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Maximizing National Product by the Choice of Industries

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The purpose of this paper is to bridge a communication gap between economists and public opinion. It is the author's answer to a stubborn misunderstanding. He shows why in the early phases of development concentration on labour-intensive activities maximizes national product and, consequently, the possibility to grow. An additional argument in favour of such a policy is that it contributes maximally to reducing unemployment.

I. INTRODUCTION

Both in the planning of a country's development and in discussing the optimal division of labour between developed and developing countries, the choice of industries constitutes an important problem for the decision-maker. The answer to this problem, given by most development economists, often meets with a certain resistance from sociologists and politicians. The resistance is against the answer that in the early phases of development labour-intensive industries should be chosen; or, to put it in terms of the international division of labour, that labour-intensive industries should be left to the developing countries and capital-intensive industries to the developed countries. Sociologists and politicians often maintain that in this way the developing countries are "given" the low-paying industries and the developed countries keep the "best", the high-paying, industries for themselves. It is the objective of this essay to show, in as simple a way as possible, that the proposed choice is in the interest of the developing countries. Simplicity (some will call it oversimplification) of exposition is chosen in order to clarify the core of the question at stake. Notwithstanding the simple set-up, to be dealt with in Section II, the model we are going to use covers several aspects characteristic for the main issue. Section III gives the proof of the main statement mentioned as well as the limits of its validity. Section IV brings out the development over time that follows from the model. Section V makes a few suggestions about how more general treatments are possible.

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II. FEATURES OF THE MODEL USED

As announced, our model is very simple, so as to enable everybody to check the calculations by redoing them. We introduce two factors of production only, raw labour (of which the quantity will be written as a) and capital (of which the quantity will be written as k). Capital is built up of three components, land, physical capital and human capital. The last term is used for the capital invested in skills beyond those of raw labour. The three components of capital are held in equilibrium throughout the development process. This means that their marginal yields are equal at any moment; this yield, r, is the price of capital, or interest. The price of raw labour will be indicated by 1.

We start by considering only two industries, 1 and 2, each of them using raw labour and capital in a fixed proportion – the "recipe" of the industry or process. For these two industries, we thus have:

$$a_1 = t_1 k_1 \quad \cdots \quad (1)$$

$$a_2 = t_2 k_2 \quad \cdots \quad (2)$$

where

$$t_1 > t_2 \quad \cdots \quad (3)$$

In our numerical illustration, we will take $t_1 = 5,\ t_2 = 3$ .. (4)

The distribution of capital within one industry over land, physical capital and human capital is left open. It is likely, however, that in k_1 human capital is a smaller part and land is a larger part than in k_2.

Each industry stands for all industries with the same value of t; but one "concrete" industry, say textiles, may be present in two versions, a more labour-intensive one with high t and a less labour-intensive one with lower t.

III. MAXIMIZING NATIONAL PRODUCT BY THE CHOICE OF INDUSTRY

The first situation we are going to consider is typical for an under developed country: since capital is scarce, all of k will be used. Labour is abundant and possibly not all of the employable labour force, a_0, will be used. The use of all capital implies that

$$k_1 + k_2 = k \quad \cdots \quad (5)$$

Using equations (1) and (2), we may write this as

$$a_1/t_1 + a_2/t_2 = k \quad \cdots \quad (6)$$

Or, in our numerical example,

$$0.2\, a_1 + 0.33\, a_2 = k \quad \cdots \quad (6')$$

where no more than two decimal places of 1/3 have been shown. Equation (6) or (6′) sets a limit to the quantity a_2 if we choose a_1, or, the other way round, to a_1 if we choose a_2. In order to find out how to maximize national product, we eliminate a_2 with the aid of (6) by solving for a_2:

$$a_2 = t_2 (k - a_1/t_1) \quad \cdots \quad (7)$$

Or, in our example,

$$a_2 = 3\,(k - 0.2\, a_1) \quad \cdots \quad (7')$$

and substitute this in the expression for national product y:

$$y = 1\,(a_1 + a_2) + r\,(k_1 + k_2) = 1a_1 (1 - t_2/t_1) + (1t_2 + r)\,k \quad \cdots \quad (8)$$

Or, again, in our exmaple,

$$y = 1a_1\ (1 - 0.6) + (31 + r)\,k \quad \cdots \quad (8')$$

From equations (8) and (8′) we see that y becomes maximum if we maximize a_1 and this is due to the fact that $t_2 < t_1$ and hence $1 - t_2/t_1$ is positive. In words, national product is largest if we invest all capital in the more labour-intensive activity.

A limit to what we can attain is set by equations (6) and (6′), however: the highest a_1 we can obtain is

$$a_1 = t_1 k = 5k \quad \cdots \quad (9)$$

and it is certainly possible that this value of $a_1 < a_0$. The remainder of the employable population $a_0 - a_1$ or $a_0 - 5k$ will remain unemployed.

Maximizing y is important because it enables the country considered to maximize investment – by private and government saving – and hence to raise k. The quicker k will grow, the earlier the unemployment will vanish.

The argument sometimes formulated – that more investment in industry 2 will raise k more because higher incomes are earned in that industry – overlooks the possibility of government saving.

At the moment that k has reached the value a_0/k_1 or 0.2 a_0, full employment will have been attained. In other words, labour will no longer be abundant and the worst situation of underdevelopment will be over. From then on, the problem is changed into one of making the best use of a_0 and k in a more symmetrical way.

This situation of our model can now be described by the system of equations:

$$a_1 = t_1 k_1 \qquad (1)$$

$$a_2 = t_2 k_2 \qquad (2)$$

$$k_1 + k_2 = k \qquad (5)$$

$$a_1 + a_2 = a_0 \qquad (10)$$

from which the solutions can be easily calculated:

$$k_1 = (a_0 - t_2 k) / (t_1 - t_2) \qquad (11)$$

$$k_2 = (t_1 k - a_0) / (t_1 - t_2) \qquad (12)$$

$$a_1 = t_1 \ (a_0 - t_2 k) / (t_1 - t_2) \qquad (13)$$

$$a_2 = t_2 \ (t_1 - a_0) / (t_1 - t_2) \qquad (14)$$

It can be easily seen that all expressions in the parentheses are positive as long as k stays between a_0/t_1 and a_0/t_2, or, in our numerical example, between 0.2 a_0 and 0.33 a_0. Both a_1 and a_2 depend linearly on k; if k moves linearly over time, a_1 and a_2 will also do so and anyway a_1 decreases while a_2 increases even if k's movement were not linear over time. With k at a_0/t_2, Industry 1 will have vanished and all labour would be employed in Industry 2.

Any deviation from the values (11) to (14), either more in Industry 1 or more in Industry 2, will result in non-utilization of a part of one of the production factors. Since, with full utilization of both factors, national product will be

$$y = l a_0 + rk$$

non-full utilization of either factor will reduce y. This implies that in the whole interval of k from a_0/t_1 to a_0/t_2, *national product is maximized.*

By way of further numerical examples, we choose, within that interval, (I) k = 0.25 a_0 and (II) k = 0.30 a_0. The reader is invited to check the following solutions of these two cases:

(I) $a_1 = 0.625\ a_0;\ a_2 = 0.375\ a_0;\ k_1 = 0.125\ a_0;\ k_2 = 0.125\ a_0$

(II) $a_1 = 0.250\ a_0;\ a_2 = 0.750\ a_0;\ k_1 = 0.050\ a_0;\ k_2 = 0.250\ a_0$

IV. EXPANSION TO MORE INDUSTRIES

A more complete and more realistic example will be obtained if we consider a larger number of industries, for instance, in addition to those already considered, t_1 and t_2, activities characterized by $t_3 = 2$, $t_4 = 1$ and $t_5 = 0.5$. Using the method shown in Section III, we may then obtain a picture in which additional phases of development are added. Taking up the numerical examples (I) and (II) just shown, we find, as already indicated, that for k = 0.33 a_0, full employment in Industry 2 will have been attained. From there on, Industry 1 remains at zero employment, but Industry 3 is introduced and grows, until it has absorbed all labour a_0 at k = 0.5 a_0; this capital stock will now be used only in Industry 3. With k growing further, Industry 4 will be started until k = a_0. From there on, Industries 4 and 5 use all manpower and all capital until the latter attains the level k = 2 a_0. If no further technologies existed, a period of under-utilization of capital would start, comparable with the under-utilization of labour in the early phase of development. Conceivably, the superfluous capital could be invested in lower-numbered industries in less developed countries. In reality, technological development creates new possibilities, although in recent times micro-processors may have introduced a new trend. It is not the intention to discuss this new trend, however, in this essay. As announced, its main objective is to show that it is *in the interest of developing countries to use their production factors as fully as possible.* This will *maximize* their *national product* at any time during their development process and, as a consequence, maximize *also* the *speed of their development* – that is the increase of their capital stock. The latter increase also implies the increase of their human capital, rightly considered important. But the importance of human capital, too, has its limitations: an equilibrium between human capital and other capital should be maintained. The component called land stands for a broader category, better called natural resources, and the clearest example in recent times is the stock of oil and gas. These forms of capital may be increased by exploration, but will be reduced by their exploitation. Moreover, their value may rise by rising prices; and the problems the OPEC countries have to solve is to keep these various activities in balance with the other components of k, so as to equalize their yields.

V. MORE GENERAL TREATMENTS OF OUR PROBLEM

In order to avoid misunderstandings about our view on what a realistic development model should deal with, we will list some features that cannot be neglected in any concrete planning operation for a given country and period. Such a model will need a total employable population, a_0, growing over time, at least for several decades to come. It will also have to be based on changing technical coefficient, t, over time. It will have to take into account the time periods needed for education and, hence, for the growth of human capital. For the interdependence between a number of concrete industries, it must use input-output analysis and international trade in a number of products. At the same time it will have to reckon with the fact that about half of total product consists of non tradables which forces us to generalize this analysis into semi-input-output analysis. We already briefly mentioned the role that can be played by particular natural resources such as oil; similar roles may have to be played by other exhaustible resources, such as metal ores. Prices of factors and of products and their expected movements as well as the elements that determine them will have to be taken into account. More general types of production functions, to begin with Cobb-Douglas and CES production functions, may have to be introduced. An example will be found in the Appendix.

Many of these features have been dealt with in the literature and several of them have been the special subject of study of other discussion papers and other publications of the Rotterdam Centre for Development Planning. As announced, however, the aim of the present paper is more restricted: it wants to remind its readers of the fundamental importance of *the fullest possible use of a country's production factors* to that country's development.

Appendix

The simplest example of introducing more general production functions consists in assuming Cobb-Douglas production functions without returns to scale for each of the two industries between which a choice has to be made. Let national product y be composed as follows:

$$y = c_1 a_1^{\lambda_1} k_1^{\mu_1} + c_2 a_2^{\lambda_2} k_2^{\mu_2} \qquad \ldots \quad (15)$$

where a_1, a_2, k_1 and k_2 have the same meaning as before and c_2, λ_i and μ_i (i = 1, 2) are constants characterizing the C-D functions, it being assumed in addition that

$$\lambda_i + \mu_i = 1 \ (i = 1, 2) \qquad \ldots \quad \ldots \quad \ldots \quad (16)$$

This national product must be maximized under the size conditions

$$a_1 + a_2 = a_0 \qquad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$k_1 + k_2 = k \qquad \ldots \quad \ldots \quad (5)$$

where, again, a_0 and k have the same meaning as before.

Using Lagrange multipliers, ϕ and σ, we have to maximize:

$$y + \phi\,(a_0 - a_1 - a_2) + \sigma\,(k - k_1 - k_2)$$

for which the conditions are

$$c_1 \lambda_1 a_1^{\lambda_1 - 1} k_1^{\mu_1} = \phi = c_2 \lambda_2 a_2^{\lambda_2 - 1} k_2^{\mu_2} \qquad \ldots \quad \ldots \quad (18)$$

and

$$c_1 \mu_1 a_1^{\lambda_1} k_1^{\mu_1 - 1} = \sigma = c_2 \mu_2 a_2^{\lambda_2} k_2^{\mu_2 - 1} \qquad \ldots \quad \ldots \quad (19)$$

Introducing

$$C_1 = c_1 \lambda_1 / c_2 \lambda_2 \qquad \ldots \quad \ldots \quad \ldots \quad (20)$$

and

$$C_2 = c_1 \mu_1 / c_2 \mu_2 \qquad \ldots \quad \ldots \quad \ldots \quad (21)$$

equations (18) and (19) can be rewritten, keeping in mind (16), as

$$C_1\,(a_2/k_2)^{\mu_2} = (a_1/k_1)^{\mu_1} \quad .. \quad .. \quad .. \quad (22)$$

and

$$C_2\,(a_1/k_1)^{\lambda_1} = (a_2/k_2)^{\lambda_2} \quad .. \quad .. \quad .. \quad (23)$$

Introducing now (1) and (2) where provisionally t_1 and t_2 are assumed to be variables, equations (22) and (23) can be transformed into

$$C_1 = t_1^{\mu_1}/t_2^{\mu_2} \quad .. \quad .. \quad .. \quad (24)$$

and

$$C_2 = t_2^{\lambda_2}/t_1^{\lambda_1} \quad .. \quad .. \quad .. \quad (25)$$

Equations (24) and (25) are two equations in two unknowns, t_1 and t_2, and may be used to express t_1 and t_2 in terms of our constants, implying that *they are constants themselves.* So we have reduced our supposedly more general case to our simplest case of Section II and III. Or, to put it the other way round, the simplest case is more general than it looks at first sight. With t_1 and t_2 as constants, a_1, a_2, k_1 and k_2 can be solved as in equations (11) through (14).

The explicit solution of equations (24) and (25) for t_1 and t_2 can best be undertaken by replacing the ts and Cs by their logs and writing logs with the aid of Greek letters:

$$\mu_1\tau_1 - \mu_2\tau_2 = \Gamma_1 \quad .. \quad .. \quad .. \quad (24')$$

$$-\lambda_1\tau_1 + \lambda_2\,\tau_2 = \Gamma_2 \quad .. \quad .. \quad .. \quad (25')$$

By substracting (25′) from (24′), we obtain:

$$\tau_1 - \tau_2 = \Gamma_1 - \Gamma_2 \quad .. \quad .. \quad .. \quad (26)$$

Equations (20) and (21) can be written:

$$C_1 = (c_1/c_2)\,(\lambda_1/\lambda_2) \quad .. \quad .. \quad .. \quad (27)$$

$$C_2 = (c_1/c_2)\,(\mu_1/\mu_2) \quad .. \quad .. \quad .. \quad (28)$$

From these relations, we derive $C_1/C_2 = \lambda_1\mu_2/\lambda_2\mu_1$ .. (27′)

and, according to (26), this equals t_1/t_2. The right-hand side of (27′) can be written as $(\lambda_1 - \lambda_1\lambda_2)\,/\,(\lambda_2 - \lambda_1\lambda_2)$.

So a relation between our previously used ratio t_1/t_2 and that between λ_1 and λ_2 can be found. If $t_1 > t_2$, also $\lambda_1 > \lambda_2$ must be assumed. In the early phase of development, Industry 1 must be chosen in order to maximize y. It is noteworthy that this is the industry with the larger λ_1, implying that the industry with the largest labour income share must be preferred. In many developing countries the labour share is smaller than in more developed countries. This may be an indication that a non-optimal choice of technology is chosen; either too capital-intensive agriculture as a consequence of the concentration in land ownership or too capital-intensive manufacturing as a consequence of "prestige projects" or of imported technologies by transnational corporations.

In one important aspect, the assumption of a Cobb-Douglas production function – and, correspondingly, its validity as a better description of reality – leads to a conclusion different from the one drawn for a fixed-ratio production function in Sections III and IV. In the first interval, where $k < a_0/t_1$, the unlimited substitution of capital by labour makes for the possibility of full employment $a = a_0$ also over this interval. In order to find out which of the two industries is preferable we now have to ask which of these industries yields the highest income y. It appears possible to prove that for $\lambda_1 > \lambda_2$

$$c_1 a_0^{\lambda_1} k^{1-\lambda_1} > c_2 a_0^{\lambda_2} k^{1-\lambda_2} \quad .. \quad .. \quad (27'')$$

The proof may be sketched by reminding ourselves that in the situation where k is just enough to employ all a_0, i.e. where $k = a_0/t_1$, we have (cf. Equation (27′) and the sentence following it),

$$t_1/t_2 = (\lambda_1 - \lambda_1\lambda_2)\,/\,(\lambda_2 - \lambda_1\lambda_2) \quad .. \quad .. \quad (27''')$$

Condition (27″) can be rewritten as

$$\frac{c_1}{c_2}\left(\frac{a_0}{k}\right)^{\lambda_1 - \lambda_2} > 1$$

or

$$(a_0/k)^{\lambda_1 - \lambda_2} > c_2/c_1 = (\lambda_1/\lambda_2)\,(t_2^{\,1-\lambda_2}/t_1^{\,1-\lambda_1})$$

If we can prove this to apply for $k = a_0/t_1$, it will apply for smaller values of k as well. Substituting this value for k our Condition (27″) becomes :

$$(\lambda_1 - \lambda_1\lambda_2) / (\lambda_2 - \lambda_1\lambda_2) > (\lambda_1/\lambda_2)^{1/(1-\lambda_2)} \quad \ldots \quad (27^{iv})$$

It can be shown to apply if we take $\lambda_2 = \lambda_1 - \in$, i.e. a bit smaller than λ_1. The left-hand side of (27^{iv}) then becomes $1 + \in / (\lambda_1 - \lambda_1^2 + \lambda_1 \in - \in)$ and the right-hand side $1 + \in (\lambda_1 - \lambda_1^2 + \lambda_2)$, smaller by a second-order amount. Repeating this reduction of λ_2 by additional small amounts we shall find (27^{iv}) to apply each time.

Another way of testing (27^{iv}) is by numerical solution of the frontier equation:

$$(\lambda_1 - \lambda_1\lambda_2) / (\lambda_2 - \lambda_1\lambda_2) = (\lambda_1/\lambda_2)^{1/(1-\lambda_2)}$$

Because of its transcendental character this equation can only be solved numerically. It is clear that one solution is $\lambda_1 = \lambda_2$: that makes both sides equal to 1. No other solutions within the area $0 < \lambda_1 < 1$ and $0 < \sigma_2 < 1$ have been found; in this whole area, with the exception of the frontier $\lambda_1 = \lambda_2$ (27^{iv}) was found to apply.

A final paragraph may be devoted to the use of CES production functions. Again a simple type may be used as a first step. Let

$$y = y_1 + y_2 \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

where y_1 and y_2 are the products of Industries 1 and 2. Let, further

$$y_1 = \left\{\alpha_1 a_1^{-\phi_1} + (1-\alpha_1) k_1^{-\phi_1}\right\}^{-\frac{1}{\phi_1}} \quad \ldots \quad \ldots \quad (29)$$

$$y_2 = \left\{\alpha_2 a_2^{-\phi_2} + (1-\alpha_2) k_2^{-\phi_2}\right\}^{-\frac{1}{\phi_2}} \quad \ldots \quad \ldots \quad (30)$$

Our problem is to maximize y under the side conditions

$$a_1 + a_2 = a_0 \quad \ldots \quad \ldots \quad (31)$$

$$k_1 + k_2 = k \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

Using Lagrange multipliers π_1 and π_2 we maximize:

$$y_1 + y_2 + \pi_1 (a_0 - a_1 - a_2) + \pi_2 (k - k_1 - k_2)$$

This requires

$$\partial y_1/\partial a_1 = \pi_1 = \partial y_2/\partial a_2 \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

$$\partial y_1/\partial k_1 = \pi_2 = \partial y_2/\partial y_2 \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

These relations can be elaborated into:

$$\alpha_1 (y_1/a_1)^{1+\phi_1} = \alpha_2 (y_2/a_2)^{1+\phi_2} \quad \ldots \quad \ldots \quad (35)$$

$$(1-\alpha_1)(y_1/k_1)^{1+\phi_1} = (1-\alpha_2)(y_2/k_2)^{1+\phi_2} \quad \ldots \quad \ldots \quad (36)$$

Dividing (36) through (35) we obtain, using (1) and (2):

$$\frac{1-\alpha_1}{\alpha_1} t_1^{1+\phi_1} = \frac{1-\alpha_2}{\alpha_2} t_2^{1+\phi_2} \quad \ldots \quad (37)$$

Moreover, (36) may be written, again using (1) and (2):

$$(1-\alpha_1)(\alpha_1 t_1^{-\phi_1} + 1 - \alpha_1) = (1-\alpha_2)(\alpha_2 t_2^{-\phi_2} + 1 - \alpha_2) \quad \ldots \quad (38)$$

and again we find that t_1 and t_2 must satisfy two equations in which only constants appear. Hence they are constants themselves.