solutions show striking similarities. Small-scale cloth production in both sets is at the maximum level permitted by existing loomage, but no further expansion of capacity is indicated in either set of solutions. In both sets of solutions the only exports are cotton and fine cloth—no exports of any commodity at any of the intermediate processing stages is indicated.

For the solutions in this Group (B) of our model, three different levels of investment ceiling were imposed—viz. Rs. 8,000 m., Rs. 10,000 and Rs. 12,000 million (and are shown separately in Tables 7, 8 and 9 respectively). The optimal basis of the two objective functions does not change with changes in the levels of the investment ceilings. However, the variation in these ceilings enables us to determine the trade—off ratios between investment, export and employment in this Group of solution.

Thus an increase of 2,000 m. rupees in investment gives an increase of 14,282 jobs in each case, i.e. the marginal capital-labour ratio is 0.14 m. rupees. The investment cost of a million dollars of “export promotion” on the other hand, is Rs. 32.8 m., i.e., the incremental capital—output ratio in export promotion is about 3.28.

Group C Solutions

The solutions in this Group (see Tables 10 and 11) minimise investment and maximize exports subject to an overall employment constraint (the ceiling for the latter).

In this Group also, the optimal bases for the two objective functions remain unchanged for all four sets of export prices (the investment model does not depend on them). The solutions to the investment minimising and exports maximizing objective functions differ in important respects. In the investment minimising function a slightly lower amount of raw cotton is exported in these solutions. However, it favours the production of coarse yarn through increase in spindle rather than rotor capacity. This may seem surprising since rotors are less costly per unit of output than spindles and in Group A results, the investment minimising solutions generally favour the installation of rotors. However since in the present case, the employment targets are high (180,000 and 150,000) and no export targets are imposed the investment solutions find it cheaper to provide targeted employment by switching from rotors to spindles, rather than by increasing employment through installation of more looms. The export maximising functions, on the other hand, favour the entire production of yarn by rotors. Although they do not recommended any expansion of small-scale production capacity they do favour the utilization of existing capacity. Both sets of solutions, however, favour the export of fine cloth among the processed products of cotton.

IV. The Search for Realism

In the preceding section a variety of solutions were presented corresponding to the different objective functions and constraints. In general the investment minimising solutions yield low values for exports and employment, while the export and employment maximising solutions yield rather high values for investment—though the differences between the two sets of solutions for exports

and employment, for given levels of investment, are rather small. We have conducted a limited amount of sensitivity analysis on the model to generate solutions which could be used to find what are the realistic limits of the different constraints and to what extent the different objectives are in conflict with each other or help to reinforce each other. This, on the other hand, will help to narrow down the search for an optimal solution which would be satisfactory to the policy maker from the different points of view i.e., which would be optimal for all three objective functions.

In order to get some idea of the degree of reasonableness of the various solutions, it is helpful to look at some of the relevant targets set by the Pakistan Planning Commission for achievement during the Fifth Five Year Plan (1975-1976 to 1980-81)¹³—the period to which our model calculations correspond. The Planning Commission gives the following export targets for 1980-81.

Commodity	Quantity	Price	Value (US \$) Millions
Raw Cotton	1.47 (Million bales*)	235.2 (US\$/bale)	345.7
Yarn	465.00 (Million Pounds)	.78 (US\$/Pound)	362.7
Cloth	1030 (Million Yds.)	.40 (US\$/Yard)	412
Total :			1120.4

*A bale of cotton weighs 392 pounds.

In addition, the Planning Commission estimates domestic requirements of cloth at 1690 m. yards (which compares with our estimate of 1961 m. yards). In order to meet this increase in export and domestic demand, the Planning Commission further estimates the amount of investment in the textile industry an addition of 900,000 spindles and 30,000 looms—of which 20,000 are to be produced domestically.¹⁴ The Planning Commission does not give a direct estimate of the amount of employment which would result from the increase in domestic and export demand. But the increase in employment can be estimated on the basis of the coefficients derived by us in an earlier section (see Section II).¹⁵ This gives an estimate of 10,250 additional workers in spinning and 20,000 additional workers in weaving. This compares with our estimates of 40,500 workers employed in spinning and 53,000 employed in weaving in the base year 1975-76—or roughly an increase of one third.

¹³Although the Plan has not officially been released, the Planning Commission has undertaken the exercise for its internal use.

¹⁴The price of an imported loom is Rs. 65 and that of a domestically produced loom is Rs. 20.

¹⁵For example we assume that each production worker in spinning can handle 400 spindles. Thus an addition of 900,000 spindles will provide employment to 7,500 additional workers (in 3 shifts) in spinning, which may be inflated by 50% to take account of peripheral workers. In a similar way employment in weaving is estimated.

If we simulate the assumptions of the Planning Commission with our model we find that they bear close resemblance to the investment-minimising solution in Table 2 for price vector P4, giving the minimum value of investment during the five year period as Rs. 2,870 m.

This Particular solution favours the export of coarse cloth and coarse yarn and meets the overall export target of \$1120 m. and an export of 1.5 m. bales of raw cotton. However, the employment increase implied by this solution is only 9,700 or only 10% increase during the five year period.¹⁶ The solution however, does not produce maximum employment or exports—which would no doubt be desirable objectives. If these objectives were used employment could rise by more than 200 percent and export could almost double. This leads us to conjecture that either the Planning Commission export price forecasts are too optimistic or its targets for total exports are too low. At less favourable prices, even the minimum investment requirements are very high. They rise to Rs. 7320 m. if the raw cotton prices remain at \$ 235.2 per bale, but prices of yarn and cloth do not rise as much. The minimum investment levels rise to Rs. 10,292 m. and Rs. 12,029 m. if raw cotton price remain at their base-year level \$164.00 per bale accompanied by high and low prices of processed goods, respectively.

Our limited exercise in sensitivity analysis shows that the objective of investment minimization is a poor choice from the point of view of realizing the potential of the textile industry in terms of its contribution to exports and employment. Our optimization model also shows that investment optimizing solutions are very sensitive to price assumptions about the export market. Since export prices are subject to a considerable degree of fluctuations and uncertainty it would seem inadvisable to do investment planning on the basis of a single price assumption. Also investment minimization solutions prove rather unsatisfactory from the employment point of view.

Although investment minimization itself may not be a very desirable objective in itself, planners cannot be oblivious to the investment implications of the other two objectives, viz. exports and employment, however attractive the latter may seem to be. It would be, therefore, desirable to impose upper limits on the volume of investment. We have chosen three such limits (Rs. 8,000 m., 10,000 m. and 12,000 m.) [see Tables 7, 8 and 9]. In a similar way it would be desirable to impose upper limits on employment and exports. For employment, we have used two ceilings, i.e., 150,000 and 180,000. For exports, only one ceiling, i.e., \$1120 m. based on Planning Commission's estimates was used (this was compensated by use of four price vectors). In all solutions exports of cotton were assured to be, at least, equal to 1 m. bale.

The solutions to the three different objective functions when employment and investment are set at a fixed level give us some idea of the region of feasibility for our model. An employment target of 150,000 (or about 60% in five years) and an investment target of Rs. 8,000 m.¹⁷ seem to be reasonable. The investment minimizing solution, when employment is constrained to be 150,000 (Table 10) gives an investment outlay of Rs. 6047.3 million. However, the export receipts of the both solutions are less than \$1120 m. at the two lower set

¹⁶The low employment increase is due to the fact that the investment minimising solution favour the use of rotors in the production of coarse yarn and the export of less labour-intensive coarse cloth.

¹⁷FAO Report [3] p. 23 estimates an investment of Rs. 7,000 m.

of prices, when the investment ceiling of Rs. 8,000 m. is put on solutions with employment and exports objective functions (Table 9). The latter almost fulfill the export target of 1120 m. dollars even at the lower set of prices and although employment generated is lower than that in the employment solution it is well above 150,000 (156,966 to be exact).

The satisfaction of a minimum employment target has an implication for the product mix for all objective functions, including investment minimization. All solutions favour the export of fine cloth as the only manufactured export. Both the employment and the investment ceilings result in high levels of raw cotton exports in all solutions (ranging from 2.75 to 3.15 m. bales). With these ceilings, however, the total export targets are higher than \$1120 m. when export prices are relatively high. A fixed export target, on the other hand, forces down the level of investment and employment when prices are favourable. It seems unlikely that public policy would abstain from taking advantage of favourable world price situation to expand output, employment, investment and export.

The solutions corresponding to a fixed export target (in dollar values) are the only ones which favour the export of coarser varieties of yarn and cloth, especially when the objective function is to minimize investment. This corresponds also to the actual situation prevailing in Pakistan. There is considerable evidence that the output and export mix of the Pakistan textile industry has been changing in favour of coarser varieties.¹⁸ The reason for this are manifold—capacity taxation, fear of labour trouble, assured markets for inferior varieties, lack of enterprise etc. It is possible that in the short-run it may not be easy to change the sub-optimal mix currently being produced by the Pakistan cotton textile industry, but in the long-run it should be possible to effect changes in public policy so as to bring the product and export mix closer to the optimal one.

V. Extension and Improvements

We have discussed the characteristics of various solutions in the previous section and have compared them with the Planning Commission's estimates. It is now tempting to ask whether we can arrive at a solution which would, simultaneously, satisfy the various constraints introduced for the sake of realism and would be optimal from the point of view of all the three objective functions. A number of interactive algorithms have been developed to solve such multiple objective linear programming problems [2]. However, these algorithms involve considerable use of human judgement for deriving a weighting system for the different objective functions. The actual use of these methods depends on the planner's judgements about the basic constraints and the relative priorities attached to the different objectives.

Another use of the model is in terms of the trade-off ratios between the various aggregative resources in the model, viz employment, exports and investment. These ratios give the marginal rates of substitution between different resources under a specific set of constraints (or regime). For any given objective function, the model also gives the shadow prices of different capacity (and others) constraints of the model. Their interpretation can be obtained in any standard treatment of function, the linear programming theory.

¹⁸[3] brings this out very clearly.

The model presented above is, inevitably, a simplified version of reality. Only three varieties each of yarn and cloth are distinguished, while raw cotton is assumed to be completely homogeneous. Only two major activities of the cotton textile industry, viz. spinning and weaving, are taken into account while finishing and dyeing—which contribute to considerable value added and employment—have been omitted. This leaves out from the realm of investment choice the activity of “balancing and modernization” which is often considered to be more important than capacity expansion in spindles and looms. The model also does not extend itself to cover garment manufacturing which could become a major source of augmenting employment and exports. However, none of these aspects is beyond the reach of the present model and given more time and sufficient data most, if not all, of them could be incorporated in the model.

Although the model suffers from the above mentioned defects it is capable of being used to answer policy questions regarding production, export and investment choices in the textile industry. The model points out the range of choices available to the planner but leaves it to his judgement the final choice which depends on his vision of the likely state of affairs in the future and the relative priorities of different economic objectives. In the particular case of the current state of textile industry in Pakistan, the model shows that the present mix of production, export and investment is substantially sub-optimal and that the industry is capable of substantially raising its export and employment targets within reasonable limits of investment. However, we would like to caution that the model in its present form should not be expected to provide detailed policy guidance.

Our estimation of many of the parameters and exogenous variables of the model has been almost cavalier. We have relied on the best technical information available. However, not being engineers or technologists ourselves, we have not always been in the best position to sift the sometimes widely varying estimates of the different coefficients. We hope that the Cotton Textile Research Centre in Karachi, which is currently undertaking a survey on many of these aspects, will be able to provide more robust estimates. One aspect of the model, which needs further detailed research, is in regard to the imposition of realistic export constraints based on the analysis of the demand for different varieties products in abroad market areas.

Appendix I
THE BASIC MODEL: ACTIVITY MATRIX

Variables →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Constraints	Y_1^a	Y_1^r	Y_2	Y_s	X_k	X_m	X_f	X_e	E_s	E_{y1}	E_{y2}	E_{y3}	E_k	E_m	E_f	ΔS	ΔR	ΔL^m	ΔL^l	ΔL^s	
Cotton	a_{cy1}^b	a_{cy1}^f	a_{cy2}	a_{cy3}					1.0												
Yarn	1.0	1.0			$-a_{y1}^k$			$-\theta a_{y1}^k$		-1.0											
Wool Yarn			1.0			$-a_{y1}^m$		$-(1-\theta)a_{y1}^m$			-1.0										
Wool							$-a_{y3}^f$					-1.0									
Cloth					1.0			θ						-1.0							
Wool Cloth						1.0		$(1-\theta)$							-1.0						
Cloth							1.0									-1.0					
Reserves	s_1		s_2	s_3													-1.0				
L-S		r																-1.0			
S-S					l_k	l_m	l_f													-1.0	
Looms Products								l_s												1.0	
FUNCTIONS																				1.0	
EMPLOYMENT	L_1^a	L_1^r	L_2	L_3	L_k	L_m	L_f	L_s													
TREATMENT																					
PRICES									P_s	P_{y1}	P_{y2}	P_{y3}	P_k	P_m	P_f		k_s	k_r	k_m	k_l	k_s

\leq C
 \leq Y_1^a
 \leq Y_2^a
 \leq Y_3^a
 \leq X_k^a
 \leq X_m^a
 \leq X_f^a
 \leq S
 \leq R
 \leq L₁
 \leq L₂
 \leq L₃
 \leq L₄
 = Max.
 = Min.
 = Max.

LIST OF VARIABLES

Endogenous Variables

- Y_1^s = Coarse yarn output per year produced by spindles (m.lbs.)
 Y_1^r = Coarse yarn output per year produced by rotors (m.lbs.)
 Y_2 = Medium yarn output per year produced by spindles (m.lbs.)
 Y_3 = Fine yarn output per year produced by spindles (m.lbs.)
 X_k = Coarse cloth output per year produced by large-scale mills (m.yds.)
 X_m = Medium cloth output per year produced by large-scale mills (m.yds.)
 X_f = Fine cloth output per year produced by large-scale mills (m.yds.)
 X_s = Cloth output per year produced by large-scale mills (m.yds.)
 E_c = Exports of raw cotton cloth (m.bales)
 E_{y_1} = Exports of coarse yarn (m.lbs.)
 E_{y_2} = Exports of medium yarn (m.lbs.)
 E_{y_3} = Exports of fine yarn (m.lbs.)
 E_k = Exports of coarse cloth (m.yds.)
 E_m = Exports of medium cloth (m.yds.)
 E_f = Exports of fine cloth (m.yds.)
 ΔS = Additional spindle capacity required (000's)
 ΔR = Additional spindle capacity required (000's)
 ΔL^m = Additional imported loom capacity of L-S Sector (000's)
 ΔL^l = Additional local loom capacity of L-S Sector (000's)
 ΔL^s = Additional local loom capacity of L-S Sector P. Loom Sector (000's)

Parameters

- a^{cy_1} = .0031 = Million bales of cotton required to produce a million pounds of coarse yarn by spindles.
 $a^r_{cy_1}$ = .0031 = Million bales of cotton required to produce a million pounds of coarse yarn by rotors.
 a_{cy_2} = .0031 = Million bales of cotton required to produce a million pounds of medium yarn.
 a_{cy_3} = .0031 = Million bales of cotton required to produce a million pounds of fine yarn.
 $a_{y_1^x}$ = .31 = Input of coarse yarn required to produce one unit of coarse cloth.
 $a_{y_2^m}$ = .22 = Input of medium yarn required to produce one unit of medium cloth.
 $a_{y_3^f}$ = .17 = Input of fine yarn required to produce one unit of fine cloth.
 θ = (1.0, 0.5, 0.3) = Proportion of coarse cloth produced by small-scale sector.
 s_1 = 2.85 = No. of spindles (in thousands) required to produce 1m.lbs. of coarse yarn per annum.
 s_2 = 4.55 = No. of spindles (in thousands) required to produce 1m.lbs. of medium yarn per annum.

s_s	= 6.67	= No of spindles (in thousands) required to produce 1m.lbs. of fine yarn per annum.
r	= .95	= No. of rotors (in thousands) required to produce one million pounds of coarse yarn.
l_k	= .021	= No. of looms required to produce one million yards of coarse cloth.
l_m	= .037	= No. of looms required to produce one million yards of medium cloth.
l_f	= .050	= No. of looms required to produce one million yards of fine cloth.
l_s	= .04	= No. of looms required to produce one million yards of cloth by small-scale industries.
Ly_1^*	= 35.71	= Employment per unit of coarse yarn output through spindles.
Ly_1^r	= 18.0	= Employment per unit of coarse yarn output through rotors.
Ly_2	= 56.8	= Employment per unit of medium yarn output.
Ly_3	= 83.34	= Employment per unit of fine yarn output.
L_k	= 13.89	= Employment per unit of coarse cloth in large-scale mills.
L_m	= 25.0	= Employment per unit of medium cloth in large-scale mills.
L_f	= 33.33	= Employment per unit of fine cloth in large-scale mills.
L_s	= 20.0	= Employment per unit of cloth in small-scale mills.
k_s	= 3.0	= Price per spindle (Rs.)
k_r	= 8.5	= Price per rotor (Rs.)
k_m	= 65.0	= Price per loom (imported) (Rs.)
k_l	= 20.0	= Price per loom (domestic) (Rs.)

Exogenous Variables:

\bar{C}	= 5.9225	= Net availability of raw-cotton (m.bales)
\bar{Y}_1^c	= 25.0	= Non-mill Consumption of coarse yarn (m.lbs.)
\bar{Y}_2^c	= 90.0	= Non-mill Consumption of medium yarn (m.lbs.)
\bar{Y}_3^c	= 0.0	= Non-mill Consumption of fine yarn (m.lbs.)
\bar{X}_k^c	= 915.4	= Domestic Consumption of coarse cloth (m.yds.)
\bar{X}_m^c	= 752.8	= Domestic Consumption of medium cloth (m.yds.)
\bar{X}_f^c	= 92.9	= Domestic Consumption of fine cloth (m.yds.)
\bar{S}	= 3600	= Spindle Capacity in 1975-76 (thousands)
\bar{R}	= 8.0	= Rotor Capacity in 1975-76 (thousands)
\bar{L}	= 30.0	= Large-scale loom capacity in 1975-76 (thousands)
\bar{L}_p	= 50.0	= Powerlooms capacity in 1975-76 (thousands)
\bar{L}_d	= 20.0	= Powerloom production capacity during 1975-76 to 1980-81 (thousands)

Table 1

Group A Solutions (Additional Constraints: $E_c = 2.5$, $\bar{X} = 1120$)

Variable		Employment(E)				Investment(I)			
		P ₁	P ₂	P ₃	P ₄	P ₁	P ₂	P ₃	P ₄
Y ₁ ^r	(X ₁)	308.77	308.77	397.32	308.77	0.0	0.0	378.75	818.75
Y ₁ ^f	(X ₂)	0.0	0.0	0.0	0.0	479.45	661.05	266.14	14.51
Y ₂	(X ₃)	554.21	488.07	690.91	255.62	255.62	255.62	255.62	255.62
Y ₃	(X ₄)	241.05	307.19	15.79	539.64	368.97	387.37	203.52	15.79
X _k	(X ₅)	40.40	40.40	326.05	163.60	290.40	290.40	290.40	1499.70
X _m	(X ₆)	1235.33	925.33	1856.42	0.0	127.80	127.80	127.80	127.80
X _f	(X ₇)	1417.95	1818.74	92.90	194.38	2170.40	2278.63	1197.19	92.9
X _s	(X ₈)	1750.00	1750.00	1750.00	1505.60	1250.00	1250.00	1250.00	1250.00
E _c	(X ₉)	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
E _{y1}	(X ₁₀)	0.0	0.0	0.0	0.0	170.67	152.27	336.12	148.97
E _{ya}	(X ₁₁)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
E _{ya}	(X ₁₂)	0.0	0.0	0.0	506.60	0.0	0.0	0.0	0.0
E _k	(X ₁₃)	0.0	0.0	285.65	0.0	0.0	0.0	0.0	1209.30
E _m	(X ₁₄)	1357.23	1047.53	1978.62	0.0	0.0	0.0	0.0	0.0
E _f	(X ₁₅)	1325.05	1725.84	0.0	101.48	2077.50	2185.50	1104.29	0.0
ΔS ^r	(X ₁₆)	1409.46	1553.90	781.37	2042.47	24.07	146.79	0.0	0.0
ΔR ₂	(X ₁₇)	0.0	0.0	0.0	0.0	447.47	429.99	244.83	5.79
ΔL ^m	(X ₁₈)	87.44	96.02	50.18	0.0	69.35	74.76	20.69	0.0
ΔL ¹	(X ₁₉)	0.0	0.0	0.0	0.0	20.00	20.00	20.00	10.87
ΔL ^s	(X ₂₀)	20.0	20.0	20.0	20.0	0.0	0.0	0.0	0.0
I (Total Investment)		10311.98	11303.00	6005.81	6527.41	8783.30	9354.64	3825.7	266.55
E (Total Employment)		176291.00	176291.00	143783.9	109367.94	158467.36	163276.93	121927.25	97433.23
\bar{X} (Total Exports)		1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0

Table 2

Group A Solutions: (Additional Constraints: $E_c = 1.5$, $\bar{X} = 1120$)

	Employment				Investment			
	P ₁	P ₂	P ₃	P ₄	P ₁	P ₂	P ₃	P ₄
Y ₁ ^o	419.55	308.77	659.14	308.77	76.22	0.0	309.11	818.11
Y ₁ ^r	0.0	0.0	0.0	0.0	761.60	781.38	628.55	337.10
Y ₂	991.27	970.87	751.67	522.51	255.62	255.62	255.62	255.62
Y ₃	15.79	146.97	15.79	595.33	332.51	389.61	233.24	15.79
X _k	397.74	40.40	1170.63	162.60	290.40	290.40	290.40	1497.40
X _m	3221.68	3128.96	2132.60	0.0	127.80	127.80	127.80	127.80
X _r	92.90	864.51	92.0	92.90	1955.94	2291.85	1372.01	92.90
X _s	1750.00	1750.00	1750.00	1505.60	1250.00	1250.00	1250.00	1250.00
E _c	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
E _{y1}	0.0	0.0	0.0	0.0	529.71	472.61	628.98	972.26
E _{y2}	0.0	0.0	0.0	266.89	0.0	0.0	0.0	0.0
E _{y3}	0.0	0.0	0.0	579.53	0.0	0.0	0.0	0.0
E _k	357.34	0.0	1130.23	0.0	0.0	0.0	0.0	1207.00
E _m	3343.89	3251.16	2254.81	0.0	0.0	0.0	0.0	0.0
E _r	0.0	771.61	0.0	0.0	1863.04	2198.95	1279.11	0.0
ΔS	2211.34	2677.74	1804.02	3628.27	0.0	161.77	0.0	0.0
ΔR	0.0	0.0	0.0	0.0	715.52	734.31	522.12	312.24
L ^m	102.20	129.84	78.13	0.0	58.62	75.42	29.43	0.0
L ^l	0.0	0.0	0.0	0.0	20.0	20.0	20.0	10.82
L ^s	20.0	20.0	20.0	20.0	0.0	0.0	0.0	0.0
I	13677.02	16872.82	10890.51	11284.81	10292.51	12029.27	7320.35	2870.43
E	196765.06	221018.88	175220.4	125786.00	156104.92	169670.17	122956.72	103205.12
X̄	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0

Table 3

Group A Solutions (Additional Constraints $E_c = 1.0$, $\bar{X} = 1120$)

		Employment				Investment			
		P ₁	P ₂	P ₃	P ₄	P ₁	P ₂	P ₃	P ₄
Y ₁ ^a	(X ₁)	630.32	308.77	790.06	308.77	119.54	0.0	274.43	818.11
Y ₁ ^r	(X ₂)	0.0	0.0	0.0	0.0	898.46	941.55	809.76	498.39
Y ₂	(X ₃)	941.78	1213.27	782.05	741.56	255.62	255.62	255.62	255.62
Y ₃	(X ₄)	15.79	65.86	15.79	537.56	314.28	390.74	248.10	15.79
Y _k	(X ₅)	1077.66	40.40	1592.92	162.60	290.40	290.40	290.40	1496.25
X _m	(X ₆)	2996.75	4230.77	2270.69	0.0	127.80	127.80	127.80	127.80
X _f	(X ₇)	92.90	387.39	92.90	92.90	1848.71	2298.46	1459.42	92.9
X ^a	(X ₈)	1730.00	1750.00	1750.00	1505.60	1250.00	1250.00	1250.00	1250.00
E _c	(X ₉)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
E _{y1}	(X ₁₀)	0.0	0.0	0.0	0.0	709.23	632.77	775.41	633.90
E _{y2}	(X ₁₁)	0.0	0.0	0.0	485.95	0.0	0.0	0.0	0.0
E _{y3}	(X ₁₂)	0.0	0.0	0.0	521.77	0.0	0.0	0.0	0.0
E _k	(X ₁₃)	1037.26	0.0	1552.52	0.0	0.0	0.0	0.0	1205.85
E _m	(X ₁₄)	3118.95	4352.98	2392.89	0.0	0.0	0.0	0.0	0.0
E _f	(X ₁₅)	0.0	294.49	0.0	0.0	1755.81	2205.56	1366.52	0.0
ΔS	(X ₁₆)	2583.88	3239.65	2315.34	4239.68	0.0	169.28	0.0	0.0
ΔR	(X ₁₇)	0.0	0.0	0.0	0.0	845.54	886.46	761.27	465.47
ΔL ^m	(X ₁₈)	108.16	146.76	92.11	0.0	53.26	75.75	33.80	0.0
ΔL ^l	(X ₁₉)	0.0	0.0	0.0	0.0	20.0	20.0	20.0	10.79
ΔL ^a	(X ₂₀)	20.0	20.0	20.0	20.0	0.0	0.0	0.0	0.0
\bar{I}		15191.04	19658.35	13333.17	13119.04	11049.12	13366.57	9067.66	4172.35
\bar{E}		2053301.75	239670.56	190938.58	133414.31	154998.52	172867.72	140398.79	106092.37
\bar{X}		1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0	1120.0

Table 4

Solutions for Group A
 (Additional Constraints: $E_c \geq 2.5$, $\bar{X} \geq 1120$)

	Employment	Investment		
	P_1 to P_4	P_1 and P_2	P_3	P_4
Y_1^s	308.77	0.0	300.35	300.35
Y_1^r	0.0	308.77	8.42	8.42
Y_2	255.60	255.62	255.62	255.62
Y_s	539.60	386.3	237.02	237.02
X_k	162.60	290.40	290.40	290.40
X_m	0.0	127.80	127.80	127.80
X_f	3174.40	2272.35	912.22	438.00
X_s	1505.60	1250.00	1250.00	1240.99
E_c	2.50	2.98	3.44	3.44
E_{y1}	0.0	0.0	0.0	0.0
E_{y2}	0.0	0.0	0.0	0.0
E_{ys}	0.0	0.0	81.95	162.56
E_k	0.0	0.0	0.0	0.0
E_m	0.0	0.0	0.0	0.0
E_f	3081.50	2179.45	819.32	345.10
ΔS	2042.50	139.66	0.0	0.0
ΔR	0.0	285.33	0.0	0.0
ΔL^m	122.40	74.44	6.44	0.0
ΔL^l	9.80	20.0	20.0	2.73
ΔL^s	10.22	0.0	0.0	0.0
\bar{I}	14483.50	8083.2	818.49	54.54
\bar{E}	208690.60	160225.50	107782.43	91976.53
\bar{X}	1303.6(for P_1, P_2)	1120.76	1120.45	1120.45
	1481.0 (P_3)			
	1974.7 (P_4)			

Table 5

Group A Solutions
 (Additional Constraints: $E_c \geq 1.5$, $\bar{X} \geq 1120$)

	Employment		Investment	
	P_1 to P_4	P_1 and P_2	P_3	P_4
Y_1^s	308.77	0.0	300.35	300.35
Y_1^f	0.0	308.77	8.42	8.42
Y_2	255.62	255.62	255.62	255.62
Y_3	862.22	386.3	237.02	237.02
X_k	162.60	290.40	290.40	290.40
X_m	0.0	127.80	127.80	127.80
X_f	5071.90	2272.35	912.22	438.00
X_s	1505.60	1250.00	1250.00	1249.99
E_c	1.5	2.98	3.44	3.44
E_{y_1}	0.0	0.0	0.0	0.0
E_{y_2}	0.0	0.0	0.0	0.0
E_{y_3}	0.0	0.0	81.95	162.56
E_k	0.0	0.0	0.0	0.0
E_m	0.0	0.0	0.0	0.0
E_f	4978.90	2179.45	819.32	345.10
ΔS	4194.00	139.66	0.0	0.0
ΔR	0.0	285.33	0.0	0.0
ΔL^m	217.3	74.44	6.44	0.0
ΔL^1	9.8	20.0	20.0	2.73
ΔL^s	10.22	0.0	0.0	0.0
\bar{I}	27106.5	8083.2	818.49	54.54
\bar{E}	298818.60	160,225	107,782	91,976
\bar{X}	1796.0 (for P_1)	1120.76	1120.45	1120.45

Table 6

Group A Solutions

(Additional Constraints; $E_c \geq 1.0$, $\bar{X} \geq 1120$)

	Employment	Investment		
	P_1 to P_4	P_1 and P_2	P_3	P_4
Y_1^s	308.77	0.0	300.35	300.35
Y_1^f	0.0	308.77	8.42	8.42
Y_2	255.62	255.62	255.62	255.62
Y_3	1023.51	386.3	237.02	237.02
X_k	162.60	290.40	290.40	290.40
X_m	0.0	127.80	127.80	127.80
X_f	6020.64	2272.35	912.22	438.00
X_s	1505.60	1250.0	1250.0	1249.99
E_c	1.0	2.98	3.44	3.44
E_{y1}	0.0	0.0	0.0	0.0
E_{y2}	0.0	0.0	0.0	0.0
E_{y3}	0.0	0.0	81.95	162.56
E_k	0.0	0.0	0.0	0.0
E_m	0.0	0.0	0.0	0.0
E_f	5927.73	2179.45	819.32	345.10
ΔS	5269.86	139.66	0.0	0.0
ΔR	0.0	285.33	0.0	0.0
ΔL^m	264.67	74.44	6.44	0.0
ΔL^1	9.78	20.0	20.0	2.73
ΔL^s	10.22	0.0	0.0	0.0
\bar{I}	23,013.7	8083.2	818.49	54.54
\bar{E}	343882	135237	82782	66976
\bar{X}	1883.04(P_1 & P_2)	1120.0	1120.0	1120.0
	1954.2(P_3)			
	2902.6(P_4)			

Table 7

Solutions for Group B

(Additional Constraints: I = 12,000)

	Employment	Exports
	P ₁ to P ₄	P ₁ to P ₄
Y ₁ ^a	300.35	0.0
Y ₁ ^r	8.42	308.77
Y ₂	255.62	255.62
Y ₃	482.75	486.40
X _k	290.40	290.40
X _m	127.80	127.80
X _r	2839.74	2861.19
X _s	1250.00	1250.00
E _c	2.68	2.67
E _{y1}	0.0	0.0
E _{y2}	0.0	0.0
E _{ys}	0.0	0.0
E _k	0.0	0.0
E _m	0.0	0.0
E _r	2746.84	2768.29
ΔS	1639.03	807.35
ΔR	0.0	285.33
ΔL ^m	102.81	103.89
ΔL ^l	20.0	20.0
ΔL ^s	0.0	0.0
I	12,000	12,000
E	192505.75	188205.8
X	1236.10 (for P ₁ & P ₂)	1239.87 (for P ₁ & P ₂)
	1426.38 (for P ₃)	1429.62 (for P ₃)
	1866.4 (for P ₄)	1874.72 (for P ₄)

Table 8
Solutions for Group B
 (Additional Constraints: $I=10,000$)

	Employment	Exports
	P_1 to P_4	P_1 to P_4
Y_1^o	300.35	0.0
Y_1^r	8.42	308.77
Y_2	255.62	255.62
Y_3	431.64	435.29
X_k	290.40	290.40
X_m	127.80	127.80
X_r	2539.06	2560.51
X_s	1250.00	1250.00
E_c	2.83	2.82
E_{y1}	0.0	0.0
E_{y2}	0.0	0.0
E_{y3}	0.0	0.0
E_k	0.0	0.0
E_m	0.0	0.0
E_r	2446.16	2467.61
ΔS	1298.09	466.41
ΔR	0.0	0.0
ΔL^m	87.78	88.85
ΔL^j	20.0	20.0
ΔL^s	0.0	0.0
I	10,000	10,000
\bar{E}	178224.38	173924.6
\bar{X}	1173.5 (P_1 & P_2)	1178.66 (P_1 & P_2)
	1374.44 (P_3)	1378.87 (P_3)
	1765.82 (P_4)	1773.68 (P_4)

Table 9

Solutions for Group B

(Additional Constraints: $I = 8,000$)

	Employment	Exports
	P_1 to P_4	P_1 to P_4
X_1^s	290.40	290.40
X_1^r	127.80	127.80
X_2	2238.39	2259.83
X_3	1250.00	1250.00
E_k	2.99	2.98
E_{y1}	0.0	0.0
E_{y2}	0.0	0.0
E_{y3}	0.0	0.0
E_r	0.0	0.0
E_m	0.0	0.0
E_f	2145.49	2166.93
ΔS	957.16	125.48
ΔR	0.0	285.33
ΔL^m	72.75	3.82
ΔL^j	27.00	20.0
ΔL^s	0.0	0.0
\bar{I}	8,000	8,000
\bar{E}	163942.88	156966.0
\bar{X}	1112.55 (P_1 & P_2)	1117.45 (P_1 & P_2)
	1325.44 (P_3)	1329.3057 (P_3)
	1668.72 (P_4)	1676.0145 (P_4)

Table 10

Solutions Set C

(Additional Constraints: $\bar{E} = 150,000$)

	Investment	Exports
	P_1 to P_4	P_1 to P_4
Y_1^s	300.35	43.1
Y_1^r	8.42	265.6
Y_2	255.62	255.62
Y_3	330.62	346.9
X_k	290.40	290.4
X_m	127.80	172.8
X_r	1944.83	2040.7
X_s	1250.00	1250.0
E_c	3.15	3.10
E_{y_1}	0.0	0.0
E_{y_2}	0.0	0.0
E_{y_3}	0.0	0.0
E_k	0.0	0.0
E_m	0.0	0.0
E_r	1851.93	1947.8
ΔS	624.30	0.0
ΔR	0.0	244.3
ΔL^m	58.07	62.9
ΔL^l	20.0	20.0
ΔL^s	0.0	0.0
\bar{I}	6047.33	6565.1
\bar{E}	150,000	150,000
\bar{X}	1053.66 (P_1 & P_2)	1073.2 (P_1 & P_2)
	1277.94 (P_3)	1293.982 (P_3)
	1574.25 (P_4)	1605.630 (P_4)

Table 11

Solutions Set C

(Additional Constraints: $\bar{E} = 180,000$)

	Investment	Exports
	P_1 to P_4	P_1 to P_4
Y_1^s	300.35	0.0
Y_1^r	8.42	308.8
Y_2	255.62	255.6
Y_3	437.99	457.0
X_k	290.40	290.4
X_m	127.80	127.80
X_f	2576.44	2688.4
X_s	1249.99	1250.0
E_c	2.82	2.76
E_{y1}	0.0	0.0
E_{y2}	0.0	0.0
E_{y3}	0.0	0.0
E_k	0.0	0.0
E_m	0.0	0.0
E_f	2483.54	2595.5
ΔS	1340.48	611.5
ΔR	0.0	285.3
ΔL^m	89.65	95.2
ΔL^l	20.0	20.0
ΔL^s	0.0	0.0
\bar{I}	10248.6	10847.6
\bar{E}	180,000	180,000
\bar{X}	1182.71 (P_1, P_2)	1205.34 (P_1, P_2)
	1383.49 (P_3)	1401.85 (P_3)
	1780.86 (P_4)	1817.13 (P_4)

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