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A Long-Range Energy Sector Plan for Pakistan

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A linear programming model for long-range energy sector planning in Pakistan has been formulated in this study. The model has been simulated to draw policy implications regarding the output of energy industry. These simulations have been carried out by changing trend in future demand, changes in costs of production and changes in the prices of output.

I. INTRODUCTION

In the post-1971 world the economic, social and environmental problems of the energy sector have become so important that careful planning of energy supply seems essential.[1] The energy sector holds a unique position in the economy. It provides an input to other industries and final consumers for further production and consumption purposes. A disequilibrium in this sector can lead to disequilibrium in the rest of the economy.

The energy sector consists of four main industries, viz. coal, oil, gas and electricity. These industries have some common and some divergent features. All of these industries need large investment funds and long time-periods to build new capacity. Their outputs are fairly good substitutes though each still has its own tied demand. The electricity industry provides as well as consumes energy. Fuels can be easily stored but the storage of electricity is not possible on any large scale. This study considers the problem of long-range energy-supply planning. This is a multi-dimensional, dynamic decision problem which can be described, in general, as a problem of deciding how to provide demanded energy efficiently over a given period of time. The decision involves numerous considerations, such as the existing demand and supply of energy, future demand forecasts, the investment costs of different

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[1] There are numerous reasons which make the planning of long-range energy supply essential. For example, the investment decisions for new capacity based on short and medium term considerations can prove very costly for the country because they may bring a discontinuity in growth, instability in supply and a disequilibrium in the economy. Moreover, the long time lags in the construction and development of new capacity make short-term optimization pointless.

alternatives and the alternatives within the alternatives, their sizes, types and time of construction, etc. All these considerations make it a complex problem.

The application of mathematical programming to the investment planning problems of the energy sector was pioneered by French economists [4; 5; 29]. They applied linear programming technique to find an optimal investment and operational programme for the electricity industry. The objective was to minimize the total discounted system costs of electricity supply, subject to conditions that the total output of the system must be sufficient at all times to meet the instantaneous power demand; that the output of each plant should be less than or equal to its available capacity; that there must be an installed capacity to meet the mean expected demand with a margin of reserve capacity; and so on.

The basic formulation has been extended in a number of directions, such as its reformulation in terms of marginal analysis [21; 32; 41; 45; 47], simulation models [3; 17], and non-linear, dynamic and integer programming models [8; 11; 20]. These formulations differed only in algorithms, but solved the same sort of problem. Some new extensions, such as replacement provisions, transmission and distribution facilities, and hydro storage provisions, have been made to make the problem more realistic [40]. The basic model has also been applied in terms of cost-benefit analysis to a number of countries [18; 19; 23; 25; 41; 42]. Later, this formulation was extended to a number of other industries, including other energy industries (viz. natural gas, coal and oil). The oil crisis turned into an energy crisis which stimulated a new interest in the energy sector, and since 1971 energy-modelling activity has grown rapidly.

Most of the energy supply models[2] have been developed as optimization models in the early French tradition. The objective function of these models is to minimize the cost of meeting established demands for energy from known sources of supplies. The models are usually large and complex because they establish links between different sources of energy and between the energy sector and the rest of the economy. The output of the models comes in terms of different patterns of supply and cost for specific situations. The usefulness of these models lies in their ability to generate alternative outcomes under changed supply and demand conditions. Some common features and limitations[3] of these models are given below:

(1) Most of these models assume that the quantities demanded are exogenous and that the objective is always to minimise system costs.

(2) The models provide only broad results, which are sometimes sensitive to even minor assumptions contained in the model. Thus the recommendations of the models are usually distrusted by the policy-makers.

(3) Most formulations are deterministic, though some make allowances for uncertainties in demand, costs and plants-and-resource availability. Work of stochastic type is fairly rare.

(4) Parametric models have a major weakness in that they are not based on firm information.

(5) Sometimes the terminal conditions are ignored in the formulation.

Examples of some recent energy models are the U.K. model [10; 15], the Birmingham model [6], the French model [2; 9; 14; 33], among others.

The Pakistan energy sector has not been studied in any systematic manner before; hence no comprehensive energy model for the country exists. However, the electricity industry has been involved in model-building exercises. Two of the studies are worth mentioning. The first detailed effort was made by the World Bank team [19]. They concentrated on the development of a mixed (hydro and thermal) system. A simulation model was used to justify the building of the Tarbela Dam. The Second study was conducted on behalf of the IAEA [16], which developed a long-term electricity model to show the importance of nuclear power for Pakistan. Both studies used trend models to estimate electricity demand.

II. THE MODEL: CONCEPTUAL BASIS: MAIN FEATURES AND ASSUMPTIONS

The energy sector consists of an integrated set of relations which operate within a complex private, semi-private and public institutional framework [36]. Thus the analysis and the long-term planning of this sector requires a comprehensive approach. Our model encompasses the whole energy sector, including all alternative resources, commercial as well as non-commercial. Its objectives are to minimize the discounted sum of capital, operational and net import costs of the system.

The main function of our model is to simulate implications, for energy industries, of any set of assumptions regarding future demand, use of resources, costs and prices, etc. These simulations also provide an evaluation of techniques and policies relating to the energy sector in the future.

The model is composed of a series of supply and demand sub-systems (viz. coal, gas, oil, and electricity), commercial and non-commercial energy demand and international trade. Any other sub-system, e.g. environment, can be incorporated into the model. The interaction between the sub-systems establishes equilibrium for the system. The fuel substitution is allowed through the demand sub-system and the process substitution through the supply sub-models.

The non-commercial energy sub-system requires a special mention. This energy is used for cooking (also some heating) purposes in the rural areas of the country. People use this form of energy because either the other fuels are not available or are too expensive to buy. It has been observed that non-commercial fuels

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[2] A good survey article related to models for the United States is that of Manne *et al.* [24].

[3] See Cook and Surrey [7, Chap. 3], for a detailed discussion on the limitations of these models.

are inferior to commercial fuels and, whenever people can afford them, they prefer commercial to non-commercial fuels. Our demand sub-system allows this substitution between these fuels. On the supply side, we include two options to meet the energy needs of rural Pakistan. These are: (a) Bio-gas and (b) wood. Our reasons for including these options in the model are based on our concern about rural living conditions and wasteful use of resources, which have high opportunity cost, and also on our desire to have a comprehensive picture of the energy sector. In planning the supplies of wood to meet rural energy needs, we have taken a special note of the limited forest base of the country, and have restricted the annual supplies to current output levels.

The whole system is linked by energy flows. Fig. 1 shows the inter-relationship between the sub-systems and also between the energy system and the rest of the economy. The supply sub-systems show the following links: the flow of coal, oil and natural gas to the power industry; the flow of coal to oil and gas industry for conversion purposes; and the flow of hydro-carbons from the external resources to domestic industries. In addition, the supply side is linked with the demand sub-systems which are also linked with the rest of the economy and the outside world.

The electricity sub-system provides greater details compared with other supply sub-systems. It incorporates the load-duration characteristics of electrical demand as well as seasonal characteristics of hydro-electric supplies.

In this model, we seek to optimize investment over the long run, i.e. from 1975 to 2005. This period has been sub-divided into six periods of five years each with the assumption that investment takes place in the middle of each period. The demand for fuels, as represented by their 5-year total demand figures used in the reference case model, is related to constant price series projections. The following assumptions are implicit in the model:

(1) The electricity transmission system is fully developed and integrated;

(2) The Water and Power Development Authority's demand load-curve represents the whole country's load-curve;

(3) The rate of discount for the reference case model is 10 percent per annum;

(4) Each plant has a life of 30 years. During this time, it is available for operation;

(5) The nuclear fuels are part of these plants and their supplies are assured; and

(6) Energy suppliers are assumed to sell in areas where they maintain maximal efficiency for their activities. At the same time, energy consumers are assumed to buy energy at the lowest possible cost. Due to this behaviour, the model seeks energy supplies at the lowest cost to meet the energy requirements.

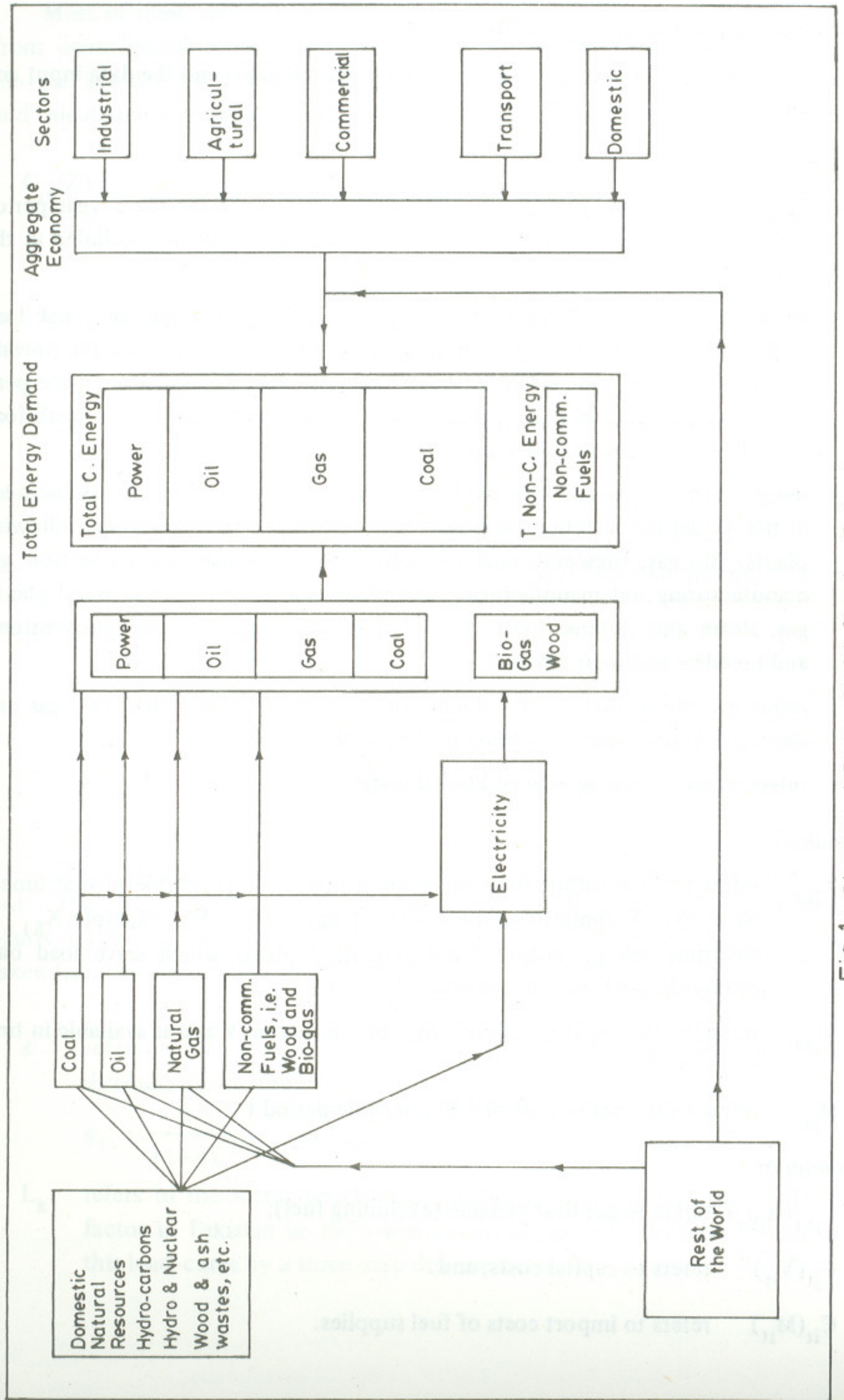

Fig.1: Diagram of Energy Model Showing Subsystems

Mathematical Formulation of the Plan

To start, we describe the indices, parameters, constants and the data input and its sources. These are given below:

Indices

t refers to the T time periods. For example, $t = 1$ relates to the 5-year period corresponding to 1976–1980. There are six 5-year periods included in the model.

k refers to the K load level; $k = 1, 2, 3$, refers to base, medium and peak load respectively. The base load is about 54 percent of the peak load and prevails 85 percent of the time. The medium load consists of 93 percent of the peak load, but prevails only 13 percent of the time, and, finally, the peak load prevails only 2 percent of the time.

j refers to the J prospective plants. There are sixteen such plants included in the model for selection purposes. $J = 1, 2 \ldots\ldots 16$ refers to the following plants: Bio-gas, firewood, coal extraction; oil extraction; gas extraction; gas manufacturing; oil manufacturing; coal-fixed electric plants; oil-fixed plants; gas, steam and turbine, hydro run of river and storage, nuclear conventional and breeders and solar plants.

i refers to the I fuels. Five fuels (viz. non-commercial, coal, oil, gas and electricity) have been considered in the model.

v refers to the V vintage year of Plant J installed.

Variables

X_{jkt} refers to fuel output from production plant j, with load level k at time t. Note that k applies for the electric plants only. For example X_{811} is electrical energy output from coal-fired plants which serve load base during the first planning period t.

V_{jvt} refers to the plant j capacity, installed in period v and is available in time period t.

M_{it} refers to the net import of fuel i, in time period t

Parameters

$C_{jkt}(X_{jkt})$ refers to per unit variable (excluding fuel);

$C_{jt}(Y_{jt})$ refers to capital costs; and

$C_{it}(M_{it})$ refers to import costs of fuel supplies.

Most of these costs have been taken from official sources but some have come from open literature, especially those which are related to new technologies, like bio-gas, breeder reactors and solar plants. For details of these costs, original sources and calculation methods, see Riaz [38];

$C_{jv}(ZY)$ refers to the terminal value of per unit of each plant capacity installed in period v and available at the end of planning period T. These values have been calculated with the assumption that all plants have 30-year lives. Plants are allowed to depreciate through a straight-line method;

b_{ij} refers to the intermediate demand for fuel, i.e. the demand arising from plant j. This consumption remains constant with the change in load level. The plants' efficiencies are assumed as such:

Plant	Efficiency (in percentage)
Oil/Gas Manufacturing	70
Oil/Gas Steam Plants	30
Gas Turbine	20
Coal Steam	28
Nuclear	38

These efficiency factors determine the intermediate demand per unit of output. The electric plants' efficiencies are based on the actual efficiencies of these plants in Pakistan. The efficiency figures for Oil/Gas manufacturing plants have been taken from the open literature [22; 40].

ϕ refers to the discount factor. It is taken as 10 percent and is equal to the discount factor used by the Pakistan Planning Commission in 1975 where

$$\phi_t = \frac{1}{(1+\phi)^{t-2.5}} \text{ and } \phi_T = \frac{1}{(1+\phi)^{T-2.5}}.$$

L_k refers to the factor which converts energy into power capacity. The load factor in Pakistan in 1975 was about 60 percent. We have approximated this load-curve by a three-step discrete curve.

a_j refers to the plant utilization factor for plant j. These are taken as such.

Plant	Utilization (in percentage)
Bio-gas, Coal/Oil/Gas Extraction	90
Oil/Gas Manufacturing	80
Coal, Oil, Gas, Hydro Storage, Solar electric plants	65
Nuclear plants	70
Hydro run of river	60

These figures represent actual utilization factors in Pakistan.

F refers to the factor which converts the '5-year' demand into 'average' or 'final' year demand in period t. F=0.20 for all fuels and equal to 0.252 for the electricity demand. The difference in F is because the electricity demand is representative of the final year of each time period t.

$\overline{F}$ refers to the factor which determines the maximum aggregate hydro capacity allowed on the system. It is assumed that total hydro capacity cannot exceed 50 percent of the total electric power capacity in any planning period. The underlying reason for the assumption is the fact that during the dry season (i.e. January to June) thermal back-up is essential to meet energy demand.

Constants

a_{ij} takes the value 1, if fuel i is being supplied by plant j in period t; otherwise its value is zero.

$\overline{a}_{jt}$ takes the value 1, if fuel i is being traded in period t; otherwise its value is zero.

λ_i refers to the factor which represents the supply losses of fuel i. The loss for non-electric plants (i.e. J = 3, 4, 5, 6, 7) is taken as 0.09 and for the electric plants it is taken as 0.2.

D_{ikt} refers to the final demand of fuel i, in load-level k (only for the electricity demand) and time period t. Note that demand is taken as five-year total demand for each fuel in each time period t. These demands which have been calculated using econometric models have been considered a constant in the model. See Riaz [34] for the details of projections.

$\overline{R}_{it}$ refers to the total reserves of fuel in the country in period t.

$\overline{Y}_{jt}$ refers to the total capacities of fuel j in the country in period t.

$\overline{X}_{jt}$ refers to the total output of fuel j in the country in period t.

Fuel reserves (i.e. $\overline{R}_{it}$) have been estimated through a resource-discovery model. Hydro and bio-gas capacity (i.e. $\overline{Y}_{jt}$) is based on total feasible hydro capacity, and maximum bio-gas capacity has been calculated by assuming that 45 Kg. of dung can support a daily capacity of 2 cubic-meter gas. Maximum fire-wood supplies (i.e. $\overline{X}_{jt}$) per annum are assumed to be equal to five times the 1975 level of output. See Riaz [36] for detailed discussion and figures.

III. THE ENERGY PLAN

The plan consists of an objective function and a number of constraints. The objective function minimizes the sum of the following costs:

(a) Discounted capital costs less discounted terminal values of technologies or plants;

(b) Discounted variable costs of outputs excluding fuels costs; and

(c) Discounted costs of net imports of fossil fuels.

The value of the objective function is, therefore, the net total discounted costs of expanding and operating domestic resources and energy imports over the entire planning period (1975–2006).

Objective Function

$$\text{Minimize Total Cost} = \Sigma_t \phi_t \left[\Sigma_k \Sigma_j \; C_{jkt} X_{jkt} + \Sigma_i C_{it} + \Sigma_j C_{jt} \; Y_{jt} \right] - \phi_T \Sigma_v \Sigma_j \; C_{jv} \; Z \, Y_{jv} \quad \ldots \quad \ldots \quad \ldots \quad .. \quad (1)$$

The objective function is subject to the following constraints :

$$\Sigma_j \; a_{ij} X_{jt} + \overline{a}_{it} \; M_{it} - \Sigma_k \; \Sigma_j \; b_{ij} \; X_{jkt} \geq (1 + \lambda_i) \; D_{ikt} \quad .. \quad .. \quad (2)$$

$i = 1, 2, \ldots 6,$

all t

all k (electric plants only)

This set of constraints relates to the operational variables and it ensures that the domestic output of each fuel i and its net import M_i in each time period must be sufficient to meet its final and intermediate demand and energy losses in the same time period.

$$X_{jt} \leq \overline{X}_{jt} \qquad j = 2 \text{ and for all } t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

This set of constraints relates to firewood supplies and requires that these supplies in each time t should not exceed a certain given miximum level. This constraint has been imposed because of the small forest base in the country.

$$\overline{F}\ \Sigma_k\ L_k\ X_{jkt} \leq \overline{a}_j\ \Sigma\ Y_{jv} \qquad \begin{array}{l} j = 1, 2, \ldots 16 \\ v = -2, -1, 0, 1, \ldots T. \\ t = 1, 2, \ldots, T \end{array} \qquad \ldots \qquad (4)$$

This set of constraints relates to investment variables, and requires that the plant capacities (both existing and newly installed) must be sufficient to generate the required levels of outputs in each time period t. Note that L_k will apply only in the case of electrical energy capacities constraints. These constraints also ensure that the electric plants capacities are sufficient to meet base, medium and peak load demand in the country in each period of time. In the case of fuels, it has been assumed that the peak demand can be met by changing the trade level or from the existing stocks. $\overline{F}$ ensures that the capacity should only be sufficient to meet the average or final year (in the case of electricity) demand and not the total five-year demand.

$$\overline{a}_j\ \Sigma_j\ \Sigma_v\ Y_{jv} \leq F.\ \overline{F}\ \Sigma_k\ L_k\ X_{jkt} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

$$\begin{array}{l} j = \text{hydro} \\ v = -2, -1, 0, 1, 2, \ldots T \\ t = 1, 2, \ldots, T \end{array}$$

This set of constraints takes account of seasonal fluctuations in hydro-electrical energy output, which is caused by unequal availability of water in Pakistan's rivers. Most hydro plants are only capable of producing full capacity output in the wet season (i.e. June to December). Energy in dry months is restricted by the storage of water. During these months, thermal output is essential for back-up purposes. As a result, there is an upper limit to the level of hydro capacity which can be allowed on the system. We have introduced this limit on hydro capacity by allowing it to produce only a fraction of total electrical energy output in each time period t. Strictly speaking, we needed a seasonal model to study this problem, but in the face of lack of information, the above given approach has been adopted.

$$\Sigma_v\ Y_{iv} \leq \overline{Y}_{jt} \qquad \begin{array}{l} v = -2, -1, 0, 1, 2 \ldots, T \\ j = \text{biogas, hydro, solar} \\ t = 1, 2, \ldots, T \end{array} \qquad \ldots \quad \ldots \qquad (6\text{–}1)$$

$$\Sigma_t\ X_{jt} \leq \overline{R}_{it} \qquad i = \text{Coal, Oil, Gas, Firewood} \qquad \ldots \quad \ldots \qquad (6\text{–}2)$$

These two sets of constraints relate to the maximum fuel reserves and capacities in the country. These ensure that total cumulative output of each fuel should not exceed the total reserves of fuels in each planning time period. Note that the constraints relating to the firewood reserves are redundant because of the second set of constraints and thus can be ignored if so wished.

IV. OPTIMAL PLAN

The optimal energy plan for Pakistan within the given framework of its objective function, constraints and data, can be described in terms of the output of the minimum problem, which provides information on the following:

(a) The optimum capacity developmet programme for the energy sector and the present worth of its total costs;

(b) The optimum output schedules for each selected technology and trade levels for each traded fuel;

(c) The dual variables of the demand constraints, which are marginal costs or, in other words, marginal savings resulting from a unit reduction in these demands;

(d) The dual variables of the reserves constraints, which reflect the marginal costs of reducing these reserves; and

(e) The dual variables of the hydro-thermal balance constraints, which reflect the economic case for new hydro capacity.

The optimum capacity development programme is given in Table 1.

A careful consideration of the above shows the following main features:

(i) A strong preference for bio-gas capacity over firewood, which comes into ranking only in the last period of the plan.

(ii) A heavy reliance on the domestic energy resources. This is reflected from the development programmes of coal mines and gas fields and hydro resources. The gradual development of these capacities reflects the increasing relative cheapness of capital costs in later years and on the non-existence of economies of scale.

(iii) All oilfields are to be developed in the first planning period. This is to take advantage of relatively cheaper domestic supplies compared with imports and also to take advantage of the increasing relative cheapness of import prices in the future. This, however, technically may not be feasible.

Table 1

Optimum Capacity Development Programme (MGJ)

Plants	Planning periods 1	2	3	4	5	6
1. Bio-Gas	362.5	39.9	44.8	50.6	56.4	37.0
2. Wood	–	–	–	–	–	26.2
3. Coal	57.1	384.9	17.5	21.1	39.8	–
4. Oil	325.6	–	–	–	–	–
5. Natural Gas	117.7	51.6	116.3	142.1	255.6	464.7
6. Synthetic Gas	–	–	–	–	–	–
7. Synthetic Oil	13.0	289.98	–	–	–	–
8. Lignite elect. plant	–	–	–	–	–	–
9. Oil elect. plant	–	–	–	–	–	–
10. Gas elect. plant	30.6	–	–	–	–	245.6
11. Gas Turbine	–	39.8	55.1	70.7	114.8	–
12. Hydro: R.	10.3	–	–	–	–	–
13. Hydro: S.	63.1	29.0	30.0	79.5	114.2	50.2
14. Nuclear conventional plants	–	–	–	–	–	–
15. Nuclear Breeder plants	–	–	–	–	–	–
16. Solar plants	–	–	–	–	–	–

Net Present Worth of Total Costs of the Programme = 155.9 billion Rs.

The estimates relate to median year in each five-year planning period.

(iv) The synthetic oil technology has become an economic option and therefore should be introduced in the country. Such a technology is not available in the country, however, it is well-developed elsewhere and can be imported.

(v) The electricity capacity programme reveals a strong preference for, and a reliance on, a balanced use of natural gas and hydro resources of the country.

(vi) The plan chooses no new capacity (over and above the existing and planned capacities for lignite, oil, nuclear or solar electricity plants. The maximum hydro (i.e. run-of-river) capacity has been recommended to be installed in the first planning period.

The optimum output schedules for each process are presented in Tables 2 and 3. Table 2 shows the schedules for the non-electric plants, and trade patterns for the traded fuels, and Table 3 shows the electric plants output schedules.

Table 2

Optimum Output Schedules for Non-electric Plants and Net Trade in Fossil Fuels (MGJ)

Plants	Total Output in Planning Period 1	2	3	4	5	6
1. Bio-Gas	1631.2	1801.6	2012.4	2240.3	2494.0	2660.4
2. Wood	0	0	0	0	0	117.8
3. Coal	453.4	2185.3	2264.14	2358.98	2472.7	873.8
4. Oil	1531.7	0	0	0	0	0
5. Natural Gas	1586.9	2062.0	2585.3	3224.9	4022.6	5761.4
6. Sy. Gas	–	–	–	–	–	–
7. Sy. Oil	52.1	1212.0	1212.0	1212.0	1212.0	0
Coal						
Import	–	–	–	–	–	–
Export	–	–	–	–	–	–
Oil						
Import	0	714.1	1053.2	1459.6	1948.1	3726.1
Export	–	–	–	–	–	–
Natural Gas						
Import	–	–	–	–	–	–
Export	–	–	–	–	–	–

The optimum output plan for non-electric plants illustrates the optimum pattern of extraction, manufacturing and trade.

The rural energy needs are mostly met from the bio-gas supplies. The plan prefers bio-gas to wood supplies. This is due to nominal running costs of family-size bio-gas plants, the free availability of dung, and maximum availability restrictions imposed on wood supplies.

In the case of fossil fuels, it is cheaper to extract them from domestic reserves than to import them. The coal and natural gas reserves are sufficient to meet the demand over the planning period. In the case of oil, the plan chooses to extract all

Table 3

Optimum Output Schedule for Electric Plants (MGJ)

Electric Plants	1977 Base Load	1977 Medium Load	1977 Peak Load	1982 Base Load	1982 Medium Load	1982 Peak Load	1987 Base Load	1987 Medium Load	1987 Peak Load	1992 Base Load	1992 Medium Load	1992 Peak Load	1997 Base Load	1997 Medium Load	1997 Peak Load	2002 Base Load	2002 Medium Load	2002 Peak Load
Lignite	0	0	0.028	0	0.474	0	0	0	0.474	0	0.	0.474	0	0	0.446	0	0	0.446
Oil	0	0	0.117	0	0	0.117	0	0	0.117	0	0	0.117	0	0	0.059	0	0	0
Gas steam	0	26.2	0	0	26.2	0	0	26.0	0	0	26.2	0	0	21.5	0	261.4	73.6	0
Gas Turbine	0	3.7	0.845			21.2	0.979		38.98	1.6		61.79	2.3	96.6	3.4		84.3	0
Hydro (run of river)	32.6	0	0	32.6	0	0	32.6	0	0	32.6	0	0	29.9	0	0	27.2	0	0
Hydro: S	224.5	0	0	382.0	0	0	541.9	0	0	747.1	0	0	1020.99	0	0	1119.98	0	0
Nuclear Conventional	12.00	0	0	12.00	0	0	12.00	0	0	12.00	0	0	12.0	0	0	12.0	0	0
Nuclear Breeder	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Solar	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

domestic oil in the first period. The model chooses to import as well as manufacture oil. Manufacturing takes place as long as the domestic supplies of coal can sustain it. Oil is being imported in all planning periods. This is due to the fact that the future is being discounted and, therefore, the present values of the future costs are reduced. Because of this, cheap fuel now, rather than later, is recommended by the plan.

The differences between the depletion patterns obtained for oil, coal and gas are due to different demand patterns for these fuels and also because of differences in their import and export prices, and the differences in their natural reserves levels.

The plan allows trade to take place only in the case of oil. This is because coal and gas reserves are sufficient, but the oil reserves cannot sustain the demand.

The optimum output schedules for each type of electric plant have been given in Table 3. The main features of these schedules are as follows:

(a) The base load is shared by nuclear, hydro and gas (produced mainly from the steam-type plants) plants where the medium load is met by gas (both steam and turbine) plants and the peak load is shared by oil, lignite and gas turbine plants.

(b) The existing nuclear capacity is fully utilized to meet the base load.

(c) The existing lignite and oil plants are used only for the peak duration and mostly act as standby plants.

The dual variables corresponding to the demand constraints show shadow prices of each form of energy. They are given in Table 4. The values are actual values corresponding to each planning period and have been calculated from the net present worth of these variables.

The shadow prices of coal, non-commercial energy and natural gas are equal to their domestic supply costs, which include an allowance for the generation, transmission and distribution losses.

Oil is imported in each planning period. Thus its shadow price is simply the assumed world price plus a small allowance for refining, transmission and distributional losses. The world oil price has been assumed to remain constant in real terms.

The shadow price of electricity is given for three demand levels (i.e. according to their time of occurrence). The difference in shadow prices reflects the standby capacity, which is essential for satisfying the medium- and peak-time demand. The shadow prices also include an allowance for losses.

The dual variables corresponding to reserves constraints are all zero in the first five planning periods. In the sixth planning period, however, the dual variables corresponding to bio-gas, coal, oil and hydro become positive, which points to the marginal savings which are possible by increasing the right-hand side of these reserves by one unit.

The dual variables of the thermal-hydro balance constraints are all zero.

Table 4

Dual Variables (Shadow Prices) Corresponding to the Demand Constraints (Rs/GJ)

Energy Form	Shadow Prices in the Median Year in Planning Period					
	1	2	3	4	5	6
1. Non-commercial Energy	3.17	3.23	3.85	2.38	3.59	3.85
2. Coal	6.76	7.21	7.97	9.17	11.61	13.3
3. Oil	23.99	23.99	23.99	23.99	23.99	23.99
4. Natural Gas	4.39	4.41	4.44	4.45	4.52	4.26
5. Electricity						
(i) Base load	45.90	46.33	47.23	48.56	50.37	47.85
(ii) Medium load	170.90	173.38	178.01	185.01	201.66	142.72
(iii) Peak load	931.71	947.54	977.81	1023.55	1133.11	748.131

V. POST OPTIMALITY ANALYSIS

The optimal plan's sensitivity has been tested for demand, reserves, cost, discount rate, overall planning period and the coefficients of the A matrix. It is found that the plan is fairly stable.

For example, the use of different demand projections (i.e. the case of a 2-percent decline and rise in prices in real terms) showed no fundamental changes either in investment or in the output plan. The results of the plan remained stable even in the face of a 25-percent rise in the reference-case demand projections. However, if an increase of 100 percent on the reference-case projections is allowed, the results change substantially. The following changes were observed in the investment plan. The model selected to install large coal capacity (i.e. 241.3 MGJ) in the first period and fairly small capacities in every other planning period. The investment in oil remained unaffected. The investment in natural gas increased manifold, and large capacities were recommended to be installed in the first four planning periods only. The synthetic-oil capacity recommended to be installed in the first planning period became about 7 times as large as that in the base-case model. The electricity investment plan recommended a greater reliance on the natural gas and hydro power stations in the early periods, and selected nuclear and solar plants for the last two periods of the model. Most of these recommendations can be explained in terms of the 'cheap-energy-now-rather-than-later' perspective.

The inclusion of variable reserves (i.e. increase in reserves equal to the historical discovery rates) consolidated the results of the reference-case model. Even with these reserves, import of oil remained essential. No significant change was observed in the investment plans for bio-gas and wood supplies. Investment capacities for coal and synthetic oil changed slightly over the years. The electricity investment plan mostly remained unaffected. Similarly, the output and the trade schedules did not show any structural changes.

An increase in the prices of oil and natural gas showed a slowing-down effect on the demand for commercial energy and on the demand for oil and natural gas. The increase in the international prices of these fuels has no significant effect on supply. However, in one run, a 100-percent increase over the 1975 prices resulted in the following changes. The trade structure changed over the plannning period. The model recommended imports of oil as well as coal which in turn necessitated a new capacity of synthetic oil in planning period No. 3 as well. The rest of the output and the investment plans showed no significant changes.

The electricity investment plan showed some sensitivity to changes in the available factors. For example, a change in the hydro (run-of-river) availability from 0.6 to 0.5 delayed the installation of new hydro (run-of-river) capacity by two planning periods. However, an increase in the availability of nuclear plants from 0.7 to 0.9 showed no effect on the model's results.

In one run of the model, the planned capacity from the given capacity at the start of the model was included and allowed the model to choose all new capacity. The model did not choose the lignite-fired electricity capacity and showed a greater reliance on hydro-gas combination. It should be recalled that restriction of the total hydro capacity in the model was necessary because of the seasonal variations in the supply of water in rivers.

Experimentation with the discount rate showed that solution is fairly stable to these changes. A 2-percent change in the discount (increase/decrease) did not change the investment, trade or output plans. A discount rate of 5 percent had no significant effect on the solution of the reference-case model.

The extension of the model from 30 years to 50 years, however, produced certain structural changes in the investment plan for electricity. In early years, the preferred combination remained the hydro-gas combination, but in later years the model recommended installation of nuclear energy.

In concluding our discussion, it is worth stressing that the sensitivity analysis is no substitute for the proper procedures [46] which have been developed to deal with uncertainty. One major limitation of our approach is that it does not take a long view under conditions of uncertainty which requires recognition of these uncertainties in terms of uncertain demand, costs, resources availability, future technologies and their efficiencies, etc. Under conditions of great uncertainty about input data and long-time horizons, our approach may be inadequate.

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