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Derived Demand for Factors in the Large-scale Manufacturing Sector of Pakistan

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Factor demand is essentially a derived demand because a profit-maximizing firm's demand for factors is derived from the demand for the final product which it produces. Since the firm's optimal choice of a bundle of factors depends on the cost-minimization strategy for a given level of output, the derived demand for factors depends on the level of output, the substitution possibilities among factors in production allowed by the production technology, and the relative prices of all factors.

Knowledge of the substitution possibilities among factors in production is particularly important if one is interested in deriving implications for policy which influence the relative price of factors. More generally, if substitution possibilities among factors in production are limited then adjustment by industry to higher factor prices will be somewhat difficult, and significant changes in the underlying technological structure may be required.

Most of the earlier work on the estimates of elasticities of substitution for Pakistan's manufacturing sector are restricted to two factors [see Battese and Malik (1988), Kazi *et al.* (1976); Kemal (1981) and Naqvi *et al.* (1983)]. These studies either use Cobb-Douglas (CD) Constant Elasticity of Substitution (CES) or Variable Elasticity of Substitution (VES) approaches. These technologies have some well-known limitations.[1] In a recent study using 'nested' CES production function Khan (1989) added energy as a third factor of production to estimate the elasticity of substitution.

To overcome some of the inherent weaknesses in CD, CES, and VES, we use a translog cost function which does not work with such a restrictive structure.[2] The estimated parameters of a translog cost function can be used to derive the elasticities of substitution and price elasticities of demand for factors of production.

All of the earlier studies dealing with Pakistani data ignored raw materials as an input in the production function. Raw materials account for about 67 percent

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[1]We report all the limitations of these technologies in Section I.

[2]Mahmood (1989a) tested the properties of the cost function for the large-scale manufacturing industries of Pakistan.

of the industrial costs of the large-scale manufacturing sector of Pakistan. In this way these studies have excluded from their concern the role of raw material prices and thus ignored the cross-substitution possibilities between raw materials and other inputs. A study which accounts for all factors will definitely characterize more completely the structure of technology.

In this paper, we attempt to include raw materials in addition to capital, labour and energy while estimating factor demand functions for the large-scale manufacturing sector of Pakistan. We estimate the parameters of the translog cost function by using the Iterative Seemingly Unrelated Equations (ISURE) technique.

In Section I, we describe the translog cost model and estimation procedures. Section II contains a brief discussion of the data. In Section III we report estimated parameters, elasticities of substitution and price elasticities of factor demand, Section IV concludes the paper.

I. TRANSLOG COST MODEL

To derive factor demand, we estimate a translog cost function. The translog function is an important one over the frequently used functions such as CD or CES in several ways. Uzawa (1964) showed that, in the multi-factor CD and CES functions, the partial elasticity of substitution between all pairs of factors are equal which rules out the possibility of complementarity between any pairs of factors. The possibility of complementarity is also absent in three-factors 'nested' CES function [see Khan (1989)].[3] The translog function, on the other hand, is sufficiently flexible to describe the substitution possibilities between all pairs of factors. 'Nested' CES, CD and CES assume strong separability among the factor levels. This is quite a stringent assumption.[4] No such assumption is required for a translog function.

To determine the cost relationships among factor prices we consider the following general cost function,

$$C \;=\; f(Q, P_K, P_L, P_E, P_M) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

where, the factor prices distinguished are P_K, P_L, P_E, and P_M of capital, labour, energy and raw materials, respectively. Assuming constant returns to scale and imposing symmetry condition on the second-order partial derivatives, gives the following translog cost function

[3]Khan (1989) outlined the criteria for nesting on the basis of complementarity (which is a logical impossibility) or low substitutability between two factors.

[4]Mahmood (1989a) tested separability assumption and found no statistical evidence in its support.

$$lnC = ln\alpha_0 + lnQ + \alpha_i lnP_i + 1/2\, \beta_{jj}(lnP_j)^2 + \beta_{ij}(lnP_i)(lnP_j) \quad \ldots \quad (2)$$

Differentiating Euqation (2) with respect to the logs of the prices gives the cost share equations

$$M_i = \alpha_i + \beta_{iK} lnP_K + \beta_{iL} lnP_L + \beta_{iE} lnP_E + \beta_{iM} lnP_M, \text{ for } i = K, L, E \text{ and } M.$$

Linear homogeneity in prices requires the following conditions on Equation (2)

$$\alpha_K + \alpha_L + \alpha_E + \alpha_M = 1 \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$\Sigma_i\ (\beta_{iK} + \beta_{iL} + \beta_{iE} + \beta_{iM}) = 0 \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

We can estimate the parameters of the four share equations from three share equations which yield a set of cross-equation symmetry restrictions.[5]

Since the four shares sum identically to unity, one must expect non-zero contemporaneous covariances between disturbances in different equations, and there is also no *a priori* reason to expect the same disturbance variance in different share equations. Under such a situation we can not apply the OLS methods. However, with the cross-equations symmetry restrictions imposed, the system of equations can be estimated by using the Seemingly Unrelated Equations (SURE) technique. Berndt and Christensen (1973), however, noted that estimates of the translog parameters with SURE are not invariant to the choice of the equation that is dropped.

Barten (1969) showed that maximum likelihood estimates (MLE) are independent of the equation omitted. On the other hand, Kmenta and Gilbert (1968) have shown that if we iterate the SURE (ISURE), the parameter estimates will converge to the MLE. This is why we have selected ISURE technique for the estimation of the translog parameters.

The elasticities of factor substitutability and price elasticities of factor demand implicit in translog parameters are of considerable significance for economic analysis and policy-making.

With the estimated parameters of translog cost function the Allen partial elasticities of substitution (AES) are

$$\sigma_{ii} = (\beta_{ij} + M_i - M_i)/M_i, \text{ for } i = K, L, E, \text{ and } M, \quad \ldots \quad \ldots \quad (5)$$

$$\sigma_{ij} = (\beta_{ij} + M_i M_j)/M_i M_j, \text{ for } i, j = K, L, E, \text{ and } M, i \neq j \quad \ldots \quad (6)$$

[5]See Mahmood (1989).

Factors i and j are substitutes if $\sigma_{ij} > 0$ and complements if $\sigma_{ij} < 0$. The price elasticity of demand for factors of production (E_{ij}) is defined as

$$E_{ij} = \partial lnX_i / \partial lnP_j = M_j \sigma_{ij}$$

where, output quantity and all other input prices are fixed. Allen (1938) has shown that the AES are analytically related to the price elasticities of demand for factors of production.

II. DATA

To estimate the translog cost function we used the data for the large-scale manufacturing industries reported in Government of Pakistan (Various Issues).[6] These data are periodically available between 1959-60 and 1984-85 for seventeen years.

We compute wages by dividing employment costs to average daily employment. Price of capital is computed by converting \$100 at the going exchange rate and is multiplied by ($r + d$), where, r is the rate of interest and d is the depreciation rate which is assumed to be 5 percent. We use prices of energy and raw materials reported in Government of Pakistan (1988), We add 'other costs' to the industrial costs to arrive at the costs of raw materials. To compute capital costs we subtract other costs and employment costs from the value-added data.

III. EMPIRICAL FINDINGS

For all the estimated share equations we compute R^2. The R^2 figures for the regressions are 0.82 for ML equation, 0.55 for MK equation, 0.89 for ME equation and 0.92 for MM equation.

The elasticities of substitution, based on the estimated parameters are reported, in Table 1 and the average share of each factor in total costs is reported in Table 2.

Table 3 reveals that most of the estimates are significant except for σ_{EE} and σ_{ML}. Our estimates also show that most of the factors are substitutes with other factors except that energy and capital are complements. Most of the elasticities of substitution turn out to be highly elastic which show that other factors may be readily substituted. This was made possible by the liberal imports of raw materials during the 70s and early 80s.

The results show that energy and labour are highly substitutable. Similarly, capital and labour are high substitutes. These results are consistent with those

[6]Due to some problems associated with energy and wage data for total manufacturing sector, we limit our analysis just to the large-scale manufacturing industries.

estimated by various traditional two-factor studies [see Malik *et al.* (1989)]. Such factor substitution also shows a consistent pattern as energy and capital have displayed substantial complementarity. This result is in contrast to the findings of Khan (1989) who found a weak substitutability instead of strong complementarity between energy and capital. Khan derived this result because of his 'nesting' pro-

Table 1

Estimates of the Translog Cost Function

Parameter	Estimates	*t*-statistics
α_L	0.085	22.19
α_K	0.178	13.13
α_E	0.030	8.12
α_M	0.707	62.31
β_{LL}	–0.041	–4.31
β_{KL}	0.025	2.02
β_{EL}	0.010	2.32
β_{ML}	0.006	1.39
β_{KK}	–0.111	–3.24
β_{EK}	–0.034	–2.89
β_{MK}	0.120	5.14
β_{EE}	–0.003	–0.23
β_{ME}	0.027	2.96
β_{MM}	–0.153	–4.40

Table 2

Means of Factor Shares in Total Costs

Factors	Means
Labour	0.0826
Capital	0.2134
Energy	0.0300
Raw Materials	0.6740

Table 3

Estimated Allen Partial Elasticities of Substitution

Elasticity	Estimates	*t*-statistics
σ_{LL}	–17.12	–4.31
σ_{KL}	2.42	2.02
σ_{EL}	5.04	2.32
σ_{ML}	1.11	1.39
σ_{KK}	–6.12	–3.24
σ_{EK}	–4.31	–2.89
σ_{MK}	1.83	5.14
σ_{EE}	–35.67	–0.23
σ_{ME}	2.34	2.96
σ_{MM}	–0.82	–4.40

cedure which ignored the possibility of complementarity. Our results are, however, in agreement with those of Hudson and Jorgenson (1974).

Raw materials turn out to be substitutable with every other factor. This result is also in agreement with Hudson and Jorgenson (1974). This result implies that with the use of more labour, energy or capital firms can economize on the use of raw materials.

Since all the elasticities of substitution turn out to be greater than one we will expect that a small increase in the factor prices would lead to a large increase in the employment of other factors. Thus removal of distortions in the capital market, say, would help the country in increasing employment opportunities. The effect of changes in factor price on the demand of factors can be seen from Table 4.[7]

The results reported in Table 4 reveal that since capital and labour are substitutes, *ceteris paribus,* a one percent increase in the price of capital would lead to a 0.52 percent increase in the demand for labour. Thus elimination of distortions in the capital market would help in generating employment opportunities in the large-scale manufacturing sector. On the other hand, elimination of distortions in the labour market would generate much lower demand for capital.

The results also show that since energy and capital are complementary, *ceteris paribus,* increase in the price of energy dampens the demand for energy and the

[7]All the own elasticities have negative signs which are required for factor stability.

Table 4

Estimated Own- and Cross-elasticities of Demand

Percent Change in Factor	With Respect to the Price			
	L	*K*	*E*	*M*
L	−1.41	0.52	0.15	0.75
K	0.20	−1.31	−0.13	1.23
E	0.42	−0.92	−1.07	1.58
M	0.09	0.39	0.07	−0.55

demand for new capital. On the other hand, energy and capital are substitutes for labour, therefore, elimination of subsidy on capital and energy would generate demand for labour.

IV. CONCLUSIONS

In this paper we have derived the elasticities of substitution and price elasticities of factor demand by using a translog cost function. The use of translog cost function instead of a conventional cost or production functions is preferred because of the restrictive structure of the latter functions.

From our estimates of the elasticities of substitution we find that like the results of the conventional techniques, labour and capital are substitutes. Our results like Malik *et al.* (1989) show $\sigma_{KL} > 1$. However, our result for σ_{KL} is higher than Battese and Malik (1988). Kemal (1981) and Khan (1989) found σ_{KL} to be lower than one.

The results show that capital and energy are strong complements, a result just opposite to the findings of Khan (1989) who found a low substitutability between energy and capital. The results also show that energy and labour are substitutes. And raw materials are substitutes with every other factor.

Our estimates of price elasticities of factor demand show that since capital and energy are complementary, and labour, capital and energy are substitutes, lifting of any subsidy on energy and capital would tend to reduce the energy and capital-intensity and, in turn, would increase the labour-intensity in the large-scale manufacturing sector of Pakistan. Adoption of such a policy can help in reducing the burden of unemployment. Moreover, investment incentives; such as, tax holidays accelerated depreciation allowances and subsidized interest rates, would imply increase in demand for both capital and energy. If energy saving and employment generation are the policy goals then investment incentives becomes less attractive as fiscal stimulants.

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Comments on "Derived Demand for Factors in the Large-scale Manufacturing Sector of Pakistan"

Zafar Mahmood has presented an interesting and provocative paper. The paper consists of specifying a transcendental logarithmic cost function for the large-scale manufacturing sectór in Pakistan and estimating it by the Iterative Seemingly Unrelated Estimation (ISURE) technique based on annual data for the years 1959-60 – 1984-85. From the estimates of the parameters of the translog cost function, Allen's partial elasticities of substitution/complementarity between the various factors are derived. Also calculated are the own and cross elasticities of demand of the factors. These factors being labour, capital, energy, and raw materials.

The translog function was first presented by Christensen *et al.*, way back in 1973. While there have been numerous studies based on this particular functional form in the U.S., this study is the first of its kind that uses it for Pakistan. In fact this paper's main contribution lies not in the results presented but in the competent application of existing methodology to the large-scale manufacturing sector in Pakistan.

This comment first takes up the issue of the choice of a particular functional form in estimating substitution elasticities. To fully comprehend the implications of the results presented, it then provides a brief historical sketch of the evolution of the large scale manufacturing and some stylized facts about it, and then goes on to sketch out the implications of the results. Finally some suggestions are offered that may improve the results and make them reflective of reality.

As Mahmood points out much of the earlier work which calculated elasticities of substitution for the manufacturing sector in Pakistan was based on the estimation of Cobb-Douglas and/or the Constant Elasticity of Substitution (CES) production functions. This was also the case in the U.S. These two functions are still widely used primarily because of their ease of use in estimating dynamic relationships: they require only the addition of past terms in relative factor prices or quantities as exogenous variables. They are also easy to work with. However they do have some limitations as noted by Mahmood i.e., strong separability, and they constrain the substitution elasticities between all pairs of factors to be equal. Indeed such stringent restrictions are not imposed by the translog function. However, since the translog function is usually estimated in an indirect form by assuming competitive equilibrium, it is difficult to estimate a dynamic version of it particularly for time-series estimation because in doing so one is assuming competitive equilibrium at each point in time. For Pakistan where clearly this has not been the case, the restrictions imposed by the Cobb-Douglas and/or CES may be the lesser of two evils. As I will show below given the peculiar nature of the development of the large-scale manufacturing sector, most of the results from time-series estimation for this sector are close to meaningless.

It would have been helpful to take into account the nature of development of the large-scale manufacturing sector in Pakistan during 1959-60 – 1984-85 before proceeding on this study since the professed objectives of the study are to help the policy-makers to take advantage of the substitution possibilities that exist among the factors. In 1955-56 when development was considered to be synonymous with industrialization/mechanization, the government of Pakistan embarked on a strategy of rapid industrialization. The policies that followed were aimed at fostering the large-scale manufacturing sector. These included artificially low interest rate loans for the importation of machinery and equipment, a system of multiple exchange rates which made the price of foreign capital and raw materials relatively low and the prices of foreign consumer goods relatively high. Through a system of licensing and differential tarriffs, monopolies were encouraged.

These policies had the effect of rapid industrialization but given the resulting relative factor prices, the structure of production that emerged was highly capital intensive. By 1970 it was common knowledge that the highly capital intensive nature of the production process had aggravated the unemployment situation and increased income inequalities. In 1973 there was widespread nationalization of a considerable portion of the large-scale manufacturing sector. More people were absorbed into these nationalized companies than could be warranted by relative factor productivities. Starting in 1978 these companies were gradually handed back to their original owners. However after the debacle of nationalization the large-scale manufacturing sector never fully recovered.

From the historical sketch given above, the period of Mahmood's study can be divided into three sub-periods: 1960–1972, 1973–1977, and 1978–1985. The first period corresponds to rapid growth and the large-scale sector can be characterized as being capital-intensive with limited substitution possibilities between capital and labour. The second period corresponds to stagnation and due to forced employment may appear capital or labour-intensive, and the final period can be characterized as the partial recovery phase where the original factor intensities as in the first period may have started to emerge. Given the peculiar nature of the evolution of the large-scale manufacturing sector a time series study which tries to estimate elasticities parameters is fraught with danger and the results should be interpreted with extreme caution if they are to be used at all.

A more general problem with time series studies is that time-series estimates of elasticities of substitution are likely to be biased if nonneutral technical changes occur that are correlated with factor price changes but not induced by them. If hicks-neutral technical change is assumed and there appears to be a correlation between the increased use of a factor and the decline in its relative price, the effect of the technological change will be mistakenly attributed to the change in the factor price. Therefore time-series estimations of the substitution parameters would be biased away from zero and greater substitution possibilities would be indicated than might be the case.

In light of the above it would have been preferable to use cross-section data for this study and the same objective could have been better served. However since I do not know the state of cross-section data that exists for the large-scale manufacturing sector, maybe this was not possible.

The estimated Allen partial elasticities of substitution indicate that labour is, in descending order, highly substitutable for energy, capital, and materials. Also the elasticity of demand for labour is greater than one in absolute value. This is the most controversial result which does not accord with either intuition nor the facts presented above. At the extreme these results imply that the large-scale manufacturing sector can get by with labour alone. All that is needed is to reduce the relative price of labour and it will be substituted for every other factor in the production process. And this is essentially what Mahmood ends up recommending to policy-makers. This is a startling example of 'results establishing the facts' rather than *vice-versa*. Instead of questioning why he got such results he takes them as valid based on statistical tests of significance.

This study can stand improvement. Firstly the data needs to be refined particularly for the user cost of capital. Mahmood uses the 'market' rate of interest as one component. Especially in Pakistan there is no such animal. Is it the rate on loans for importation of capital or purchase of land or construction of buildings or rate on government bonds? Berndt (1976) has shown that minor differences in the measurement of the price of capital can cause the estimates of substitution elasticities to vary sharply. Similarly Mahmood uses 5 percent as the depreciation rate on capital as the other component of the user cost of capital. Again no rationale is provide as to why he chooses 5 percent and whether it is an average depreciation on the principle value of capital or it is an annual rate on the remaining value of capital. Assuming it is the average depreciation rate it implies that capital has a useful life of 20 years. This is much longer than what is the case in Pakistan where in recent years machines are being replaced much faster. In the absence of any firm number on the depreciation several rates could be used to check on the robustness of the elasticity estimates. Finally, given the three distinct periods in the history of the large-scale manufacturing sector incorporation of intercept and/or slope dummy variables is called for which may improve the results.

It is important for predicting the effects of policy changes on demand for factors to have empirical estimates of elasticities of substitution at a disaggregated level. This paper has attempted to obtain such estimates. While the estimates appear to be biased for reasons indicated above, nevertheless the techniques are competently applied. Further studies using this methodology are called for which preferably rely on cross-section data and reliable estimates of factor prices.

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