

# Estimation of Elasticities of Substitution for CES and VES Production Functions using Firm-level Data for Food-processing Industries in Pakistan

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Analyses involving CES and VES production functions indicate that the constant-returns-to-scale CES model is an adequate representation of large-scale firms in the wheat-flour milling, rice husking, sugar refining and edible-oil processing industries in Pakistan. The hypothesis that these four food-processing industries have the same elasticities of substitution is not rejected. The pooled elasticity estimate for the food-processing industries is significantly different from zero, but not significantly different from one

## 1. INTRODUCTION

A recent study by Battese and Malik (1986b) has shown that there are considerably greater labour-capital substitution possibilities in most of the major industries in Pakistan at the aggregate level than earlier studies had shown e.g. Kazi *et al.* (1976) and Kemal (1978). However, the study highlighted the need for analyses at a more disaggregate level, using firm-level data.

The elasticity of substitution parameter is generally estimated in available literature using a Constant Elasticity of Substitution (CES) production function. However, this is restrictive and theoretically, there is no justification for the elasticity of substitution to be a constant. A number of function forms are available that permit the estimation of a Variable Elasticity of Substitution (VES).

This study attempts to estimate the elasticity of substitution using disaggregate firm-level data for both CES and VES type production functions. Careful statistical

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testing is undertaken to determine the adequacy of the particular type of production function to explain the underlying data.

The difficulties associated with estimation of elasticities of substitution, using aggregative data for firms within specified asset-size categories, are discussed in Battese and Malik (1986a & 1986b). In order to identify and estimate the elasticity of substitution for CES and VES production functions, defined in terms of firm-level data, it is necessary that values of inputs of production be the same for firms within specified categories. Further, there is a problem associated with the interpretation of an elasticity of substitution for a product that is defined in a highly aggregative form. For example, the aggregate two-digit-level industry, Food, consists of twenty-eight quite diverse components, such as meat preparation, ice cream, fish canning, vegetable and fruit canning, bakery products and salt refining. An aggregate estimate for its elasticity of substitution does not necessarily imply that the elasticities for all of the component industries are the same. Moreover, given the heterogeneous nature of the products involved, it is quite possible that the aggregate elasticity of substitution measures, not only the substitution of labour for capital to produce a given homogeneous product, but also the substitution of one product for another.

The above discussion suggests the desirability of estimating elasticities of substitution for well-defined products using firm-level data. This paper presents estimates of elasticities of substitution based upon data obtained from a survey of large-scale firms in the wheat flour milling, rice husking, sugar refining and edible oil processing industries in Pakistan. These four industries are responsible for nearly ninety percent of the value added in the aggregate two-digit-level industry, Food, based upon Government of Pakistan (1983). The output of the firms in each of these industries is fairly homogeneous, although rice husking and edible oil processing produce a wider variety of products and by-products than flour milling and sugar refining. Rice husking produces a range of different quality rice with the output composed of varying proportions of fine, broken and powdered rice and bran, while edible-oil processing produces cottonseed, rapeseed and mustard and sesamum oils, cakes and meal. Flour milling produces a fairly standard quality of flour and bran, while sugar refining produces only white sugar and molasses.

## 2. DATA ON FOOD-PROCESSING FIRMS

During 1980-81, a survey was conducted of large-scale firms within manufacturing industries in Pakistan. From the list of large-scale firms available for the Census of Manufacturing Industries, firms were selected in this survey according to the following criteria:

- (a) all firms in a particular three-digit-level category if their number was less than forty; or

- (b) twenty-five percent of the firms in a particular category if their number was more than forty.

There were sixty-eight firms in the flour milling, rice husking, sugar refining and edible oil processing industries. Firms with these four industries are estimated to comprise about six percent of the total number of large-scale firms covered by the Census of Manufacturing Industries. The percentages of sample firms within the four food-processing industries were 25.0, 30.9, 16.2 and 27.9 for flour milling, rice husking, sugar refining and edible oil processing, respectively. For the 1976-77 Census of Manufacturing Industries, the percentages of food-processing firms within these four food-processing industries were 39.9, 2.2, 11.2 and 46.6, respectively: Government of Pakistan (1982, p. 1). While there may have been changes in the relative percentages of firms within the different food-processing industries, between the 1976-77 Census and the 1980-81 Survey, the significant differences between the two sets of percentages are likely to be due to the criteria by which the sample firms were selected. It is also noted that information supplied to the census is voluntary and the number of firms reported therein does not necessarily represent the true proportions of firms in the total population. For example, it was reported that in the 1976-77 Census only sixty-five percent of the total number of large-scale firms on the census lists actually completed the census: Government of Pakistan (1982, p. ix).

In the 1980-81 Survey, information was obtained on the value of output, value of input, changes in stocks, employment costs and the number of persons employed. Of the sixty-eight firms within the four food-processing industries, two firms reported data showing that value added was negative and four firms reported employment costs that were greater than value added. Since this situation could arise only in the very short-run or have resulted from reporting, or recording errors, these six firms are omitted from our analyses. Data on the book value of different types of capital equipment were obtained for only forty-two of these firms because the remaining firms did not complete the questions on capital assets in the survey. Summary statistics for selected variables are presented in Table 1.<sup>1</sup>

For the seventeen firms in rice husking, the sample mean wage rate, Rs 5,410 and the sample mean value added, Rs 836,000, are the lowest among the four industries considered. For the eleven sample firms in sugar refining the sample mean of value added, Rs 80,600,000, is the highest. The overall sample mean of the wage rate is Rs 11,280, the highest being in edible oil processing, Rs 17,230. The sample mean of employment is highest in sugar refining and its coefficient of variation is significantly lower than those for the other three industries. The coefficients of variation for the wage rate and the number of persons employed are much lower in sugar refining and flour milling than for rice husking and edible oil processing.

<sup>1</sup> The firm-level data on the variables cannot be presented for reasons of confidentiality.

Table 1

Sample Means and Sample Standard Deviations for Selected Variables from the Survey Data

Industry	Number of Firms	Value added <sup>1</sup> (Rs 1,000)	Wage Rate <sup>2</sup> (Rs 1,000)	Share of Wages in Value added (Percent)	Number of Persons Employed	Book value of Capital Assets <sup>3</sup> (Rs 1,000)
Flour Milling	15	1,707 (1,386)	10.31 (2.42)	0.30 (0.29)	27.7 (19)	1,454 (1,111)
Rice Husking	17	836 (863)	5.41 (3.33)	0.32 (0.16)	40.7 (52)	1,066 (2,152)
Sugar Refining	11	80,600 (57,732)	11.38 (3.36)	0.22 (0.15)	1,165.4 (325)	94,931 (85,688)
Oil Processing	19	58,948 (97,835)	17.23 (10.22)	0.17 (0.14)	324.3 (384)	14,565 (28,607)
Food Processing	62	33,007 (67,222)	11.28 (7.6)	0.25 (0.20)	324.0 (483)	29,505 (58,615)

Notes. <sup>1</sup>The figures in parentheses are sample standard deviations.

<sup>2</sup>The wage rate is calculated as the total employment cost divided by the number of persons employed.

<sup>3</sup>The data on capital assets are obtained from the 8, 5, 11 and 18 firms within the flour milling, rice husking, sugar refining and edible oil processing industries, respectively, which answered the appropriate questions in the survey questionnaire.

### 3. ANALYSES INVOLVING CES PRODUCTION FUNCTIONS

We first assume that for the observations on individual firms the stochastic constant-returns-to-scale CES production function [cf. Arrow, *et al.* (1961)],

$$Y_i = \gamma \left[ \delta K_i^{-\rho} + (1-\delta)L_i^{-\rho} \right]^{-1/\rho} e^{U_i}, \quad i = 1, 2, \dots, n, \quad \dots \quad (1)$$

applies, where  $Y_i$ ,  $K_i$  and  $L_i$  represent value added, book-value of capital equipment and total number of persons employed for the  $i$ th sample firm;  $\gamma$ ,  $\delta$  and  $\rho$  are the efficiency, distribution and substitution parameters; and the random errors,  $U_1$ ,  $U_2, \dots, U_n$ , are assumed to be independently and identically distributed as normal random variables with means zero and variances,  $\sigma_U^2$ , and  $n$  represents the number of sample firms involved.<sup>2</sup>

Given the assumption of perfect competition in the factor and product markets, the elasticity of substitution for the CES production function (1),  $\sigma = (1+\rho)^{-1}$ , can be estimated from the indirect form:

$$\log(Y_i/L_i) = \beta_0 + \beta_1 \log w_i + U_i, \quad i = 1, 2, \dots, n, \quad \dots \quad (2)$$

where  $w_i$  denotes the wage for labourers in the  $i$ th firm; and  $\beta_1 = (1+\rho)^{-1}$ . The least-squares estimator for  $\beta_1$  in the indirect form (2) is the minimum-variance, unbiased estimator for the elasticity of substitution.

The indirect form (2) of the CES production function is specified for each of the four different food-processing industries being considered. The numbers of sample firms involved in each industry, the coefficients of determination ( $R^2$ ) for the regression analyses involved and the estimated elasticities of substitution are presented in Table 2. The coefficients of determination for flour milling and sugar refining are very low and the estimated elasticities are not significantly different from zero. However, for rice husking and oil processing, the coefficients of determination are moderately large and the estimated elasticities are significantly different from zero. Further, the estimated elasticities for all four food-processing industries are not significantly different from one. This implies that the Cobb-Douglas production function is likely to be a reasonable model for these food-processing industries.

<sup>2</sup>For the sake of simplicity we assume that there are only two factors of production, homogeneous, capital and labour. We postpone the analysis of intermediate inputs in the production structure to a later study. Moreover, while accepting the simultaneity problem associated with using endogenous variables on the right-hand side in the estimating forms of the equations used, simultaneous equations estimation was not undertaken because of reasons of simplicity and non-availability of relevant data.

Table 2

Estimated Elasticities of Substitution for Food-processing Industries, under the Assumptions of the Constant>Returns-to-Scale CES Production Function

Industry	Number of Firms	R <sup>2</sup>	Elasticity
Flour Milling	15	0.112	1.57 (1.23)
Rice Husking	17	0.682	0.97** (0.17)
Sugar Refining	11	0.003	0.10 (0.66)
Oil Processing	19	0.351	0.70** (0.23)
Food Processing	62	0.599	0.82** (0.16)

Notes: Figures in parenthesis denotes estimated standard errors.

\*\*denotes significant at the one-percent level.

Although the estimated elasticities for the four industries are different, it is of interest to consider if the CES production functions have the same elasticities of substitution. We consider the hypothesis that the four industries have indirect forms (2) with the same coefficient of the logarithm of wages (i.e. the same elasticity) but permit the functions to have different intercept (or efficiency) parameters. If this hypothesis is true, then the relevant test statistic has F-distribution with degrees of freedom 3 and 54, respectively. For the given sample data, the test statistic has value 0.70, which is not statistically significant. Thus the hypothesis that the four food-processing industries have the same elasticities is not rejected.<sup>3</sup> The estimated elasticity of substitution, under the assumption that the four food-processing industries have the same elasticities, is 0.82, which is not significantly different from one. The coefficient of determination for the associated indirect form for the four industries is equal to 0.599.

<sup>3</sup>The hypothesis that the four food-processing industries have identical indirect forms (2) is also accepted at the five-percent level of significance, because the associated F-statistic, with parameters 6 and 54, respectively, is equal to 1.97. The estimated elasticity under this assumption is 1.11, with an estimated standard error of 0.14, and so is significantly different from zero, but not significantly different from one.

Suppose that the stochastic variable-returns-to-scale CES production function [cf. Brown and de Cani (1962)],

$$Y_i = \gamma \left[ \delta K_i^{-\rho} + (1-\delta) L_i^{-\rho} \right]^{\nu/\rho} e^{U_i} \quad i = 1, 2, \dots, n, \quad \dots \quad (3)$$

applies, where, in addition to the parameters and assumptions defined for the CES model,  $\nu$  is the homogeneity parameter. A possible indirect form for the CES production function (3), based upon the assumption of perfect competition in the factor and product markets is given by.

$$\log(Y_i/L_i) = \beta_0 + \beta_1 \log w_i + \beta_2 \log L_i + U_i, \quad i = 1, 2, \dots, n, \quad \dots \quad (4)$$

where  $\beta_1 = \nu(\nu+\rho)^{-1}$  and  $\beta_2 = (\nu-1)(1-\beta_1)$  [cf. Behrman (1982, p. 161)]. For this production function, the elasticity of substitution,  $\sigma = (1+\rho)^{-1}$ , is not identically equal to the coefficient of the logarithm of wages. However, the elasticity of substitution and the parameters of the indirect form (4) are functionally related by  $\beta_1 = (1+\beta_2)\sigma$ . Thus, if  $\beta_2 \neq -1$  and the observations on the model (4) satisfy basic regularity conditions, then a consistent estimator for the elasticity of substitution is defined by

$$\hat{\sigma} = \hat{\beta}_1 (1+\hat{\beta}_2)^{-1}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

where  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are the least-squares estimators for the parameters,  $\beta_1$  and  $\beta_2$ , in the indirect form of the variable-returns-to-scale CES production function (4). This estimator does not have a finite mean (or variance) because the least-squares estimators,  $\hat{\beta}_1$  and  $\hat{\beta}_2$ , are normally distributed, under the assumptions of the model (3). However, the estimator (5) is such that the random variable,  $n^{-1/2}(\hat{\sigma} - \sigma)$ , converges in distribution, as  $n$  approaches infinity, to a normal random variable with mean zero and a finite variance. By using a Taylor-series expansion of the estimator (5), a consistent estimator can be obtained for its asymptotic variance, in terms of the variances and covariance for  $\hat{\beta}_1$  and  $\hat{\beta}_2$ .

The estimated elasticities of substitution for the four food-processing industries, under the assumptions of the variable-returns-to-scale CES production function (4), are presented in Table 3, together with the values of the coefficient of determination and estimates for the homogeneity parameter (discussed below). The elasticity estimates are different from those presented in Table 2 for the constant-returns-to-scale CES production function. Except for rice husking, all the estimates are not significantly different from zero. However, the relatively large standard errors imply that all the elasticity estimates are not significantly different from one.

The estimated elasticities for the four food-processing industries, under the assumption of the variable-returns-to-scale CES production function, are not significantly different. If the hypothesis that the four industries have the same elasticities is true, then the traditional test statistic involved has F-distribution with degrees of freedom 6 and 50. The value of this test statistic for the given sample data is 1.07, which is not significant at the ten-percent level. The estimated elasticity, under the assumption that the four food-processing industries have the same elasticities, is 1.09, as reported at the bottom of Table 3. This elasticity is significantly different from zero at the one-percent level, but is not significantly different from one.

Table 3

*Estimates for the Elasticities of Substitution and the Homogeneity Parameter for Food-processing Industries, under the Assumptions of the Variable>Returns-to-Scale CES Production Function*

Industry	Number of Firms	R <sup>2</sup>	Elasticity	Homogeneity Parameter
Flour Milling	15	0.267	3.03 (6.45)	-1.51 (9.93)
Rice Husking	17	0.700	0.82** (0.21)	3.04 (5.36)
Sugar Refining	11	0.003	0.10 (0.74)	1.01 (0.77)
Oil Processing	19	0.382	1.42 (1.12)	3.92 (14.64)
Food Processing	62	0.609	1.09** (0.33)	-1.25 (7.99)

Notes: Figures in parenthesis denotes estimated standard errors.

\*\*denotes significant at the one-percent level.

The homogeneity parameter,  $\nu$ , is expressed in terms of the parameters  $\beta_1$  and  $\beta_2$  of the indirect form (4) of the variable-returns-to-scale CES production function by  $\nu = 1 + \beta_2 (1 - \beta_1)^{-1}$ , provided  $\beta_1 \neq 1$ . From this it follows that a consistent estimator for the homogeneity parameter is defined by

$$\hat{\nu} = 1 + \hat{\beta}_2 (1 - \hat{\beta}_1)^{-1} \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

where  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are as defined for (5). Although this estimator does not have a finite mean or variance, a consistent estimator for its asymptotic variance can be obtained by standard methods.

Values of the consistent estimator (6) for the homogeneity parameter are presented in Table 3. The values obtained for flour milling, rice husking and edible-oil processing are unreasonable. However, estimates of the asymptotic variances are sufficiently large that the hypothesis of constant returns to scale is not rejected. The results reported in Table 3 suggest that a more precise analysis of the degree of homogeneity of the CES production function (3) may require additional data or alternative estimators for the homogeneity parameter than that defined by (6).

If the coefficient of the logarithm of labour  $\beta_2$  is zero for the indirect form (4) for the variable-returns-to-scale CES production function, then the  $t$ -ratio for the estimator for that parameter has  $t_{n-3}$  distribution, where  $n$  is the number of sample firms in the given industry. The values of the  $t$ -ratios for flour milling, rice husking, sugar refining and edible oil processing are  $t_{12} = -1.60$ ,  $t_{14} = 0.91$ ,  $t_8 = 0.02$  and  $t_{16} = -0.89$ , respectively, which are not significant at the five-percent level. Thus the hypothesis of constant returns to scale is not rejected, given the assumptions of the variable-returns-to-scale CES production function (3) - (4).

#### 4. ANALYSES INVOLVING VES PRODUCTION FUNCTIONS

In this section we consider the estimation of the elasticity of substitution under the assumption that a variable-elasticity-of-substitution (VES) production function applies. We initially consider that the stochastic constant-returns-to-scale VES production function [cf. Lu and Fletcher (1968)],

$$Y_i = \gamma \left[ \delta K_i^{-\rho} + (1-\delta)\eta L_i^{-\rho} (K_i/L_i)^{-c(1+\rho)} \right]^{-1/\rho} e^{U_i}, \quad i = 1, 2, \dots, n, \quad (7)$$

applies, where the variables  $Y_i$ ,  $K_i$  and  $L_i$  and the random errors  $U_1, U_2, \dots, U_n$  are as defined for the constant-returns-to-scale CES production function (1).

The indirect form of this VES production function is defined by

$$\log (Y_i/L_i) = \beta_0 + \beta_1 \log w_i + \beta_3 \log (K_i/L_i) + U_i, \quad i = 1, 2, \dots, n, \quad \dots \quad (8)$$

where  $\beta_1 = (1+\rho)^{-1}$ ; and  $\beta_3 = c$ .

It is evident that if the coefficient of the logarithm of the capital-labour ratio  $\beta_3$  is zero, then the model reduces to the indirect form of the constant-returns-to-scale CES production function (2). Given the assumption of the VES production function (7), it follows that a test of the hypothesis that the production function has constant elasticity of substitution is obtained by a  $t$ -test on the least-squares estimator for the coefficient of the logarithm of the capital-labour ratio.

Given the assumptions of perfect competition, the elasticity of substitution for the constant-returns-to-scale VES production function (7) is expressed in terms of the parameters of the indirect form (8) by

$$\sigma = \beta_1 (1 - \epsilon \beta_3)^{-1} \dots \dots \dots \dots \dots \dots (9)$$

where  $\epsilon = (wL + rK)/rK$  is the ratio of total factor costs to the rental cost of capital for the firm involved [cf. Lu and Fletcher (1968, p. 450)].

A consistent estimator for the elasticity is defined by

$$\hat{\sigma} = \hat{\beta}_1 (1 - \epsilon \hat{\beta}_3)^{-1} \dots \dots \dots \dots \dots \dots (10)$$

where  $\hat{\beta}_1$  and  $\hat{\beta}_3$  denote the least-squares estimators for  $\beta_1$  and  $\beta_3$  in the indirect form (8) and the value of  $\epsilon$  is taken to be the ratio of the sample mean of value added to the sample mean of value added minus employment cost for the firms in the industry concerned. The asymptotic variance of this estimator for the elasticity is estimated by standard methods.

The elasticity estimates for the four food-processing industries are presented in Table 4, together with the number of firms involved, values of the coefficient of

Table 4

*Estimated Elasticities of Substitution for Food-processing Industries, under the Assumptions of the Constant>Returns-to-Scale VES Production Function*

Industry	Number of Firms	$R^2$	Elasticity	$\epsilon$ -values
Flour Milling	8	0.616	3.70 (12.60)	1.19
Rice Husking	5	0.989	0.99** (0.07)	1.34
Sugar Refining	11	0.281	0.70 (0.67)	1.19
Oil Processing	18	0.181	0.62 (0.42)	1.17
Food Processing	42	0.611	0.79** (0.22)	1.18

Notes: Figures in parenthesis denotes estimated standard errors.

\*\* denotes significant at the one-percent level.

determination ( $R^2$ ) for the least-squares fit of the indirect form (8), and the respective  $\epsilon$ -values. The elasticity of substitution for the four industries combined is also estimated. If the hypothesis that the four food-processing industries have the same slope parameters ( $\beta_1$  and  $\beta_3$ ) for their indirect forms (8) is true, then the appropriate test statistic has F-distribution with degrees of freedom 6 and 30, respectively. For the data available, this test statistic has value 0.32, which is not significant, at the ten-percent level. It is noted, however, that even if the hypothesis that the indirect forms (8) for the four industries have the same slope parameters is true, the elasticities for the four industries are likely to be different under the assumption of the VES production function. Differences are expected to arise because of different levels of capital and labour in the different industries (i.e. the value of  $\epsilon$  in (9) generally varies from industry to industry).

The estimated elasticity for rice husking is significantly different from zero at the one-percent level. The estimated elasticities for the other industries are not significantly different from zero. The large estimated elasticity for flour milling, 3.70, is due to the value of  $\epsilon \beta_3$  being close to one, making the denominator in (10) small relative to  $\beta_1$ . The elasticity estimates reported for rice husking and edible oil processing in Table 4 are not significantly different from those reported for these industries in Table 2. The coefficients of determination ( $R^2$ ) reported in Table 4 are generally higher than those reported for the respective categories in Tables 2 and 3.

If the hypothesis that the coefficient of the logarithm of the capital-labour ratio  $\beta_3$  is zero, is true for each industry, then the  $t$ -ratio associated with the estimator for the parameter has  $t$ -distribution with degrees of freedom  $n-3$ , where  $n$  is the number of sample firms in the given industry. The values of the  $t$ -ratio for flour milling, rice husking, sugar refining and edible oil processing are  $t_5 = 1.78$ ,  $t_2 = -0.12$ ,  $t_8 = 0.94$  and  $t_{15} = 0.97$ , respectively, which are not significant at the five-percent level. Thus the hypothesis of constant elasticity of substitution is not rejected, given that the constant-returns-to-scale VES production function (7) – (8) applies.

We now consider the stochastic variable-returns-to-scale VES production function, derived by Yeung and Tsang (1972)

$$Y_i = \gamma \left[ \delta K_i^{-\rho} + (1-\delta) \eta L_i^{-\rho} (K_i/L_i)^{-c(1+\rho)} \right]^{-\nu/\rho} e^{U_i}, \quad i = 1, 2, \dots, n, \quad (11)$$

where, in addition to the parameters and assumptions defined for the constant-returns-to-scale VES production function (7),  $\nu$  is the homogeneity parameter. The associated indirect form of this CES production function is defined by

$$\log(Y_i/L_i) = \beta_0 + \beta_1 \log w_i + \beta_2 \log L_i + \beta_3 \log(K_i/L_i) + U_i, \quad \dots \quad (12)$$

where  $\beta_0 = \nu(\nu+\rho)^{-1}$ ;  $\beta_2 = (\nu-1)(1-\beta_1)$ ; and  $\beta_3 = c$ .

This indirect form is applied to each of the four food-processing industries and a test obtained for the hypothesis that the four industries have the same slope parameters. Given that the random errors in the indirect forms (12) have the same variances for all industries, the appropriate test statistic has F-distribution with degrees of freedom 9 and 26. The value of this statistic for the given data is 0.25, which is not significant at the ten-percent level. Thus, the hypothesis, that the indirect forms of the variable-returns-to-scale VES production function (12) for the four food-processing industries have the same slope parameters, is not rejected. As stated for the constant-returns-to-scale VES production function, this does not necessarily imply that the four industries have the same elasticities.

It is evident that the variable-returns-to-scale VES production function (11) is equivalent to the constant-returns-to-scale CES production function (1) if the parameters,  $\beta_2$  and  $\beta_3$ , in the indirect form (12), are both zero. Under the assumptions of the VES production function (11), it follows that if these two parameters are zero, then the appropriate test statistic has F-distribution with degrees of freedom 2 and  $n-4$ , where  $n$  is the number of sample firms in the industry involved. The values of this F-statistic are 4.33, 2.36, 0.55 and 0.46 for the four respective food-processing industries. These values are not significant at the five-percent level and so the hypothesis that the constant-returns-to-scale CES production function (1) – (2) is adequate, is not rejected, given that the assumptions of the variable-returns-to-scale VES production function (11) – (12) apply. Thus, we do not proceed to obtain estimates for the elasticities of substitution for the variable-returns-to-scale VES production function.

## 5. CONCLUSIONS

The foregoing analyses, based upon firm-level data, suggest that the constant-returns-to-scale CES production function (1) – (2) is an adequate representation of the data, given the assumptions of the models considered. Given the available data and the assumptions of this production function, the hypothesis that the four food-processing industries have the same elasticities is not rejected. Thus, these data may be aggregated to efficiently estimate the elasticity for the two-digit-level industry, Food Processing. The estimated elasticity is significantly different from zero at the one-percent level, but not significantly different from one. In fact, none of the elasticity estimates obtained are significantly different from one. These analyses suggest strongly that the Cobb-Douglas production function is an adequate representation of the firm-level data. Given the problems of estimation with inadequate capital data, the indirect form (2) of the CES production function (1) provides a convenient framework for estimating the elasticity of substitution. However, the usefulness of the results obtained is limited by the extent to which the assumptions underlying the analyses are likely to be true.

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