A Note on the Fertilizer Ratio Concept

JOHN REUSS AND SAM JOHNSON III*

The concept of plant nutrient ratios has long been utilized by agronomists and appears to be particularly prevalent in Pakistan. Production experts often express concern that the nutrient ratios utilized by the farmers are not optimum for a particular crop [4]. These concerns are reflected in the press and are considered by those charged with planning production and imports. Agronomists interested in plant nutrients may thus find themselves designing experiments to determine the optimum ratios of plant nutrients for different crops grown under different conditions.

For the sake of simplicity we will limit this discussion to two nutrients, nitrogen and phosphorus and the resulting N/P ratios. This is appropriate because these fertilizer elements are the most widely used in Pakistan and crops often respond to their use. However, combinations involving other nutrients or crop inputs are subject to the same principles.

THE RATIO CONCEPT

First we must recognize that the concept of an optimum ratio does not simply refer to a single rate of N and P_2O_5. Suppose we recommend that 90 lb. N and 45 lb. P_2O_5 be applied. The N/P ratio of this dose is, of course, 2 : 1. If this ratio only applies to this single rate or dose the concept of ratio would be trivial. In order for N/P ratio to be a meaningful concept it must apply more generally over the range of rates that may be effectively utilized.

Thus there are certain principles inherent in the concept of an optimum ratio for a given crop that are often not understood or at least are not clearly stated. First, an optimum ratio requires that a generalized N—P response surface exists that is reasonably valid for the population of fields with which we are concerned. Secondly, the economic optimum ratio will depend on the relative price of the two inputs. Finally, the concept of an optimum ratio would place severe constraints on the geometry of the response surface. Let us first examine the effect of relative prices.

*The authors are Professor of Agronomy and Assistant Professor of Economics, Colorado State University, Water Management Project, Lahore, Pakistan.

ECONOMIC CONSIDERATIONS

The relationship between optimum ratio and relative price of the nutrients is a key concept. Suppose we have experimentally determined under a given set of conditions the combinations of N and P, applications that will result in a maize yield of 40 mds/acre. Plotted on a graph such as Fig. 1, these combinations constitute a contour line on the response surface, or what is known as an isoquant [2]. The two straight lines with negative slopes are isocost lines [2]. The broken line has a slope of —2 and all points along that line represent an equal total fertilizer cost if the price of N is twice that P_2O_5. The solid isocost line has a slope of —1 and all points on this line represent an equal total fertilizer cost if the price per lb. of N and P_2O_5 is the same.

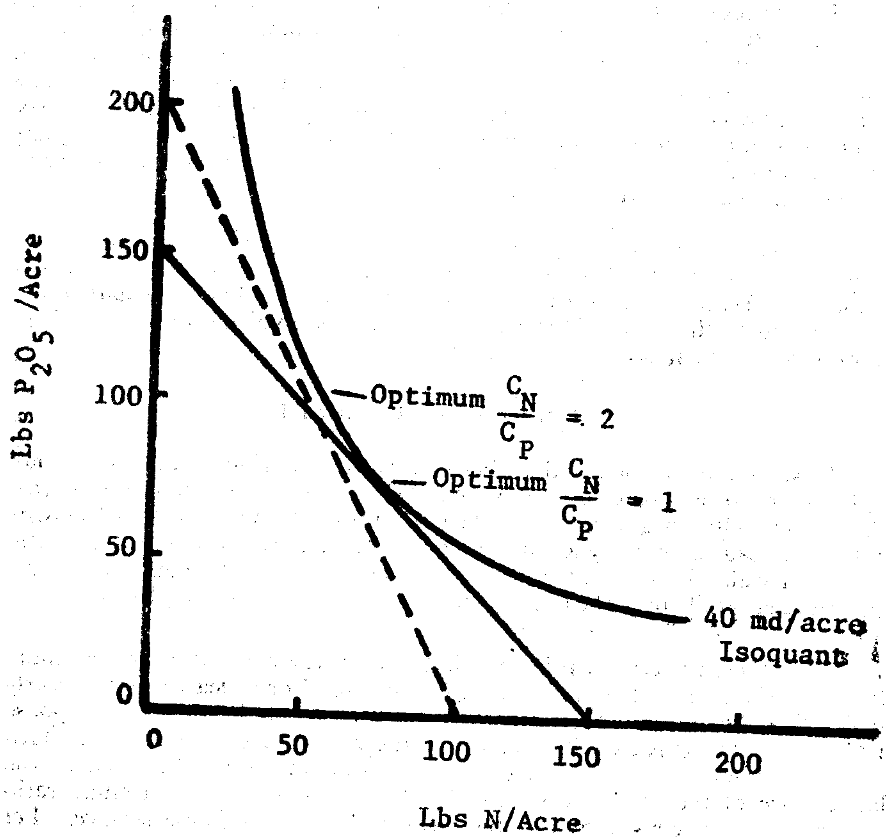

Figure 1

Illustration of determination of optimum combination of N and P for two different N/P price ratios for a sample 40 mds/acre isoquant

For any N/P cost ratio the least cost combination to produce 40 mds/acre occurs at the point where the slope of the 40 md. isoquant is equal to the N/P cost ratio. In mathematical terms this optimum is described as the point where:

$$\frac{dP}{dN} = \frac{C_N}{C_P} \qquad \text{.. .. (1)}$$

Where P and N are the ratio of nutrients applied at any constant yield and C_N and C_p are the costs per lb. of the two nutrients. Thus equation (1) states that the first differential of nitrogen applied with respect to phosphorus applied at constant yield is equal to the N/P cost ratio. At this point the optimum ratio of the two inputs is achieved. If these isoquants take the usual shape as shown in Fig. 1, the optimum ratios of N and P are obviously dependent on the relative N and P price. The previously stated principle, that optimum ratios are dependent on the cost price ratios of the inputs, arises from these relationships.

Next we shall examine the constraints these relationships impose on the nature of the N—P response surface if the concept of an optimum N/P ratio is to be valid. If this response surface is represented by yield contours on a graph such as that shown in Fig. 2, any straight line passing through the 0.0

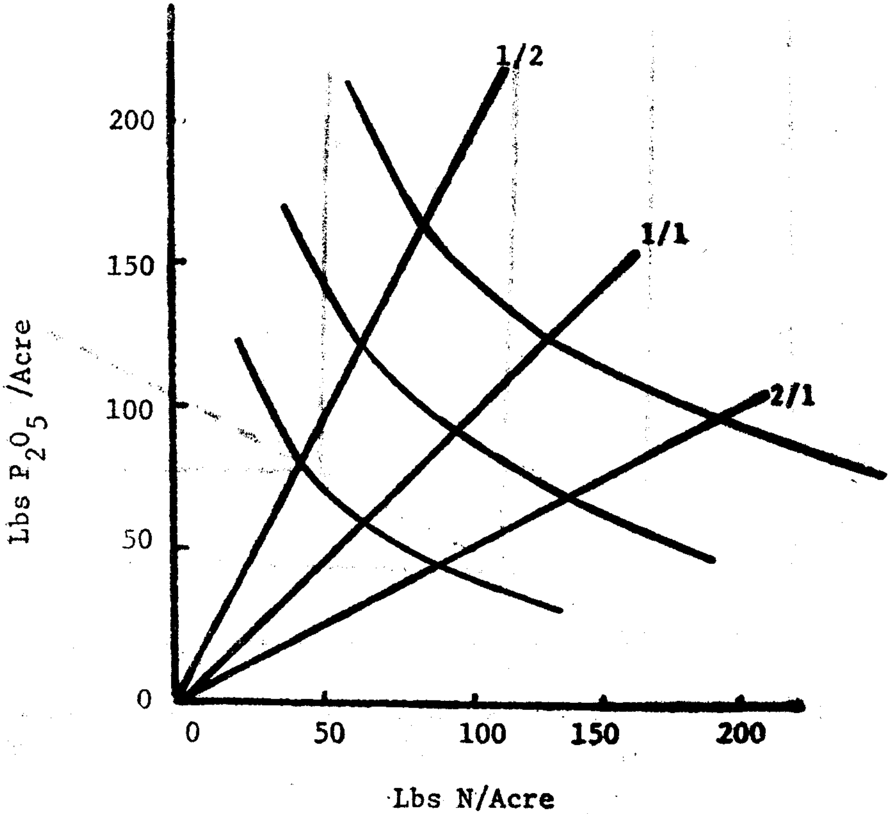

Figure 2
Graphically constructed yield isoquants that appear to satisfy the optimum ratio constraints over the range of N/P ratio from 1/2 to 2/1

point represents a fixed N/P ratio. For a complete validity of the optimum ratio concept, all isoquants (contours) that any such line might cross, must be crossed at the same slope. If the slope of the contours are not equal along any such line, it means that for the price ratio represented by these slopes the optimum ratio of N and P varies with the nitrogen (or phosphorus) cost, and the concept of an optimum ratio is not valid. These considerations give rise to the principle stated above relative to the very rigid constraints imposed on the geometry of the response surface by this concept. An interesting exercise would be to define these constraints in mathematical terms so that mathematical response surface models could be similarly constrained and the effect on goodness of fit examined with various data sets. This exercise is unfortunately beyond the scope of the present analysis.

One special case to be considered is the possibility of a response surface of the type shown in Fig. 3 [1]. If the response is of this type the two nutrients

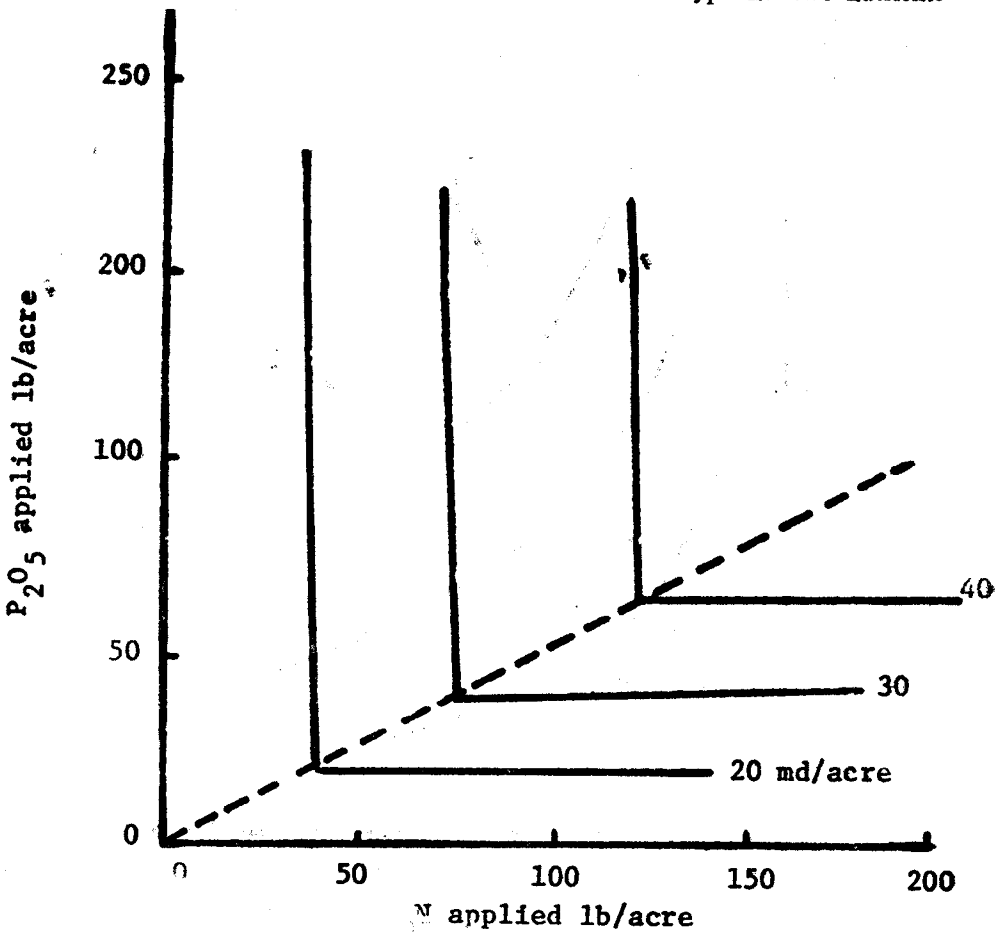

Figure 3

Yield isoquants for an N/P response surface for which no substitutes are allowed and the optimum N/P ratio is independent of rate applied.

cannot substitute one for the other. The particular surface shown in Fig. 3 assumes the bends in the yield isoquants occur at a constant N/P ratio represented by the dashed line. Even surfaces that do not allow substitution may not have this property, in which case the optimum ratio would not be independent of fertilizer rate.

Most published two nutrient response surfaces are not in fact of the type shown in Fig. 3, but are rather of the type where substitution dependent on price ratios is allowed. However, we must recognize that most published response surfaces have been determined by a least squares fit of mathematical models that would not allow the non-substitution type of surface. Even if the true surface were of this type a good fit might be obtained from one of the commonly used second order polynomials that would indicate that a moderate amount of substitution was permissible. This possibility should be considered in the interpretation of any data set. The model shown in Fig. 3 will be further discussed below.

It should also be emphasized that in an economic sense whenever the isoquants "bend back upon themselves" or have positively sloped segments they are no longer within the economic range of production [3]. The parallel dashed lines in Fig. 4, indicate the points at which the isoquants bend back upon

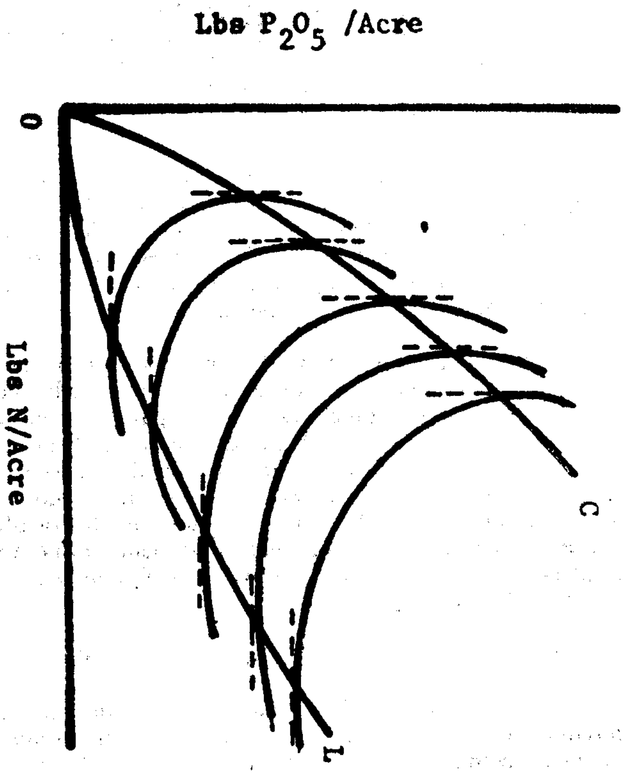

Figure 4
Isoquant map and the relevant range of production

themselves. The lines OC and OL join these points and form the boundaries for the economic region of production. These isoclines (lines, OC and OL) then define the boundaries outside of which our agronomic experiments have no relevancy in terms of economic optimum productions. Such regions are not uncommon in nutrient response surfaces, as when high nitrogen rates results in lodging and loss of yields in small grains.

Fortunately the ratio concept does not necessarily need to be strictly valid for all possible N/P cost ratios to be agronomically useful. The regular contours shown in Fig. 2 would probably meet the criteria sufficiently well to be useful in the range of N/P cost ratios from 1/2 to 2.

AGRONOMIC CONSIDERATIONS

For the moment let us reconsider the response surface shown in Fig. 3 that does not allow for substitution between the two nutrients. In fact this is the surface that would be generated if the system strictly follows one of the oldest concepts in plant nutrition, that of limiting factors. According to this concept if one factor or nutrient is limiting additional increments of other nutrients will not increase production. Published response surfaces typically have regions where substitution is effective and other regions where it is not. Thus, if one input is in sufficiently short supply it becomes a limiting factor. As mentioned above, it behooves us to be extremely careful in interpreting data from field experiments that we do not in fact create an area in which substitution is allowable by fitting a mathematical model that could not accommodate the surface shown in Fig. 3.

Considering the above discussion it seems anticlimatic to question the agronomic validity of the ratio concept. Many plants use the elements N and P in about a 10/1 ratio, or on the basis of N to P_2O_5 would be about 4.4/1. Since the efficiency of uptake is usually less for applied phosphorus than for nitrogen, particularly in the season of application, we often find recommended N/P ratios for non-leguminous crops of about 2/1.

One of the assumptions stated above that must be met for the concept to be valid is that there is a general response surface that will be valid for a particular crop over some geographic area or some particular group of soils. This includes not only the general form of the surface but that the initial levels of N and P, available to the crop without additional fertilizer should be reasonably constant. In practice they are far from constant and the apparent response surfaces obtained from different fields may be drastically different. Thus, the response surface defined from a field experiment will be that starting from some initial N and P values which we will call N_i and P_i. The actual surface we plot is in fact that of:

$$\text{Yield} = f\,[(N_i + N_a),\ (P_i + P_a)]$$

Where N_a and P_a are amounts applied. Thus from different fields we are examining different regions of the surface. As N_i and P_i are usually unknown, conclusions drawn concerning the optimum ratios or the validity of the ratio concept may vary drastically from site to site.

The above assumes that the response surface is the same across sites and only the origin varies. In fact the amount of fertilizer required to produce a given response may vary drastically on different soils. Consider for instance the phosphorus system which can be schematically represented by Fig. 5.

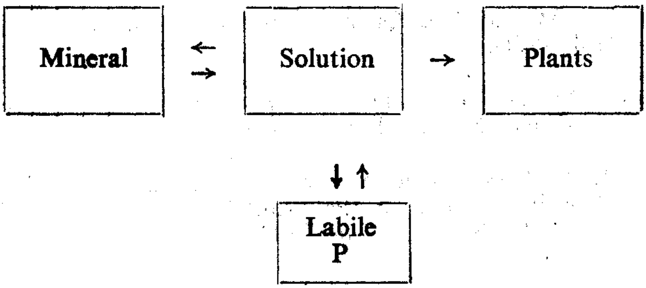

Figure 5
Schematic representation of the soil phosphorus system

Only a very small fraction of the total P in the soil in solution form at any one time. This is in equilibrium with a somewhat larger pool of so called labile P. This equilibrium is different in different soils. In some soils the labile pool may be relatively small but will maintain a relatively high solution concentration. This is adequate for plant needs until the pool is depleted, but this condition may occur rapidly. Thus, the capacity to supply P may be low, but it can be supplied at a relatively high intensity, *i.e.*, solution concentration. Such soils could be expected to give large response to a minimum application of P but the applied P would be rapidly depleted.

The equilibrium in other soils may be such that the labile pool must be very large before the solution concentration is sufficiently high to maintain maximum plant growth. Thus, these soils have a large capacity to supply P, but at low intensity. In these systems a large application would be required to obtain maximum response per unit of P applied to that crop would be small, and the response surface would appear quite different from that of the soils having a low P capacity associated with a high intensity.

This paper, of course, cannot cover all aspects of the supplying power of the soils for plant nutrients but should be sufficient to illustrate some of the technical and economic limitations of the ratio concept. Raising an alarm over the Pakistan farmers' improper fertilizer ratio not only serves no useful purpose but also illustrates a lack of knowledge of the technical-economic relationships involved. A blanket recommendation for the whole country such as a N/P ratio of 2/1 will not be valid for all areas or farmers.

Farmers in different stages of adoption of improved practices will be applying widely different rates. There is no sound basis for assuming that ratios should be the same for different doses. Actually most fertilizer experiments would have to be drastically redesigned in order to adequately test the ratios concept, even for fixed cost/price relationships.

REFERENCES

1. Ferguson, C. E. *Microeconomic Theory.* (Homewood, Ill.: Richard D. Irwin, 1972).

2. Henderson, J. M. and R. E. Quandt. *Microeconomic Theory.* (New York: McGraw-Hill, 1971).

3. Intriligator, M. D. *Mathematical Optimization and Economic Theory.* (Englewood Cliffs, N. J.: Prentice-Hall, 1971).

4. Salam, Abdul. "Socio-economic and Institutional Factors Influencing Fertilizer Use in the Punjab (Pakistan)." *The Pakistan Development Review.* Vol. XIV, No. 4. Winter 1975.