Short Run Forecasts of the Money Stock in Pakistan

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Until recently it was conventional to treat the money stock as a policy variable exogenously determined by the central bank of a country. However, the notion that the money stock is jointly determined by the central bank, the commercial banks, and the nonbank public has gained general acceptance among economists. Thus regarded as an endogenous variable, the problem of choosing that model of money stock determination which provides the best predictions of its values in the immediate future assumes importance. It is the purpose of this paper, then, to formulate alternative models of the Pakistani money stock process and determine which of them yields the best short-run predictions.

The existing models are of two basic types: money multiplier models of various degrees of sophistication, in which the money stock is obtained as the product of a stock of base money and an appropriate multiplier; and an empirical money supply equation used by Gibson [9]. These are discussed in first section of the paper.

In section second we develop our own models which are also money multiplier models. However, to take account of an institutional feature of the money supply process in Pakistan, in these models the money stock is obtained as the product of the sum of certain liquid assets held by the commercial banks (rather than a stock of base money) and an appropriate multiplier. Our reason for developing these models is that in Pakistan the commercial banks must observe not one but two required reserve ratios. Assets equal to a certain proportion of their deposits must be held as cash or as deposits with the central bank, the State Bank of Pakistan (SBP). The exact proportion, the conventional required reserve ratio, is a policy variable determined by the SBP. In this, Pakistan follows the practice of many other countries. But the SBP sets a

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second ratio.   A certain proportion of deposits, the "required liquidity ratio", must be held in the form of liquid assets: cash, deposits with the SBP, and government securities which have not been used as collateral for borrowing from the SBP.   Porter [12] has   shown that as long as the required liquidity ratio is greater than the conventional required reserve ratio, either the former or a deficiency of loan demand, rather than the latter will constitute the effective constraint on monetary expansion.   Thus, it is important to develop a money multiplier model utilizing the required liquidity ratio.   Such a model uses liquid assets as the scale variable.

Section third presents estimates of all the models and assesses their short term forecasting pereformance.

EXISTING MONEY STOCK MODELS

At the outset it is important to note that throughout this paper we have limited ourselves to models and forecasts of the narrowly defined money stock, defined as currency in the hands of the public plus demand deposit liabilities of commercial banks.   The money multiplier approach to determination of this narrowly defined money stock has been the stuff of money and banking textbooks for decades.[1]   The stock, M, is derived as the product of a stock of high-powered money or monetary base, MB, defined as currency in the hands of the public and bank holdings of reserves against their deposit liabilities, and a multiplier, m.

$$M = mMB \qquad .. \qquad .. \qquad (1)$$

Naive Money Multiplier Models

One approach to making *ex-post* forecasts of the money stock requires no analysis of the determinants of m, but merely predicts its value using an autoregressive scheme:

$$\hat{m}_{t+j} = \sum_{i=1}^{p} b_i m_{t-i+j} \qquad .. \qquad .. \qquad (2)$$

where t is the date of the forecast made for date $t+j$.   The predicted value of m is then substituted into equation (1) to obtain a prediction of the money stock. A very simple special case of equation (2) is the no-change multiplier in which b_1 is set equal to one and all other b's are set equal to zero:

$$\hat{m}_{t+j} = m_{t-1} \qquad .. \qquad .. \qquad (2a)$$

An alternative to this no-change multiplier is the same-percentage-change multiplier model which assumes equal percentage changes in the multiplier will occur in successive forecast periods:

$$\hat{m}_{t+j} = (1+\alpha_t)^j m_t \qquad .. \qquad .. \qquad (2b)$$

where $\alpha_t = (m_t - m_{t-1})/m_{t-1}$.   Another special case of the auto-regressive multiplier model, equation (2), was proposed by Burger, Kalish, and Babb [4].

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[1]As Clower [6] has pointed out that while it is customary to attribute the working out of the relation between bank reserves and bank deposits to C.A. Phillips, F.W. Crick, and A. Hahn, based on works published by these authors between 1920 and 1930, Marshall indicated in testimony before the Royal Commission on the Values of Gold and Silver [11] that he was aware of this relationship.

They set b_i equal to zero for i greater than three and constrained b_1, b_2, and b_3 to equality. They then estimated the common single value of these parameters anew each period as new data became available. Thus they implicitly restricted the value of j to one.[2]

Subsequently Burger [3] added as an independent variable the lagged percentage change in the treasury bill rate along with monthly dummy variables.

$$\hat{m}_t = \overset{*}{b}_0 + \overset{*}{b}_1 1/3 \left(\sum_{i=1}^{3} m_{t-1}\right) + \overset{*}{b}_2 (TB_{t-1} - TB_{t-2})/TB_{t-2}$$

$$+ \sum_{i=1}^{11} b^*_i +_2 D_i \qquad .. \qquad .. \qquad (2c)$$

where it is understood that the parameters b_i^* are estimated anew each month adding the data for the latest month and dropping the observation which is now thirty-seven months old.

In Burger's formulation, the money stock is obtained as the product of the *adjusted* monetary base (monetary base minus commercial bank borrowings from the central bank) and an appropriate multiplier, rather than as the product of the monetary base and a multiplier as in equation (1). This is because he was proposing a money stock control procedure and felt that the adjusted base is more likely to be under the control of the central bank than is the monetary base. However, for our purposes, it may well be that a version of (2c) relating to the monetary base rather than to the adjusted base gives better forecasts. Equations (2), (2a), (2b), and versions of (2c) using both the monetary base multiplier and the adjusted monetary base multiplier, then, will be utilized to obtain predictions of the multiplier which can be used to make *ex post* predictions of the money stock.[3]

Models Using the Determinants of the Money Multiplier

An alternative approach to money multiplier models deals with the economic determinants of the value of the multiplier, m. Early expositions of the approach implicitly assumed that the public wishes to hold no currency or time deposits, and that the banks desire to hold no excess reserves. In such circumstances the multiplier, m, is simply the reciprocal of the required ratio of reserves to deposits. Later authors, most notably Brunner and Brunner and Meltzer [1, 2], have greatly enriched the analysis. Typically their money multiplier equations (for the narrowly defined money stock) take the form

$$M = \frac{1+k}{(r+e)\ (1+t)+k}\ MB \qquad .. \qquad (3a)$$

and

$$M = \frac{1+k}{(r+e-b)\ (1+t)+k}\ MB^a \qquad .. \qquad (3b)$$

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[2]They also included on the right hand side of their estimating equation the reserve adjustment magnitude calculated by the Federal Reserve Bank of St. Louis, Burger and Rasche [5], and monthly dummy variables.

[3]A very elementary study to estimate the relationship between the money supply, and the monetary base was undertaken by Hamdani [10a] Several equations were tried for the period 1972 to 1974 to predict changes in the money supply.

where k is the public's desired ratio of currency to demand deposits, r is the required reserve ratio, e is the banks' desired excess reserve ratio, b is the banks' desired ratio of borrowings from the central bank to total deposits, t is the public's desired ratio of time deposits to demand deposits, MB is the monetary base, and MB^a is the adjusted monetary base (monetary minus commercial bank borrowing from the central bank).

Equations (3a) and (3b) are typical of what Brunner and Meltzer call their "nonlinear money supply hypothesis." Brunner's "linear money supply hypothesis" consists of writing the narrowly defined money stock, M, as a linear function of the appropriate parameters and appropriate measure of the base:

$$M = c_0 + c_1 k + c_2 r + c_3 e + c_4 t + c_5 MB \qquad \cdot \cdot \qquad (4a)$$

$$M = c_0' + c_1' k + c_2' r + c_3' e + c_4' t + c_5' MB^a + c_6' b \qquad (4b)$$

Adding error terms we obtain regression equations whose estimated coefficients can be used for forecasting purposes. While Brunner's derivation of the linear money supply hypothesis [1] is, in his own words, "admittedly tedious" [2, p. 248] and cannot be repeated here, it should be noted that much more is involved in the derivation of equations (4) than an arbitrary change in the functional form of equations (3). Nevertheless, it is possible to discuss the expected signs of the coefficients of the former equations by examining the latter. If the monetary base and adjusted monetary base are independent of the parameters of the multiplier,[4] then changes in r, e, and t are inversely related to changes in M, changes in b are directly related, and changes in k are inversely related unless (r+e—b) (1+t) and/or (r+e) (1+t) are greater than one. Changes in the monetary base and adjusted monetary base, *Ceteris Paribus* are of course directly related to changes in M.[5] Equations (4a) and (4b) will be estimated and used to forecast the Pakistani money stock.

The Gibson Model

Gibson [9] has used an equation in which the money supply depends on the total reserves against deposits held by the banks and on two interest rates. One measures banks' return and the other measures their costs.

$$M = a_0 + a_1 TR + a_2 r + a_3 r_d \qquad \cdot \cdot \qquad \cdot \cdot \qquad (5a)$$

$$M = a_0 + a_1 TR + a_2 (r - r_d) \qquad \cdot \cdot \qquad \cdot \cdot \qquad (5b)$$

[4]The only parameter for which this might not be true is B. If the central bank is successfully miantaining a policy-determined value of MB, then a change in b will change MB^a. On the other hand, if the policy-determined variable is MB^a then it is presumably independent of changes in b. Considerations of this sort were first raised by the De Leeuw and Kalchbrenner [8].

[5]It is possible that changes in the parameters of the multiplier do not change the value of MB or MB^a, as discussed in the previous footnote, and yet changes in MB or MB^a, through their effects on interest rates, do change the Values of some of the parameters of the money multipliers, so that the *cet. par.* qualification of this sentence is not valid empirically. After conducting an extensive survey of empirical studies of the U.S. monetary mechanism regarding this matter, Rasche observed that "the evidence suggests quite conclusively that the short run feedbacks through interest rate changes, which would be generated by policy changes in reserve aggregates are very weak" [13, p. 19]. For the Pakistani economy, in which a market determined interest rate on government securities does not exist, it is hard to see how interest rate feedbacks could be a problem.

In equation (5b) the second independent variable is explicitly the interest rate banks earn minus the interest rate they pay, while in equation (5a) these two interest rates are allowed to have separate coefficients. These equations will be estimated and used to predict the Pakistani money stock.

MODELS BASED ON THE REQUIRED LIQUID ASSETS RATIO

The Linear Model

As explained in the introduction, commercial banks in Pakistan must not only hold a certain proportion of their deposits as reserves but must also hold another proportion as liquid assets: cash, deposits whith the SBP, or government securities not pledged as collateral. And Porter [12] has shown that as long as the latter proportion is higher than the fromer, the latter proportion will be the binding constraint on monetary expansion. Teherefore it is important to develop a money multiplier model in which the money stock is obtained as the product of the total of these liquid assets, L, and an appropriate multiplier.

$$M = m_L L \qquad \qquad (6a)$$

We now derive the appropriate expression for m_L. Deposits, D, are the sum of demand deposits and time deposits:

$$D = DD + TD$$

The money stock is the sum of currency in the hands of the public and commercial banks' demand deposit liabilities:

$$M = C^p + DD$$

The total of liquid assets, L, can be expressed as:

$$L = (l+e')\ (DD+TD)$$

where l is the required liquidity ratio and e' is the banks' desired ratio of excess liquid assets to total deposits. Then

$$m_L = \frac{M}{L} = \frac{DD+C^p}{(l+e')\ (DD+TD)}$$

Dividing the numerator and denominator of the right hand side of this equation by DD, we get

$$m_L = \frac{1+k}{(l+e')\ (1+t)}$$

so

$$M = \frac{1+k}{(l+e')\ (1+t)}\ L \qquad \qquad (7a)$$

This is the analogue of Brunner and Meltzer's nonlinear money supply hypothesis, equations (3). The analogue of the linear hypothesis, equations (4) is:

$$M = d_0 + d_1k + d_2l + d_3e' + d_4t + d_5L. \qquad .. \qquad (8a)$$

Unfortunately, the SBP did not begin reporting a single figure for L on a quarterly basis before 1967. We attempted to calculate L from the published data on assets of the banking system but our constructed series did not agree with data on L published after 1967. Therefore, let

$$L' = L + L''$$

where L' is our constructed series, L is the correct but unobservable series and L'' is the difference between the two
Let.

$$L'' = e''(DD+TD)$$

where e'' is the banks' desired ratio of the subset of assets, L'', to deposits. Then L' can be used as the scale variable in a money multiplier formulation:

$$M = m_L'L'. \qquad .. \qquad .. \qquad (6b)$$

An expression for m_L' can be derived exactly as equation (7a) was derived from equation (6a):

$$M = \frac{1+k}{(l+e'+e'')(1+t)} L' \qquad .. \qquad . \qquad (7b)$$

The analogue Brunner and Meltzer's linear hypothesis is:

$$M = d_0' + d_1'k + d_2'l + d_3'(e'+e'') + d_4't + d_5'L' \qquad (8b)$$

Adding an error term we obtain a regression equation whose estimated coefficients can be used for forecasting purposes. If we simply proceeded as before we would use equation (7b) to evaluate the expected signs of the coefficients in equation (8b). The expected signs of the coefficients of k and L' would be positive, and those of l, $(e'+e'')$, and t negative.

An Alternative Formulation

We will, in fact, estimate the parameters of equation (8b), and use it to forecast the Pakistani money stock. However, it is necessary to note that the expected positive sign on k in equation (8b) surely should run counter to one's intuition. Why does an increase in the public's desired currency-deposit ratio lead to an expected increase in the money stock when it is elementary

that an increased leakage of monetary base into currency leads to a decrease in the money stock? The paradox is resolved when we note that the coefficient d'_1 is a partial derivative. It tells us the effect on the money stock of a given change in k, all other variables, in particular L', held constant. While it may have been appropriate in discussing equations (3) and (4) to assume that the monetary base and/or adjusted monetary base will not change when the parameters composing the multiplier change (footnote 4), this assumption is much shakier here. In particular L' will be constant when k rises only if the central bank obligingly puts more currency into circulation as if it were manna from heaven. A more reasonable assumption is that when k rises, the banks must oblige the public by reducing L'.

As an alternative to equation (8b), therefore, we obtain equation (9) by logarithmic differentiation of equation (7b):

$$\frac{dM}{M} = \Sigma\epsilon\, \frac{m_L'}{j}\, \frac{dj}{j} + \frac{dL'}{L'} \qquad j = k, l, (e'+e''), t, \;\; .. \quad (9)$$

where

$$\epsilon_k = \frac{m_L'}{\partial k} \cdot \frac{k}{m_L'} = \frac{k}{1+k} \qquad\qquad .. \qquad\qquad .. \quad (9k)$$

$$\epsilon_l = \frac{m_L'}{\partial l} \cdot \frac{l}{m_L'} = -\frac{l}{(l+e'+e'')} \qquad .. \qquad\qquad .. \quad (9l)$$

$$\epsilon_{(e'+e'')} = \frac{m_L'}{\partial (e'+e'')} \cdot \frac{(e'+e'')}{m_L'} = -\frac{(e'+e'')}{(l+e'+e'')} \quad .. \quad (9e)$$

$$\epsilon_t = \frac{m_L'}{\partial t} \cdot \frac{t}{m_L'} = -\frac{t}{(1+t)} \qquad .. \qquad\qquad .. \quad (9t)$$

The percentage change in the money stock equals the sum of percentage changes in the parameters of the multiplier, j, each percentage change multiplied by the elasticity of the multiplier with respect to that parameter, $\epsilon_j m_L'$ plus the percentage change in the stock of liquid assets, dL'/L'. Since our basic reason for proposing equation (9) as an alternative to equation (8b) is that L' (and hence changes in it) cannot be assumed independent of the parameters, j, we must express dL'/L' as a function of these parameters. We begin by noting that from previous definitions:

$$L' = (l+e'+e'')\ (DD+TD)$$
$$DD = \frac{1}{k} C$$
$$TD = tDD.$$

Substituting the latter two relations into the first and collecting terms gives:

$$L' = (l + e' + e'') (\frac{1+t}{k}) C.$$

For ease of exposition define:

$$l' = (l + e' + e'') (1 + t) \qquad .. \qquad .. \qquad (10)$$

so that:

$$L' = \frac{l'}{k} C. \qquad .. \qquad .. \qquad (11)$$

Taking the total differential of (11) we get:

$$dL' = \frac{l'}{k} dC + Cd\frac{l'}{k}$$

As discussed above, we assume that when (for example) k rises, the banks provide the additional currency by reducing their liquid assets. Specifically assume that for each additional rupee in the hands of the public, the banks' holdings of liquid assets decline by a rupee:

$$dC = -dL'.$$

Thus:

$$dL' = Cd \frac{l'}{k} - \frac{l'}{k} dL'$$

$$= C \left[\frac{1}{k+l'} dl' - \frac{l'}{k(k+l')} dk \right]$$

Making use of (11) and dividing by L' we get:

$$\frac{dL'}{L'} = \left[\frac{1}{k+l'} dl' - \frac{l'}{k(k+l')} dk \right] \frac{k}{l'}$$

and substituting (10) for dl' gives:

$$\frac{dL'}{L'} = \left[\frac{1}{k+l'} d[(l+e'+e'')(1+t)] - \frac{l'}{k(k+l')} dk \right] \frac{k}{l'} \quad .. (12)$$

It is tedious but straightforward to manipulate equation (12) to show that:

$$\frac{dL'}{L'} = \sum_j \varepsilon \frac{L'}{j} \frac{dj}{j} \qquad .. \qquad .. \qquad (13)$$

where $j = k, l, (e' + e''), t$ and

$$\varepsilon \frac{L'}{k} = -\frac{\partial L'}{\partial k} \cdot \frac{k}{L'} = -\frac{k}{k + l'} \qquad \cdots \quad (13k)$$

$$\varepsilon \frac{L'}{l} = \frac{\partial L'}{\partial l} \cdot \frac{l}{L'} = \frac{l\,k}{(k + l')\,(l + e' + e'')} \qquad (13l)$$

$$\varepsilon \frac{L'}{(e' + e'')} = \frac{\partial L'}{\partial (e' + e'')} \cdot \frac{(e' + e'')}{L'} = \frac{(e' + e'')\,k}{(k+l')\,(l+e'+e'')} \quad (13e)$$

$$\varepsilon \frac{L'}{t} = \frac{\partial L'}{\partial t} \cdot \frac{t}{L'} = \frac{t\,k}{(k + l')\,(1 + t)} \qquad \cdots \quad (13t)$$

Substituting equation (13) into equation (9) will yield:

$$\frac{dM}{M} = \sum_j \varepsilon_j \frac{dj}{j} \qquad \cdots \qquad \cdots \qquad (14)$$

where

$$\varepsilon_j = \varepsilon_j^{m_L'} + \varepsilon_j^{L'}$$

The percentage change in the money stock is now given by the sum of percentage changes in the multiplier parameters each weighted by the appropriate elasticity. What are the signs of these elasticities? Comparing (9k) with (13k) we see that the multiplier elasticity, (9k) will always be smaller in absolute value as long as l, is less than unity. Therefore the negative sign of (13k) prevails and we obtain the intuitive result that an increase in k, *cet. par.*, reduces the growth rate of the money stock in contrast to the expected positive sign of the coefficient, dl' in equation (8b). The latter, it will be recalled, was our motivation for developing the alternative approach embodied in equations (9)-(14). A comparison of the other elasticities in the set (9) with those in the set (13) reveals that they are linked by the relation:

$$\varepsilon_j^{L'} = -\varepsilon_j^{m_L'} \left(\frac{k}{k + l'} \right)$$

Since $k/(k+l')$ is less than unity it is apparent that the multiplier elasticities will be greater than the liquidity elasticities in absolute value. Hence, the negative signs of the former cause the signs of the corresponding ε_j's to be negative.

It would be possible to use regression techniques to estimate the elasticities in equation (14). However, the use of regression commits the investigator to the maintained hypothesis that the unknown values of the parameters he is estimating have remained constant over the sample period. Since we have derived the analytical expressions (9k), etc., it is easy to see that this hypothesis is untenable. An alternative is to use the analytical expressions to calculate the values of the elasticities period by period, calculate their means and standard deviations, substitute the means in equation (14) and use it to forecast percentage changes in the money stock. Rather than using the mean values of the elasticities we may also try the values of the elasticities prevailing during the period in which the forecast is made.

Since, in any application, the variables in equation (14) will be discrete percentage changes, the analytical expressions for the elasticities, (9k)—(9t) and (13k)—(13t), which are point elasticities based on infinitesimal changes are not strictly appropriate. Rather, two sets of analytical expressions for the elasticities when discrete changes are involved were derived[6]. One set uses the orginal values of the variables as the base from which to measure percentage changes:

$$\varepsilon_{j1}^{i} = \frac{\Delta i}{\Delta j} \cdot \frac{j_{t-1}}{i_{t-1}} \qquad .. \qquad .. \qquad (15a)$$

$$\text{for } j = k, l, (e' + e''), t \text{ and}$$
$$i = m_{L'}, L'$$

The other set uses the new values of the variables as the base from which to measure percentage changes for each j and i:

$$\varepsilon_{j2}^{i} = \frac{\Delta i}{\Delta j} \cdot \frac{j_{t}}{i_{t}} \qquad .. \qquad .. \qquad (15b)$$

Using, alternatively, the definitions (15a) and (15b) in equation (14), then yields four alternative models:

$$\frac{dM}{M} = \Sigma \bar{\varepsilon}_{j1} \frac{dj}{j} \qquad .. \qquad .. \qquad (16a)$$

$$\frac{dM}{M} = \Sigma \bar{\varepsilon}_{j2} \frac{dj}{j} \qquad .. \qquad .. \qquad (16b)$$

$$\frac{dM}{M} = \Sigma \varepsilon_{j1, t-1} \frac{dj}{j} \qquad .. \qquad .. \qquad (16c)$$

$$\frac{dM}{M} = \Sigma \varepsilon_{j2, t-1} \frac{dj}{j} \qquad .. \qquad .. \qquad (16d)$$

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[6]The expressions were derived using analogues of equations (9) and (12) in first difference form.

$$j = k, l, (e^l + e^r), t$$

$$\varepsilon_{jh} = \varepsilon_{jh}^{{}^{m}L'} + \varepsilon_{jh}^{L'} \quad h = 1, 2$$

Models (16a) and (16b) use the means of the calculated elasticities to forecast the percentage change in the money stock while models (16c) and (16d) are similar to the Burger model (2c), in that the values of the elasticities used in making the forecasts are revised each period. In fact, they are the calculated values of the previous quarter's elasticities.

The forecasts of percentage changes are converted into forecasts of levels in order that these models can be compared with all the others, which predict levels of the money stock.

RESULTS

Estimates of Coefficients

In this section we first report and discuss the coefficients of those forecasting equations estimated by regression methods or calculated utilizing the analytical expressions for elasticities derived from the first difference analogues of equations (9) and (12).[7] We then examine the quality of the forecasts provided by our diverse collection of money supply models.

To assess the degree to which any or all of our equations are plagued by short-run instabilities, we adopted the method used by Goldfeld in a recent major study of the U.S. demand for money function [10]. The set of coefficients in each equation was estimated (or calculated, in the case of equations (16a) and (16b) over four sample periods, each starting in 1961:1 and differing in that the terminal point was systematically moved from the end of 1967 to the end of 1971, in steps of four quarters. Based on the estimates or calculated values obtained for each sample period, each equation was dynamically simulated for the next four quarters. The only exceptions were the Burger model, equation (2c), which, following his procedure, was reestimated each quarter, and equations (16c) and (16d) in which forecasts of the percentage change in the money stock were obtained utilizing actual values of the relevant elasticities lagged one quarter.

The first model discussed in the first Section was the auto-regressive money multiplier, equation (2). However, in four of the five samples, the coefficient of the one-period lagged multiplier was not significantly different from one, and in all samples, the coefficients of higher order lagged multipliers were not significantly different from zero. Thus, we conclude that for the Pakistani economy, the no-change multiplier model, equation (2a), is a reasonable empirical approximation to the general autoregressive model, and we do not report the estimates of the latter or use them to obtain *ex post* forecasts. The no-change multiplier model and the equal-percentage-change multiplier model, equation (2b), involve no estimation or calculation of coefficients to be reported here. The Burger model, equation (2c), was, as noted at the end of the previous

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[7]The data in this study are quarterly observations running from 1961:1 to 1971:4. All observations were taken from various issues of the *Annual Report on Currency and Finance* and of *Annual Banking Statistics*, both published by the State Bank of Pakistan.

paragraph, estimated over sixteen separate sample periods. Furthermore, two versions were estimated in each sample period, one relating to the adjusted monetary bases and one relating to the monetary base, so there are thirty-two separate sets of regression coefficients. We have chosen not to report this voluminous set of results since our purpose in the present discussion is to convey a succinct impression of the degree of stability of the coefficients in the various models, as we vary the end point of the sample period. We also do not report the calculated values of the elasticities for the sixteen quarters 1967.4 to 1971.3 utilized in predicting percentage changes in the money stock according to models (16c) and (16d).

Tables 1 through 5, then, report the coefficients associated with the remaining models. In Tables 1 through 4, the variable S_6 is a seasonal dummy whose value is unity in quarters one through three and is zero in quarter four. Our maintained hypothesis that the only systematic seasonal disturbance occurs in the fourth quarter was dictated by the very striking quarterly behaviour of the money stock series. There is always a dramatic rise in the money stock during the fourth quarter. In the same tables, the parameter ρ is the estimated first-order correlation coefficient of the disturbances. Estimates of ρ were obtained by the use of the Cochrane-Orcutt procedure for dealing with serially correlated disturbances [7].

Table 1 presents the estimated coefficients of equation (4a), the Brunner-Meltzer linear money supply model with the monetary base. The model clearly fits the Pakistani data very well. All coefficients except that of the excess reserve ratio have the expected sign and in most cases are highly significant. The incorrect positive sign on the excess reserve ratio is never significant. The seasonal dummy is generally not significant, suggesting that the model fits so well without it, that it is not needed. However, the progressive addition of each year's quarterly observations does seem to cause the values of the coefficients to change appreciably.

Our estimates of equation (4b), the Brunner-Meltzer linear money supply model with the adjusted monetary base, rather than the monetary base itself, produced money stock predictions inferior to those produced by the estimtes obtained for equation (4a), just discussed, and are not reported here. Tables 2 and 3 present the estimated coefficients of equations (5a) and (5b) respectively, the Gibson model. While this model appears to fit the Pakistani data much less well, the seasonal dummy accounting for most of the high value of R^2, we should mention that when the models were estimated without adjusting for serial correlation of the disturbances, the estimated coefficients of the reserves and interest rate variables were generally significantly different from zero (although the estimated coefficient of r_d also had the incorrect positive sign in that case, a result obtained by Gibson, in his study, as well). The problem was that the values of the Durbin-Watson statistic were unsatisfactorily low. Furthermore, we found that the models adjusted for serial correlation gave better predictions of the money stock so we have chosen not to report either the coefficients of the unadjusted models, which would look better than those in Tables 2 and 3, or the predictions given by those models, which would be worse than those obtained using the coefficients in these tables. Again, it seems clear that the progressive addition of each year's quarterly observations causes

Table 1

Summary Statistics for Equation (4a) for Alternative Sample Periods

End Point	Constant	MB	k	t	r	e	S_5	ρ	DW	R^2	SE
				Coefficients[a]							
1967	2950.5*	1.6*	—1510.5*	—312.9	— 4953.5*	313.2	—38.2	.63	1.61	.99	55.8
	(9.7)	(42.5)	(12.7)	(1.6)	(3.4)	(0.1)	(1.6)				
1968	3481.4*	1.6*	—1857.9*	—417.6	— 7958.9*	2103.3	—40.0	.68	2.19	.99	99.6
	(6.7)	(26.3)	(9.5)	(1.3)	(3.3)	(0.3)	(1.0)				
1969	4025.1*	1.7*	—2066.8*	— 763.0*	—10382.9*	1237.2	—59.6	.67	2.12	.99	119.6
	(7.1)	(26.6)	(9.3)	(2.2)	(3.8)	(0.2)	(1.3)				
1970	4428.5*	1.7*	—2200.3*	— 730.7*	—12 595.3*	5985.5	—89.9*	.53	2.52	.99	130.7
	(7.4)	(32.7)	(9.6)	(2.2)	(4.2)	(0.7)	(1.8)				
1971	6693.1*	1.7*	—3059.9*	—1785.7*	—16197.8*	—16335.2	14.1	.22	2.00	.98	412.4
	(4.4)	(17.7)	(5.1)	(2.9)	(1.8)	(0.7)	(0.1)				

[a]The figure in parentheses is the t-ratio.
*Coefficient significantly different from zero at the 5 percent level.

Table 2

Summary Statistics for Equation (5a) for Alternative Sample Periods

End Point	Coefficients[a]					ρ	DW	R²	SE
	Constant	TR	r	r_d	S_s				
1967	5775.0	1.5	394.9	115.4	—337.6*	.94	1.90	.96	354.8
	(0.3)	(1.5)	(0.7)	(0.3)	(3.1)				
1968	—4287.6*	2.0	1415.7*	638.5	—356.0*	.62	1.76	.95	406.8
	(2.0)	(1.5)	(3.7)	(1.5)	(2.6)				
1969	—5396.0*	2.0	1665.8*	547.4	—355.4*	.63	1.82	.96	426.0
	(2.6)	(1.5)	(4.8)	(1.3)	(2.6)				
1970	—6144.5*	2.0	1833.0*	485.9	—403.2*	.57	1.78	.97	431.7
	(3.3)	(1.5)	(6.4)	(1.2)	(3.1)				
1971	—8709.9*	0.2	2378.2*	467.9	—367.5*	.64	1.81	.97	539.1
	(3.7)	(0.1)	(7.4)	(0.9)	(2.3)				

a The figure in parentheses is the t-ratio.
*Coefficient significantly different from zero at the 5 percent level.

Table 3

Summary Statistics for Equation (5b) for Alternative Sample Periods

End Point	Coefficients[a]				ρ	DW	R²	SE
	Constant	TR	$(r-r_d)$	S_s				
1967	9537.1* (5.3)	1.6 (1.7)	55.3 (0.2)	—324.1* (3.1)	.95	1.85	.95	349.8
1968	11829.0* (5.1)	1.3 (1.3)	48.7 (0.2)	—350.8* (3.2)	.96	1.85	.95	379.1
1969	12768.8* (5.1)	1.4 (1.3)	81.7 (0.2)	—376.8* (3.5)	.97	1.89	.96	398.9
1970	14357.0* (4.8)	1.5 (1.3)	151.5 (0.4)	—435.9* (3.9)	.97	1.85	.97	431.1
1971	17262.5* (4.7)	0.1 (0.1)	103.9 (0.2)	—431.0* (3.3)	.97	2.0	.97	517.1

[a]The figure in parentheses is the t-ratio.
*Coefficient significantly different from zero at the 5 percent level.

appreciable change in the estimated coefficients. This was equally true in the estimates of the model unadjusted for serial correlation of the disturbances.

Table 4 presents estimates of equation (8b), the liquidity regression model. This model appears to fit the data almost as well as the Brunner-Meltzer linear model. As explained in section second, although the positive coefficient on k runs counter to our intuition, it is what we should expect if we fail to take account of the dependence of L^I on k, as, indeed, we fail to do in a single equation linear regression. The positive seasonal dummy is a little disturbing but perhaps it, too, can be attributed to the inappropriate presence of L^I in the regression. There do not seem to be as many instances of noticeable shifts in the estimated coefficients when another year's quarterly observations are added to the samples, as occurred in both the Brunner-Meltzer and the Gibson models.

Finally, Table 5 presents the means and standard deviations of the elasticities calculated according to first difference analogues of equations (9) and (12). In all cases the mean values of the elasticities are at least twice the standard deviations. In sharp contrast to the coefficients in Tables 1 to 4, these coefficients display remarkable stability as the sample size increases in steps of four quarterly observations. Also, the figures shown in columns 1 to 4, which are means of elasticities calculated according to (15a), in which the previous period's values are used as the base to measure percentage changes, are virtually identical to their counterparts in columns 5 to 8, which are means of elasticities calculted according to (15b), in which the current period's values are used as the base.

Prediction Tests

We now turn to a discussion of the relative efficacy of our various models in making short-term predictions of the money stock. Goldfeld's procedure was to use a set of estimates for a given period to predict the money stock over each of the next four quarters. In making these predictions, *ex post*, he used the realized (rather than predicted) values of the independent variables. He then calculated the root mean square error of the predictions for the four quarters. The procedure was repeated for each sample, where, as we have seen, each sample differed from the previous one by the addition of another year's quarterly observations.

Goldfeld, of course, was only dealing with a single model. We followed him in using realized rather than predicted values of the independent variables. However, his procedure of calculating the root mean square error of the four quarterly predictions for each year was not very helpful for our purposes of comparing models. It gave us four sets of rankings of mean square error statistics, one relating to the predictions for each of the years 1968 to 1971. None of the models was consistently near the top or the bottom of the rankings. Models that had the lowest root mean square error of prediction for the four quarters of 1968 were not among those with the lowest root mean square errors of prediction for the four quarters of the other years, etc.

A more reasonable procedure for our purposes was to calculate separately the root mean square errors of each model's first quarter forecasts, second

Table 4

Summary Statistics for Equation (8b) for Alternative Sample Periods

End Point	Coefficients[a]							ρ	DW	R^2	SE
	Constant	L'	l	$(e'+e'')$	k	t	S_δ				
1967	5880.4* (15.1)	4.2* (25.0)	−26140.9* (15.5)	−19673.6* (15.5)	3729.5* (12.7)	−4844.9* (8.9)	151.8* (2.6)	.27	2.00	.99	117.7
1968	5828.9* (15.9)	4.2* (26.8)	−25584.6* (15.8)	−19415.0* (16.6)	3638.2* (13.2)	−4743.7* (10.0)	153.7* (2.8)	.28	2.10	.99	116.3
1969	8411.7* (11.8)	3.4* (19.7)	−25595.9* (7.9)	−22112.8* (15.6)	2480.9* (8.3)	−3740.0* (8.0)	39.1 (0.8)	.69	2.00	.99	127.9
1970	6190.2* (9.6)	3.6* (29.8)	−24359.6* (8.3)	−20731.6* (10.8)	3634.7* (8.0)	−3718.6* (8.1)	31.4 (0.3)	.19	1.81	.99	203.4
1971	5743.5* (5.3)	3.6* (27.4)	−23407.8* (4.9)	27364.7* (11.1)	5020.0* (7.8)	−4708.4* (7.2)	256.8* (1.9)	.25	1.73	.98	355.3

[a]The figure in parentheses is the t-ratio.
*Coefficient significantly different from zero at the 5 percent level.

Table 5

Means and Standard Deviations of the Elasticities in Equations (14a) and (14b) for Alternative Sample Periods

End Point	ε_{k1}	ε_{l_1}	$\varepsilon_{(e'+e'')1}$	ε_{t1}	ε_{k2}	ε_{l_2}	$\varepsilon_{(e'+e'')2}$	ε_{t2}
1967	—0.10 (0.04)	—0.18 (0.05)	—0.14 (0.03)	—0.14 (0.03)	—0.08 (0.38)	—0.19 (0.06)	—0.14 (0.03)	—0.15 (0.03)
1968	—0.10 (0.04)	—0.20 (0.07)	—0.14 (0.03)	—0.15 (0.03)	—0.09 (0.04)	—0.21 (0.07)	—0.14 (0.03)	—0.16 (0.04)
1969	—0.10 (0.04)	—0.21 (0.07)	—0.14 (0.03)	—0.16 (0.04)	—0.09 (0.03)	—0.22 (0.07)	—0.14 (0.03)	—0.16 (0.04)
1970	—0.10 (0.04)	—0.22 (0.07)	—0.14 (0.03)	—0.16 (0.04)	—0.09 (0.04)	—0.22 (0.07)	—0.13 (0.03)	—0.16 (0.04)
1971	—0.10 (0.05)	—0.23 (0.07)	—0.13 (0.03)	—0.16 (0.04)				

quarter forecasts, third quarter forecasts, and fourth quarter forecasts over the four years. We would again have four separate sets of rankings, but here knowledge of differences in rankings would be more helpful. It is much more useful to know that model A is far superior to model B in making predictions one quarter ahead (say) while the opposite is true for predictions of what will occur four quarters ahead, than it is to know that model A predicted 1968 better and model B predicted 1969 better.

Table 6, then, presents rankings of the models according to the root mean square error of prediction criterion. As noted earlier, all predictions obtained with the Burger model and with equations (16c) and (16d), designated in the Table as Liquidity-Elasticity (c) and (d) respectively, were based on the values of coefficients lagged only one quarter, so that all forecasts obtained with these models represent forecasts one period ahead. For all other models, the first quarter forecasts represent forecasts one period ahead, the second quarter forecasts two periods ahead, etc. As one would expect, the root mean square errors tend to increase for forecasts further into the future. Root mean square errors of forecasts obtained with the Burger model using the *monetary* base are reported in Table 6. Except for the fourth quarter, the root mean square errors of forecasts obtained with the Burger model using the adjusted base were much higher than for any of the models in Table 6, and we have chosen not to report them.

It appears from Table 6 that there is little to choose between the Brunner-Meltzer model and the linear version of the liquidity model. The Brunner-Meltzer model comes in first quarter forecasts and second in second and fourth quarter forecasts, but ninth in third quarer forecasts. The linear version of the liquidity model comes first in second quarter forecasts, second in third quarter forecasts, and fourth in first quarter forecasts, but eighth in fourth quarter forecasts. It should be noted that with respect to third quarter forecasts it is second only to the liquidity-elasticity model, equation (16d), whose mean square error is for a prediction only one quarter ahead.

The performance of the liquidity-elasticity models is comparatively good for first and third quarter predictions but comparatively poor for second and fourth quarter predictions. Of all the models these are the only ones which did not allow for seasonal variation. Failure to take account of the marked increase in the Pakistani money stock, which occurs annually in the fourth quarter, provides a plausible explanation for the poor fourth quarter performance of these models. The high root mean square errors of their second quarter forecasts are entirely attributable to very bad second quarter forecasts in 1971. The error of second quarter forecasts in the other three years is less than 1 percent in ten out of twelve cases, and is less than 3 percent in the other two cases. However, the predictions of all four models for the second quarter of 1971 are in error by between 9 and 10 percent. This raises their root mean square errors of prediction for the second quarter above that of the no-change multiplier model.

In general, the Gibson model's performance is quite poor. The model in which the absolute values of the coefficients of the two interest rates are not constrained to equality [Gibson (a)] performs consistenlty poorly. This is

Table 6

Root Mean Square Errors for Predictions One, Two, Three and Four Quarters Ahead (Billions of Rupees)

Rank	First Quarter Predictions		Second Quarter Predictions	
	Model	RMSE	Model	RMSE
1	Brunner-Meltzer	.10	Liquidity-Linear	.24
2	Liquidity-Elasticity (b)	.20	Brunner-Meltzer	.29
3	Liquidity-Elasticity (d)	.26	Gibson (b)	.43
4	Liquidity-Linear	.27	Burger	.42
5	Liquidity-Elasticity (a)	.29	No Change Multiplier	.43
6	Liquidity-Elasticity (c)	.30	Liquidity- Elasticity (d)	.74
7	Burger	.31	Liquidity-Elasticity (a)	.75
8	No-Change Multiplier	.48	Liquidity-Elasticity (c)	.76
9	Gibson (a)	.66	Liquidity-Elasticity (d)	.77
10	Same % Change Multiplier	.92	Same % Change Multiplier	.83
11	Gibson (b)	1.11	Gibson (a)	1.50

Rank	Third Quarter Predictions		Fourth Quarter Predictions	
	Model	RMSE	Model	RMSE
1	Liquidity-Elasticity (d)	.09	Gibson (b)	.48
2	Liquidity-Linear	.50	Brunner-Meltzer	.52
3	Liquidity-Elasticity (a)	.57	Burger	.56
4	Liquidity-Elasticity (b)	.59	No-Change Multiplier	.58
5	No Change Multiplier	.80	Liquidity-Elasticity (d)	1.01
6	Gibson (b)	.83	Liquidity-Elasticity (a)	1.03
7	Same % change Multiplier	1.12	Gibson (a)	1.04
8	Liquidity-Elasticity (d)	1.20	Liquidity-Linear	1.07
9	Brunner-Meltzer	1.32	Liquidity-Elasticity (c)	1.09
10	Gibson (a)	1.35	Liquidity-Elasticity (b)	1.15
11	Burger	1.46	Same % Change Multiplier	1.67

perhaps not surprising, since, as noted, we duplicated Gibson's finding of an incorrect positive sign on the discount rate. The model in which the absolute values of the coefficients of the two interest rates were constrained to equality [Gibson (b)] performs comparatively well in its second and fourth quarter predictions but poorly in its first and third quarter predictions, exactly the opposite pattern of the liquidity-elasticity models. And its good fourth quarter predictions can plausibly be attributed to the same thing that caused the poor fourth quarter predictions of the latter models. The annual behaviour of the Pakistani money stock is characterized by a marked seasonal increase in the fourth quarter. While the liquidity-elasticity models do not take this behaviour into account, the seasonal dummy contains most of the explanatory power in the estimates of the Gibson model and thus exerts a strong influence on its predictions. This point is supported by the fact that the fourth quarter forecasts of the unconstrained Gibson model, in which the seasonal dummy also provides most of the explanatory power, while comparatively poor, are appreciably better than its second and third quarter predictions.

The naive multiplier models give mediocre predictions although the comparative performance of the Burger model's second and fourth quarter predictions are respectable. With the exception of third quarter predictions, where it comes in dead last, the Burger model is superior to the no-change multiplier model and the latter is superior to the same-percent-change multiplier model.

To sum up, there is no clear winner. It appears that the Brunner-Meltzer, Gibson, linear liquidity, and liquidity-elasticity (d) models should each be used in making a different quarter's prediction. Alternatively the Gibson and liquidity-elasticity (d) models could be eliminated and the Brunner-Meltzer model used for first and fourth quarter predictions and the linear-liquidity model for second and third quarter predictions.

CONCLUSIONS

We have reviewed a number of existing models of the money supply process and have built two models of our own to depict the workings of this process in Pakistan. The parameters of all these models were estimated, and the estimated models were used to obtain out-of-sample *ex-post* predictions of the money stock, one, two, three, and four quarters ahead. We concluded that the Brunner-Meltzer linear money supply hypothesis and an analogous model derived from a relationship between the stock of money and the stock of banks' holdings of liquid assets provided the best short-term predictions of the Pakistani money stock. Since the latter model, while not applicable to the money supply process in the United States, is applicable to the money supply process of a number of other countries, it would be worthwhile to estimate its parameters with data from those countries and compare its predictive performance with that of the existing models of the money stock reviewed in the first part of this paper.

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