Some Comparative Aspects of Production and Profit Functions: Empirical Applications to a Punjab District

MIAN M. ASLAM*

The estimation of technical coefficients of production is a prerequisite for the construction of an agricultural production model. The production functions, introduced by the classical school and developed extensively by the neo-classical writers, have been frequently used in deriving the technical coefficients. But the problems posed by the management factor [8] and simultaneous determination of variable inputs and outputs by the farm firm [5, 10] are quite serious and have generally led to biased estimates. On the other hand, the profit function [14, 19] has been developed as an alternative to the production function. Since the arguments of this function are normalized input prices and fixed inputs which are exogenously determined, the bias of simultaneous equations is avoided. However, the profit function may be difficult to estimate due to data problems. For example, what is the wage rate of agricultural labour and how is it imputed? Also, data on product prices are not easily accessible and some products may not be marketed at all, especially, in developing countries.

This paper is aimed at comparing the results of the two approaches and elaborating the empirical difficulties in the use of the alternative approach of profit functions. The study is based on the data from the agriculturally developing area of Faisalabad Pakistan. The methodological implications of this paper are not only relevant to Pakistan but also to other countries which are at a stage of agricultural development similar to that of Pakistan.

---

*The author is a Lecturer in Farm Management at the University of Agriculture, Faisalabad. This paper is based on the author's unpublished Ph.D. dissertation which was submitted to the University of Hawaii and was funded by the East-West Centre. The author has benefitted from the guidance and suggestions of Professor Peter V. Garrod, Gary R. Vieth, Perry F. Philipp, Frank S. Scott, Jr. and James A. Silva of the University of Hawaii. The author alone, however, is culpable for any errors and views expressed in the paper.

THE SAMPLE AND SURVEY DESIGN

The study is based on a farm survey conducted in Faisalabad District. From the Punjab, Faisalabad was selected on the basis of being likely to provide typical data for the least cost. Wheat, cotton and sugarcane are the dominant crops of this district. From the district of Faisalabad, four villages located at a distance of about six to eight miles from Gojra, were selected. The selected area had no special government projects like Salinity Control and Reclamation Projects and Integrated Rural Development Centres. Canal irrigation is commonly supplemented by sweet water from private tubewells located along the canal. A preliminary survey of these villages was conducted to obtain information from each farmer regarding area owned, area rented out, area rented in and sources of irrigation. The farming population was stratified into two strata according to size of land holding. A farm was considered small if it had less than or equal to 12.5 acres, otherwise large. A random sample of eighty cases was selected using proportionate allocation. Nine cases were dropped as they were either incomplete or inconsistent resulting in a usable sample of 71 farms. The sample comprised 21 large farms (30 percent) and 50 small farms (70 percent). The sample average size of land holding is 14.8 acres. The data were collected on a pretested questionnaire. The data pertain to the year, 1974-1975.

THEORETICAL FRAMEWORK AND EXPLANATION OF VARIABLES

The decision making process underlies the production of goods and services. The decision maker has available several processes by which "output" is produced from certain other specified elements [7]. The decision maker then selects the appropriate "process" or "processes" based on his own criterion. A generic statement of a process pertaining to agriculture could be:

$$[Q(t), R(t), I(t), M(t), W(t); L(t), K(t), H(t)]$$

where Q, R, I ,M, W denote flows of products, natural resources manufactured articles, maintenance supply and waste, L, K and H represent the services of Ricardian land, capital and labour and t stands for time.

Basic theory assumes that for a given product, the representation of all elementary processes form some surface in the corresponding functional space; the equation of this surface expressed in terms of the product coordinate is the production function:

$$[Q(t) = R(t), I(t), M(t), W(t); L(t), K(t), H(t)]$$

This function is functional, it relates functions to functions, not numbers to numbers. This is an important distinction; the fact that a production function relates functions to functions and not just things is crucial to the understanding of formation of outputs from inputs. An example would be the role of fertilizer in the production of beef. The amount of beef produced per acre is partially a function of the amount of grass per acre which is in turn partially a function of the amount and type of fertilizer applied per acre, *i.e.*, the output, beef, is functionally related to the quantity and quality of an input, fertilizer, through a series of functional relationships, not *via* a single process which is an obvious misspecification of the relationship. In the case of a multiproduct farm firm, one ends up by having to deal with not just a single production equation but with a system of several equations in which many of the variables interact and are endogenously determined.

McFadden, Lau and Yotopoulos [14, 15, 18, 19, 26] developed the theory and Cobb-Douglas formulation of "unit-output-price" profit function and demonstrated that production coefficients can be derived from U.O.P. profit functions. Since profit functions involve only normalized input prices and the quantities of fixed inputs, variables that are exogenously determined, the simultaneous equations bias is avoided if management does not affect the technical coefficients of other factors.

The derivation of the profit function is based on the assumption of profit maximizing behaviour by the farm firm, and many economists harbour suspicions regarding this tenet of rational behaviour with regards to traditional and modern agriculture. Fine examples of the examination of firm behaviour are given by Lin, Dean, Moore and Nurdin [16, 21].

The Variables

The variables used in the production and profit functions are defined and explained as follows:

OPT_i = Total physical output of ith enterprise. The output is measured in pounds in the case of sugarcane, cotton, maize and wheat. The outputs of fodders are measured in tons.

LND_i = Area under the ith crop on a farm measured in acres.

MLB_i = Man days of labour used on ith enterprise. This includes family labour, permanent hired labour and casual hired labour. Child labour is converted into man-equivalents by treating two children equal to one man. The labour input was measured by two methods:

(1) Four groups of experienced farmers, one from each village, were selected. Each group consisted of five farmers. They were asked; how much time will it take to perform a given operation of an enterprise, assuming an average output per acre of the village. The summation over all operations yielded a measure of labour used.

(2) The data on frequency and intensity of various farm operations performed on each crop were collected from each farmer. Questions regarding the number of labourers (including family, casual hired and permanent hired labourers) and their time devoted to an operation were also asked. Labour time spent on individual farm operations was summed over all operations of an enterprise to obtain total labour input. This measure was checked against (1) and used as an explanatory variable in the regression equation.

BLB_i = A measure of flow of bullock power going into the production of the ith crop. This measure was obtained in the form of days in the same manner as in MLB_i (2).

TWR_i = Hours of water used on the ith crop on a farm. This includes both canal and tubewell water. This variable was determined by using the following arithmetic expression:

$$TWR_i = (LND_i * NCI_i * TAC_i) + (LND_i * NTI_i * TAT_i)$$

where

LND_i = Defined already.

NCI_i = Number of canal irrigations given to a crop.

TAC_i = Hours of time used to irrigate one acre with canal water.

NTI_i = Number of tubewell irrigations given to a crop.

TAT_i = Hours of time used to irrigate one acre with tubewell.

This approach is justified on the ground that along a water course, land is uniformly levelled and water courses and tubewells are of the same capacity in the sample area. The drawback of this method is that it does not take into account the farm to farm variability in depth of irrigation, changes in water flow due to breaches, quality differences in canal and tubewell water. This approach may not be the most accurate and may have caused aggregation bias in the estimates. However, quantity of irrigation water used in a particular crop cannot be accurately measured in a survey method. Data on total number of irrigations applied to a particular crop did not show significant variation among farms. Accessibility to tubewell water and a more or less regular canal water flow may be responsible for this lack of variation. Moreover, farmers were unable to tell depth of each irrigation given to each crop and, therefore, acre inches of water could not be used. Farmers had fairly good knowledge of time taken to irrigate an acre of his ith crop, hence, this variable was used as a basis for the calculation of irrigation hours. Similar discharge of tubewells and water courses in the area also lend support to this measurement procedure.

NTN_i = Pounds of available nitrogen used on the ith crop. This includes nitrogen applied in the form of chemical fertilizers such as Urea and Diammonium phosphate.

CTL_i = A measure of flow of capital services plus cash expenses on seed, farm yard manure and chemical fertilizers going into the production of the ith crop. A flow of services is derived for each durable input that is used in farming. It includes depreciation charges, interest charges and maintenance expenses. The rates of depreciation for different capital items were adopted from Chaudhary *et al.* [3] and used for the calculations in this study. Interest costs were estimated at a uniform interest rate of 12 percent per annum. The asset-specific amount of depreciation, interest and operating expenses were allocated to individual crops on the basis of hours for which a capital item was used on a particular crop. The cash expenses on seed, farm yard manure and fertilizer are included in CTL_i because, for a given range of output, these expenses do not vary widely and hence their separate representation in the production equation may not capture the explained variation significantly.

CWN_i = CTL_i minus expenses on chemical fertilizers. This is an alternative form of capital which does not include expenses on chemical fertilizers.

PFT_i = Profit from ith enterprise which is defined as total revenue minus costs of seed, farm yard manure, plant protection means and materials, chemical fertilizers, tubewell water and payments to hired labour. However, payments to family labour are treated as part of profits. Land rent and bullock labour have been assumed as fixed cost items and have not been deducted from total revenue to arrive at profit. Canal water rates are also assumed as fixed expenses. Neoclassical theory maintains that fixed variables drop out in the profit maximization decision equation.

MWR_i = Money wage rate per day for the ith crop. This is derived from the sum of payments (cash plus kind) to casual hired labour and permanent labour used on the given enterprise divided by the number of labour days used. The money value of payments to permanent labour were apportioned to enterprises on the basis of labour time spent on different enterprises. Thus, wage rate was calculated and used in the profit equation only in case of crops for which either permanent, casual or both types of labour were hired.

P_i = Price of output of ith enterprise per unit.

RWR_i = MWR_i/P_i; real wage rate per day.

D_i = Dummy variable for farm size taking the value of 1 if cultivated land 12.5 acres and zero otherwise.

$MLBA_i = MLB_i/LND_i$

$BLBA_i = BLB_i/LND_i$

$TWRA_i = TWR_i/LND_i$

$NTNA_i = NTN_i/LND_i$

$CTLA_i = CTL_i/LND_i$

$CWNA_i = CWN_i/LND_i$

SPECIFICATION OF PRODUCTION AND PROFIT FUNCTIONS

Production Functions

The production function may be defined as the relationship between the variable productive services, and the output under the assumption that durable inputs do not vary during the time period considered, *e.g.*, there exist a given stock (fund) or building, machine and hand tool, services that are not used up in the production process. This means that an empirical production function is defined in relation to a given set of durable inputs. Assuming a firm producing a single commodity, this relationship may be expressed in a mathematical form:

$$Y_i = F(X_{1i}, \ldots, X_{ji}, \ldots, X_{mi}) \quad (i = 1, \ldots, n \text{ and } j = 1, \ldots, m)$$

where Y_i denotes total quantity of output obtained by a firm from ith enterprise, X_{ji} stands for the quantity of the ith variable productive service used by a firm on the ith enterprise.

In empirical studies, it is noted generally that some points in a given sample lie on the estimated line expressing the functional relationship while others lie off the line. In other words, there are substantial differences of output among firms using the same variable factors. That is, some or all of the assumptions in the foregoing paragraph may be violated, and errors of sampling and measurement confound the understanding of these differences. Thus, it may not be sufficient to specify the behavioural function as a relation between inputs and outputs. Some other explanatory variables which are immeasurable or are difficult to measure, may have to be specified. Also, the relevant assumptions may have to be made. In this situation, the following assumptions are made:

(1) It is assumed that perfect competition prevails, farmers' input supply and output demand functions are infinitely elastic. This may be taken as a fair approximation of the farming situation in Pakistan as farmers do not have any influence in either the input or product markets.

(2) The spread of productive technology at any point in time differs from firm to firm. These differences are not incorporated. This may be a limitation of the methodology.

(3) The differences in physical environment are assumed to be captured by the error term.

(4) Farm firms are different with regard to the possession and exercise of managerial ability. Given the same technology, farmers may have varied success in its use in production. One factor, among others, that impinges upon this success is farm size. Farm size is indeed an important variable and has been incorporated by dummy variables. In this study, a dummy variable which takes the value of one if farm size is greater than 12.5 acres and zero otherwise was used.

Are single equation estimates consistent? Hoch's assumptions [10] provide the justification for the single equation estimation. A more rigorous statement of these assumptions in the context of Cobb-Douglas production function has been furnished by Zellner, Kmenta and Dreze [27]. Instead of "anticipated profit" they used a more exact term of mathematical expectation of profit. They assumed that: entrepreneurs maximize the mathematical expectation of profit, *i.e.*, they know that the production function is stochastic;

and input prices are known with certainty or are statistically independent of the production function disturbance. They maintain the premise, "...whenever the production process is not instantaneous, the effect of the disturbance on output cannot be known until after the pre-selected quantities of inputs have been employed in production". They have shown that inputs do not depend on the disturbance in the production function. The argument is that disturbances in production function result largely from "acts of nature" such as weather conditions and machine performance. Errors in profit maximization equations are "human errors". The two are uncorrelated, hence, single equation estimates are consistent and that under the normality assumption they are unbiased.

How does the above set of assumptions apply to this study? The production environment of the present study (Faisalabad, Pakistan) is not different from the specific requirements of the studies cited above. There is a lag between input application and production, and the sample farmers have their inputs predetermined independently of the forthcoming weather changes or other such disturbances. Thus, the set of assumptions described above holds good for this study as well.

The Algebraic Form of Production Functions

Many empirical studies have employed the Cobb-Douglas form of production function. This algebraic model is computationally feasible and has provided adequate fit of data more often than do other forms [4]. However, one point needs consideration in the choice of Cobb-Douglas form; that is, will such a function represent conditions of production correctly? The answer to this question requires a discussion of substitution possibilities between different inputs. The Cobb-Douglas form requires the assumption that the elasticity of substitution between any pair of inputs is unity. It seems desirable to examine the empirical value of the elasticity of substitution before adopting this form of production function. Hayami [9] used inter-country cross-section data and found the results consistent with unity elasticity of substitution. Lau and Yotopoulos [15] fitted Indian data to a CES production function directly with non-Linear methods and found that the elasticity of substitution was not significantly different from one. Sidhu [23] used four years data regarding wheat production in Indian Punjab (this area is approximately similar to the sample area of this study) and fitted a CES production function. The results indicate that the hypothesis that the Cobb-Douglas form represents the data adequately cannot be rejected. The cited evidence indicates that this form should be appropriate for this study. Hence, the general form of Cobb-Douglas production function was used and may be written as:

$$Y_i = A^{m}_{j=1} X_{ji}\, j \exp^{(d+u)} \quad (i = 1, \ldots, n \text{ and } j = 1, \ldots, n)$$

where Y_i is total physical output, A represents the intercept term and S_{ji} is the amount of input j used by a firm on ith enterprise. $_j$ is the elasticity of output with respect to input j. The error has been decomposed into two parts; a measure of neutral variation in technical efficiency among farms which will be captured by the coefficient of dummy variable, d, and the residual term, u, which is the rendom disturbance term independently distributed with a zero mean and a finite variance. This approach assumes that there are no non-neutral differences in the respective technologies.

Using this functional form results in computational difficulties when sample data contain zero observations. The logarithmic transformation for zero values is not defined in real space. Johnson and Rausser [12] have shown that replacement of zero values by a small positive constant results in a small bias in estimated parameters. Accordingly, zero observations were replaced by unity in this study.

Profit Functions

The generalized theory of UOP profit function was developed by Mc-Fadden, Lau and Yotopoulos [14, 19]. It is readily available in the literature and will not be presented in this paper. However, UOP profit function may be written as:

$$\frac{PFT}{p} = \Pi^* = G^* (C_1, \ldots, C_h; f_{h+1}, \ldots, f_m) \quad (J=1, \ldots, m)$$

where PFT stands for total profit from an enterprise, p is the price of the output per unit, Π^* is the normalized profit, C_j is the normalized price per unit of the jth variable input and f_j stands for the jth fixed input.

The Estimated Equation of Profit Functions

It is not deemed necessary to present the Cobb-Douglas approximation of the profit function as it can be readily found in the literature [14]. However, an explanation of the estimated equations is in order.

First note, that by definition:

$$LN\ (PFT/p) = LNA + \sum_{j=1}^{h} \beta_j\, LN\, C_j + \sum_{j=h+1}^{m} \theta_j\, LN\, f_j$$

$$= LNA + \sum_{j=1}^{h} \beta_j\, LN\, (C_j'/p) + \sum_{j=h+1}^{m} \theta_j\, LN\, f_j$$

where A is the technical efficiency parameter, β_j is the coefficient of normalized price of the jth variable input and θ_j is the coefficient of the jth fixed input. Other variables were defined in the previous section it follows that:

$$LN\ PFT = LN\ A + \sum_{j=1}^{h} (1-\beta_j)\, LN\, p + \sum_{j=1}^{h} \beta_j\, LN\, C'_j$$

$$+ \sum_{j=h+1}^{m} \theta_j\, LN\, f_j$$

and if there is only one variable input, then:

$$\text{LN PFT} = \lambda + \text{LN } C^{j} + \sum_{j=2}^{m} \theta_j \text{ LN } f_j$$

where $\lambda = \text{LN A} + (1-\beta) \text{ LN p}$

The last equation is the form that will be used in estimating the actual equations. Using the coefficients of the respective profit functions, the estimates of the input elasticities were obtained by following the procedure described in Lau and Yotopoulos [14].

The specification bias arising from the ommission of the management factor was taken care of by the approach indicated previously for the production function, *i.e.*, it is assumed that management inputs are related to farm size.

Assumptions in Estimating Profit Functions

Water, animal power and chemical nitrogen have been treated as fixed inputs in the estimation of profit function equations. The price of canal water is fixed by the government and does not vary from farm to farm for a unit quantity of water used on a crop. The price of tubewell water per hour when adjusted for capacity, exhibits no significant variation, too. The absence of variation in the price of water precludes its use as a variable input in the profit equation. The case of chemical nitrogen is similar; its price is also controlled by the government. Even the transportation cost of chemical nitrogen did not cause any variation in its price from farm to farm as the sample villages are located approximately at the same distance from the supply town. Bullock labour does not have any price tag. The opportunity cost of bullock labour is hard and difficult, if not impossible, to determine. Thus, as a computational alternative, these inputs are assumed as fixed inputs. These assumptions are relevant in the context of the sample of the study.

EMPIRICAL RESULTS

The Production Functions

The results from the ordinary least squares regressions linear in natural logrithms are presented in Table 1. In all the functions estimated, the size of the adjusted coefficient of determination[1] (R^2) suggests that a major proportion of the interfarm variation in output of these crops is explained by the included expalanatory variables. Except in the case of maize, all output elasticities of the functions do not have expected signs; some are positive while others are negative. Land input has a negative coefficient in the cases of sugarcane and wheat which are also the functions in which bullock labour was included as an explanatory variable. Bullock labour is related to land size and this collinearity may have been responsible for these wrong signs. In general, it is

[1]The adjusted coefficient of determination is a more conservative estimate of the percent of variance explained than the unadjusted co-efficient.

and if there is only one variable input, then:

$$\text{LN PFT} = \lambda + \text{LN } C^{l} + \sum_{j=2}^{m} \theta_j \text{ LN } f_j$$

where $\lambda = \text{LN A} + (1-\beta) \text{ LN p}$

The last equation is the form that will be used in estimating the actual equations. Using the coefficients of the respective profit functions, the estimates of the input elasticities were obtained by following the procedure described in Lau and Yotopoulos [14].

The specification bias arising from the ommission of the management factor was taken care of by the approach indicated previously for the production function, *i.e.*, it is assumed that management inputs are related to farm size.

Assumptions in Estimating Profit Functions

Water, animal power and chemical nitrogen have been treated as fixed inputs in the estimation of profit function equations. The price of canal water is fixed by the government and does not vary from farm to farm for a unit quantity of water used on a crop. The price of tubewell water per hour when adjusted for capacity, exhibits no significant variation, too. The absence of variation in the price of water precludes its use as a variable input in the profit equation. The case of chemical nitrogen is similar; its price is also controlled by the government. Even the transportation cost of chemical nitrogen did not cause any variation in its price from farm to farm as the sample villages are located approximately at the same distance from the supply town. Bullock labour does not have any price tag. The opportunity cost of bullock labour is hard and difficult, if not impossible, to determine. Thus, as a computational alternative, these inputs are assumed as fixed inputs. These assumptions are relevant in the context of the sample of the study.

EMPIRICAL RESULTS

The Production Functions

The results from the ordinary least squares regressions linear in natural logrithms are presented in Table 1. In all the functions estimated, the size of the adjusted coefficient of determination[1] (R^2) suggests that a major proportion of the interfarm variation in output of these crops is explained by the included expalanatory variables. Except in the case of maize, all output elasticities of the functions do not have expected signs; some are positive while others are negative. Land input has a negative coefficient in the cases of sugarcane and wheat which are also the functions in which bullock labour was included as an explanatory variable. Bullock labour is related to land size and this collinearity may have been responsible for these wrong signs. In general, it is

[1]The adjusted coefficient of determination is a more conservative estimate of the percent of variance explained than the unadjusted co-efficient.

Table 1

Estimates of Production Functions for Selected Crops, 1974-1975, *Faisalabad, Pakistan*

Crops		N	Intercept	Coefficient of				
				D_i •	LND_i	MLB_i	BLB_i	TWR_i
Sugarcane		66	2.918	—0.041 (0.095)	—0.122 (0.247)	0.759** (0.177)	0.225 (0.148)	0.163 (0.204)
Cotton		64	4.397	—0.013 (0.095)	0.292 (0.185)	0.681** (0.125)	—	—0.113 (0.160)
Maize		45	5.427	—0.046 (0.118)	0.377 (0.284)	0.295* (0.147)	—	0.204 (0.232)
Wheat	(1)	69	3.472	—0.162* (0.072)	—0.375 (0.189)	0.570** (0.157)	0.483** (0.128)	0.244* (0.116)
	(2)	69	3.731	—0.112* (0.078)	—0.423 (0.195)	0.626** (0.162)	0.465** (0.133)	0.324** (0.117)
'Rabi' Fodder		71	2.110 —	0.276** (0.096)	0.498* (0.196)	0.319** (0.077)	—	0.039 (0.151)
'Kharif' Fodder		71	1.868	0.206** (0.075)	0.581** (0.097)	0.426** (0.056)	—	0.018 (0.077)

—*Continued*

Table 1—*Contd.*

Crops		Coefficient of				F-ratio	Function Co-efficient	t Value
		NTN_i	CTL_i	CWN_i	R^2			
Sugarcane		0.079* (0.034)	—	0.016	0.922	110.02	1.120	0.029
Cotton		0.070** (0.023)	—	0.005 (0.032)	0.933	146.81	0.935	—1.413
Maize		—	0.034 (0.062)	—	0.796	34.26	0.910	0.04
Wheat	(1)	—	0.143** (0.043)	—	0.958	256.51	1.065	—
	(2)	0.040* (0.016)	—	0.005 (0.037)	0.954	203.03	1.037	0.008
'Rabi' Fodder		—	—0.002 (0.071)	—	0.853	81.10	0.854	—
'Kharif' Fodder		—	—0.065 (0.045)	—	0.905	133.65	0.960	—

Notes: Regressions linear in natural logrithms are estimated by ordinary least squares.
Standard errors of coefficients are in parentheses.
Intercept is in natural logarithms of output. The output of sugarcane, cotton, maize and wheat is measured in pounds. The output of fodders is measured in tons.

R^2 is the coefficient of determination adjusted for degrees of freedom.

t value is calculated to test the assumption of constant returns.

— means variable not included in the equation or the statistic not calculated.

MTN_i was used as an explanatory variable in those crops in which it showed a positive and statistically significant coefficient. In such cases, CWN_i measure of capital was used. MTN_i was dropped from those equations in which it showed negative and non-significant coefficient and CTL_i form of capital was included. This has been done with a view to point out the importance of MTN_i in the production of important crops like sugarcane, cotton and wheat.

*Significant at 95 percent level.
**Significant at 99 percent level.

logical to expect a high correlation between land input and other factors of production as there exists a sympathetic relationship between land and other inputs. In cross-sectional data, this phenomenon makes the interpretation of the estimated coefficients very difficult for structural or behavioural purposes.

Do the production functions show the same coefficients for the two classes of small and large farms? To answer this question, separate regressions for small and large farms were estimated for sugarcane, cotton and maize. Another regression of all farms without the dummy variable was also estimated. The SSEs from these regressions and the one from the corresponding equation in table 1 were used to perform the analysis of covariance and calculate the F-statistics [13]. The hypothesis of homogeneity of slopes with respect to various inputs in separate regressions of small and large farms was not rejected at 95 percent level of significance, provided the intercept term is allowed to differ, for sugarcane and maize. In the case of cotton, this hypothesis was rejected at the same level of significance.

The significance of differences in intercepts between farm size classes was tested, assuming constant slopes for all classes. The F-statistics were calculated for sugarcane, cotton and maize. The comparison of these values with the corresponding table F-values revealed that none of them is significant at the 95 percent level. This provides the evidence for accepting the hypothesis that the intercepts are the same between small and large farms growing these crops.

It can be observed from the estimated regression equations that, except in the case of fodders, all other crops show a negative coefficient for farm size. This implies that small farms have a higher intercept term than large farms. This indicates that the small farmers are better managers than the large farmers. However, the coefficient of farm size variable was significantly different from zero in the case of wheat only where the intercept for small farms was found to be 3.09 percent higher than that of large farms. The reason why small farms produce more than large farms cannot be pinpointed exactly as a host of factors, in addition to physical inputs, determine output. One plausible explanation may be that the small farmer has a small, usually self-operated area and thus he can pay greater attention to such factors as land preparation, time of sowing and intercultures, etc. Contrary to this, the large farm is dependent on hired labour which is either insufficient or too poorly supervised to do farm jobs effectively and on time. In addition, small farms operate with relatively cheap labour (since the opportunity cost of family labour in a rural sector is low), relatively expensive capital (because of imperfections in the capital market) and by definition a small piece of land. A large farm has more land, the farmer has access to credit on more favourable terms and can hire labour in the market at the going wage rate. These characteristics explain the differences in inputs and outputs. In the extreme cases of excess labour on small farms and excess land on large farms, the maximization rule would lead the former to maximize output per unit of land, the latter to maximize output per unit of labour.

The assumption of constant returns to scale was tested using the t distribution [13]. The computed t values for four major crops are given in table 1. None of the calculated t values is significant at the 95 percent level, thus, the hypothesis of constant returns to scale cannot be rejected in these enterprises. One possible reason why the hypothesis of constant returns is not rejected may be the large variances of the coefficients which may have resulted from the collinearity phenomenon. However, the assumption of constant returns is a reasonable assumption and has not been rejected in many studies; one of them is by Sidhu [24].

The assumption of constant returns to scale was imposed on the production function estimates of sugarcane and wheat. The land is assumed as fixed input and its output elasticity is restricted to zero. All other inputs are considered as variable inputs and the sum of their elasticities is constrained to one. The results are shown in Table 2. As compared to the estimates given in table 1 the coefficient of bullock labour has dropped significantly and that of manual labour has increased slightly in the case of sugarcane. In the case of wheat, the coefficients manual labour, bullock labour and irrigation water decreased significantly as compared to their corresponding estimates in table 1.

The Per Acre Production Functions

In order to overcome the problem of collinearity of independent variables, production functions on the per acre basis were estimated. The results of log-linear per acre functions of major crops, estimated by ordinary least squares, are presented in Table 3. These functions yielded significant F-ratios and reasonable values of R^2 in sugarcane, cotton and wheat. The F-ratio of the function of maize is significant at the 95 percent level, however, the percent of variance explained is very small. This poor fit may be due to firstly, lack of significant variation in per acre yield and explanatory variables, secondly, the small number of observations and thirdly, a more or less constant technology of production between farms and the narrow range of variation in per acre yield.

The Profit Functions

The results obtained from log-linear profit functions for selected crops are shown in Table 4. These functions were estimated by ordinary least squares. The corresponding indirect estimates of input elasticities of output are presented in Table 5. The available data permitted the estimation of functions for four crops only. The data problems in estimating profit functions are discussed in the next section. A perusal of the two tables (table 1 and table 5) brings out the following points:

First, the coefficient of farm size variable (D_i) is negative in three out of four equations; this may imply better economic efficiency of small farms. Nevertheless, none of the dummy coefficients is significantly different from zero (table 4).

Second, whereas production functions resulted in many negative input elasticities, profit functions yielded coefficients having signs consistent with economic theory. The elasticity of manual labour dropped significantly in the case of sugarcane, cotton and wheat. The production functions of some

Table 2

Linearly Restricted Estimates of Production Functions for Sugarcane and Wheat, 1974-1975, *Faisalabad, Pakistan*

Crops	N	Intercept	Coefficient of							
			D_i	LND_i	MLB_i	BLB_i	TWR_i	MTN_i	CTL_i	CWN_i
Sugarcane	66	3.940	0.042	0	0.820	0.045	0.105	0.083	—	—0.055
Wheat	69	5.031	—0.054	0	0.440	0.360	0.149	0.049	—	0.001

Notes: The Cobb-Douglas form of the production function has been used. Regressions linear in natural logarithms are estimated by the ordinary least squares method. The restrictions imposed on each of the two equations are:

(1) the elasticity with respect to LND_i is equal to zero, and

(2) all other estimated elasticities sum to one.

Table 3

Estimates of Production Functions (on Per Acre Basis) for Selected Crops 1974-1975, *Faisalabad, Pakistan*

Crops	N	Intercept	Coefficient of							R^2	F-Ratio
			D_i	$MLBA_i$	$BLBA_i$	$TWRA_i$	$NTNA_i$	$CTLA_i$	$CWNA_i$		
Sugarcane	66	2.957	0.057	0.835**	0.124	0.220	0.115**	—	−0.039	0.474	10.45
			(0.081)	(0.172)	(0.138)	(0.195)	(0.034)				
Cotton	64	4.163	−0.089	0.734**	0.084	−0.184	0.061*	—	0.032*	0.481	10.39
			(0.078)	(0.132)	(0.127)	(0.166)	(0.023)		(0.015)		
Maize	45	5.412	−0.096	0.348**	—	−0.115	—	0.054	—	0.153	2.67
			(0.107)	(0.137)		(0.216)		(0.059)			
Wheat (1)	69	3.479	−0.075	0.610**	0.452**	0.270**	—	0.134**	—	0.573	18.78
			(0.050)	(0.157)	(0.128)	(0.116)		(0.043)			
(2)	69	3.790	−0.059	0.604**	0.461**	0.306*	0.068	—	0.005	0.574	15.89
			(0.051)	(0.158)	(0.128)	(0.114)	(0.021)		(0.036)		

Notes: As under Table 1.

Table 4

Estimates of Cobb-Douglas Profit Functions for Selected Crops, 1974-1975, *Faisalabad, Pakistan*

Crops	N	Inter-cept	Coefficient of D_i	LND_i	MWR_i	BLB_i	TWR_i	NTN_i	CTL_i	CWN_i	R^2	F-Ratio
Sugar-cane	66	7.496	−0.169	1.001*	−2.029**	—	0.090	0.215*	—	0.239	0.672	22.72
			(0.287)	(0.596)	(0.763)		(0.567)	(0.095)		(0.263)		
Cotton	64	7.094	0.047	0.917**	−0.306	—	0.031	0.041	—	0.138*	0.789	41.64
			(0.209)	(0.308)	(0.513)		(0.300)	(0.041)		(0.062)		
Maize	45	5.288	−0.377	0.719**	−0.660*	0.848**	—	—	−0.363	—	0.470	8.40
			(0.384)	(0.329)	(0.275)	(0.292)			(0.183)			
Wheat	69	6.671	−0.142	0.913**	−0.314*	—	0.175	0.031	—	−0.020	0.889	90.56
			(0.134)	(0.206)	(0.154)		(0.190)	(0.026)		(0.061)		

Notes: Long-linear regressions are estimated by ordinary least squares.
Standard errors of coefficients are in parentheses.
*Significant at 95 percent level.
**Significant at 99 percent level.
Intercept is in natural logarithm of profit measured in Rupees.
R^2 is the coefficient of determination adjusted for degrees of freedom.
— means variable not included in the equation.
For the definition of variables see The Variables.

Table 5

Estimates of Inputs Elasticitles Derived From Profit Functions or Selected Crops, 1974-1975, Faisalabad, Pakistan

Crops	Coefficient of							Returns to Scale
	LND_i	MLB_i	BLB_i	TWR_i	NTN_i	CTL_i	CWN_i	
Sugarcane	0.330	0.669	—	0.029	0.070	—	0.078	1.176
Cotton	0.702	0.234	—	0.023	0.039	—	0.105	1.103
Maize	0.433	0.397	0.511	—	—	—0.218	—	1.093
Wheat	0.695	0.238	—	0.133	0.023	—	—0.015	1.074

Notes: Variables are defined in The Variables.

crops showed negative coefficient for land but, contrary to this, the profit function approach produced significant and positive response coefficients for the same input. Moreover, the input elasticity of water also decreased in comparison to its counterpart in production equations. The drop in the cited coefficients and improvement in those of the land may be attributed to reduction or elimination of the simultaneous equations bias in estimates of production functions.

Third, with the profit function approach, slightly increasing returns to scale are observed in sugarcane and cotton, however, the function coefficient approaches unity in maize and wheat.

THE COMPARATIVE ASPECTS OF PRODUCTION AND PROFIT FUNCTIONS

Theoretically, the production function is a simple relation, easy to intuit, straightforward to estimate and parsimonious in terms of data requirements. The profit function, on the other hand, is a more sophisticated construct and is comparatively difficult to estimate requiring more data than the production function. In addition to data on other variables, data on prices of inputs and outputs, which are hard to obtain or arrive at, are pre-requisites for estimating a profit function. The reward from this relatively difficult to estimate function, is the avoiding of the simultaneous equations bias in the estimates. The results depicted in table 5 are encouraging as compared to those obtained from production functions (table 1). Not only has it been possible to obtain positive and thus more realistic elasticities of output for inputs including, but also estimates of other elasticities changed.

Whereas the production function is free of the riddle of computation of wage rate of labour, the profit function depends on it if interest lies in knowing the degree of responsiveness of output to labour input. But the computation of wage rate is not so direct and straightforward in economies characterized by payment in kind to labour. The wage rate is obtained by evaluating payments in the form of food, clothing, other kind payments and cash payments, if any. The sum of all such payments to all hired labourers divided by the number of days for which labour was hired will yield the overall money wage rate per day on a farm. Should this vector of money wage rates enter the overall profit (sum of profit from all enterprises) function as an explanatory variables? Yes, a priori, it seems that this variable will explain variation in profit. But this wage rate may not be used in the profit function of a given crop. It may or may not explain the variation in profit of that crop. In the context of a crop, the appropriate wage rate is one computed from payments made to the labour employed for accomplishing different operations of the crop. Intuitively, it is easy to see that the wage rate so computed will be correlated with the profit and therefore, will explain variation in profit. Under the farming conditions prevailing in Pakistan, not every farmer hires permanent labour. But where no permanent labour is hired, casual labour is hired to do operations, for example, harvesting, threshing and winnowing of wheat, and kind payments to that casual labour may help us obtain an approximation of wage rate. This approach is helpful only in the case of major crops like sugarcane, cotton, maize and wheat. All the operations of minor crops like oilseeds, fodders, etc., are completed by the small farmer himself. What is his wage rate? Can we use the overall wage rate in such a situation? Perhaps no. Here we reach a dead end caused by the non-availability of measurements on a proper variable. It may be concluded that production functions can be estimated for all enterprises but profit functions for minor enterprises may not be estimated practically for want of data. This holds good under farming conditions similar to those of the sample area of this study.

The estimation of a profit function is confronted with another problem when per unit prices of inputs like fertilizer and water are more or less the same for every farmer. This happens when government controls prices. The computational trick is to treat them as fixed inputs. When these inputs are fixed in the same sense as capital, the inclusion of money wage rate in the estimated equation becomes crucial. If money wage rate is not included, the sum of elasticities of variable inputs is zero which implies that $(1-u)^{-1}=1$ (u is the sum of elasticities of variable inputs. See Lau and Yotopoulos [14]) and, therefore, the coefficients of the profit function are the elasticities of output with respect to inputs.

What defines total profit from an enterprise? It is one simple thing to say that profit is total revenue minus variable costs but it is quite another thing to compute it empirically. Different studies indicate different approaches to arrive at profit figure of an enterprise. Some researchers deduct payments to labour from total revenue. Others have subtracted expenses on chemical fertilizer, farm yard manure and seed in addition to payments to labour. Still others treat payments to family labour as part of profit and deduct payments to hired labour only. Perhaps, different approaches have been necessitated by the non-availability of data on certain variable farm expenses. Such conceptual problems and the methods of handling them do impinge upon the results expected from the profit function of any given enterprise. Output, as a regress and in a production function, does not exhibit such definitional problems but does suffer from measurement errors.

At a given point in time, product prices received by the farmers do not vary widely. But if a large number of farmers stagger the sales of their surplus produce over post-harvest months or if sales are effected in distant markets then the price becomes a function of time and place of sale. This price when used to arrive at total revenue, may distort the correlation between profit and the independent variables such as money wage rate resulting thereby in unexpected results. This may happen also when there are markedly significant qualitative differences in product from farm to farm. Small samples are likely to portray such data tendencies.

CONCLUSIONS

Whereas production functions resulted in many negative output elasticities, profit functions yielded coefficients having signs consistent with production theory. The elasticity with respect to manual labour dropped significantly in case of sugarcane, cotton and wheat. The profit function yielded significant and positive response coefficients for land. Moreover, the output elasticity with respect to water also decreased in comparison to its counterpart in the production function equations. The drop in cited coefficients and improvement in those of land may be attributed to the reduction or elimination of simultaneous equations bias in the estimates of production functions. Here, a hasty conclusion regarding the superiority of the profit function approach over that of the production function is very tempting. But it must be maintained that the production function model is capable of doing equally well if the production relationship is specified and tested in terms of simultaneous equations rather than a single equation. Thus, there is a compelling need to examine this model in this framework before its superiority or inferiority to an alternative approach is adjudged.

REFERENCES

1. Bessel, J.E. "A Development of an Interfarm Input-Output Analysis for Intrafarm Interpretation". *Incorporated Statistician*. 10(3). 120, 1960.

2. ————. "The Measurement of Managerial Efficiency in Agriculture". *J. Agr. Econ.* 21(3): 391-406, September 1970.

3. Chaudhary, A.M. and others. *Man, Water and Economy*. West Pakistan Agricultural University, Lyallpur and Water and Power Development Authority, Lahore. 1970.

4. Dillion, J.L. and E.O. Heady. *Agricultural Production Functions*. Ames, Lowa : Iowa State University Press, 1969.

5. Fishman, G.S. "Bias Considerations in Simultaneous Experiment". *Operations Research*. 24: 253-263, July-August 1972.

6. Garrod, P.V. and Mian M. Aslam. *Agricultural Production Models: Methods and Considerations Part I*. Hawaii State University, Agricultural Experiment Station, forthcoming.

7. Georgescu-Roegen, N. "Process Analysis and Neo-Classical theory of Production". *American Journal of Agricultural Economics*. 54: 279-94, May 1972.

8. Griliches, Z. "Specification Bias in Estimates of Production Functions". *Journal of Farm Economics*. 39: 8-20, February 1957.

9. Hayami, Y. "On the Use of Cobb-Douglas Production Function on the Cross-country Analysis of Agricultural Production". *American Journal of Agricultural Economics*. 52: 327-329, May 1970.

10. Hoch, I. "Simultaneous Equations Bias in the Context of Cobb-Douglas Production". *Econometrica*. 26: 566-578, October 1958.

11. ————. "Estimates of Production Function Parameters Combining Time Series and Cross-Sectional Data". *Econometrica*. 30: 39-53, January 1962.

12. Johnson, S.R. and G.C. Rausser. "Effects of Misspecification of log-linear Functions when Sample are Zero or Negative". *American Journal of Agricultural Economics*. 53: 120-124, February 1971.

13. Johnston, J. *Econometric Methods*. New York: McGraw Hill Book Company, 1972.

14. Lau, L.J. and P.A. Yotopoulos. "A Test for Relative Efficiency and its Application to Indian Agricultute". *American Economic Review* 61: PP. 94-109, March 1971.

15. ——————. "Profit, Supply and Factor Demand Functions". *American Journal of Agricultural Economics.* 54: PP. 11-18, February 1972.

16. Lin, W., G.W. Dean and G.V. Moore. "An Empirical Test of Utility Versus Profit Maximization in Agricultural Production". *American Journal of Agricultural Economics.* 56: PP. 497-508, August 1974.

17. McCallum, B.T. "Relative Asymptotic Bias from Errors of Omission and Measurement". *Econometrica.* 40: PP. 757-758, July 1972.

18. McFadden, D. "Further Results on CES Production Function". *Rev. Econ. Studies.* 30: PP. 73-83, June 1963.

19. ——————. "Cost, Revenue and Profit Functions". in D.L. McFadden (Eds.). *An Econometric Approach to Production Theory.* Amsterdam. Forthcoming.

20. Mundlak, Y. "Empirical Production Functions Free of Management Bias". *Journal of Farm Economics.* 43: PP. 44-56, February 1961.

21. Nurdin, T. *Factors Affecting Farm Decision Making: A Case Study in West Sumatra, Indonesia.* Unpublished Ph. D. Dissertation, University of Hawaii, August 1975.

22. Rasmussen, K. and M.M. Sandilands. *Production Function Analysis of British and Irish Farm Accounts.* Sutton Bonington: Dep. Agr. Econ. Univ. Nottingham, 1962.

23. Sidhu, S.S. *Economics of Technical Change in Wheat Production in Punjab (India).* Unpublished Ph. D. Dissertation, University of Minnesota, 1972.

24. ——————. Economics of Technical Change in Wheat Production in Indian Punjab". *American Journal of Agricultural Economics* 56: PP. 127-226, May 1974.

25. Upton, M. "The Influence of Management on Farm Production on a Sample Nigerian Farms". *The Farm Economist.* 11: PP. 526-536, 1970.

26. Yotopoulos, P.A and L.J. Lau. "A Test for Relative Economic Efficiency: Some. Further Results". *American Economics Review* 63: PP. 215-224, March 1973.

27. Zellner, A., J. Kmenta and J. Dreze. "Specification and Estimation of Cobb-Douglas Production Function Models. *Econometrica.* 34: PP. 784-795, October 1966.