The Pakistan Development Review
31 : 2 (Summer 1992) pp. 165 – 188

Technical Change, Efficiency, and Capital-labour Substitution in Pakistan's Large-scale Manufacturing Sector

SHAHID N. ZAHID, MOHAMMAD AKBAR and SHABBAR A. JAFFRY

Pakistan's manufacturing sector is characterised by relatively high capital intensity and the level of absorption of labour in industry is low. This paper estimates the elasticities of substitution in Pakistan's large-scale manufacturing sector to determine the potential for switching to relatively more labour-intensive production techniques. Data for the years 1960 to 1986 have been used and a total of seventeen industry groups have been analysed. This involved the aggregation of data from the Census of Manufacturing Industries (CMI). Industry groups were aggregated while keeping in mind the structural and economic similarities within the groups. The functional form used for the estimation is the CES production function and direct estimation procedures have been used. Industries in Pakistan are generally considered to be characterised by low substitution between capital and labour, near-constant returns to scale, high capital intensity, and low exogenous technical change. The results of this study bear this out with a few exceptions and the policy implications are interesting. The level of capital intensity in the manufacturing sector is not commensurate with the relative factor endowments, and there is a need to redirect the industries towards greater use of labour-intensive technology. In the short term, there appears to be little scope for altering the capital-labour ratios in the manufacturing sector. In the long run, however, measures aimed at the gradual replacement of capital with labour in production techniques may come to fruition.

I. INTRODUCTION

Like many other developing economies, Pakistan is faced with an acute problem of unemployment. With an increasing population, this problem is more

---

At the time of writing of this paper Shahid N. Zahid, Mohammad Akbar and Shabbar A. Jaffry were Senior Research Economist, Project Economist, and Project Economist, respectively, at the Applied Economics Research Centre (AERC), University of Karachi. Shahid N. Zahid is presently an Economist with the Asian Development Bank, Manila, and Shabbar A. Jaffry is a Lecturer in Economics at Southampton Polytechnic, England.

Authors' Note: We would like to thank Oldrich Kyn for his help with the methodology and the statistical procedures used to clean the data and perform the estimations; Hanid Mukhtar for his invaluable help with computer estimations; and Kaiser Bengali for his help with data collection. This study is based on the AERC Macro-econometric Model of the Pakistan Economy. Thanks are due to the AERC for the use of their model. We would also like to thank an anonymous referee of this journal for valuable comments that helped improve this paper. We alone, however, are responsible for any remaining errors.

than likely to be compounded further. Basically an agrarian economy, Pakistan has a large endowment of human resources. Unfortunately, its modern sector has failed to absorb in sufficient quantities the labour that was in surplus in the traditional sector and has over the years migrated to the urban centres.

One of the major reasons cited for this is the factor market distortions and the adoption of capital-intensive techniques of production in manufacturing ill-suited to the economy's factor endowments [Amjad (1982)].[1] The matter of the reasons and policies that led to the choice of these techniques is a separate issue. However, as a result of the adoption of these techniques, various problems arose, not least among them being the relatively insufficient absorption of labour in the productive industrial sector. If more attention had been diverted towards the adoption of production techniques commensurate with the relative factor endowments, the problem of urban employment may not have been as severe as it is as present.

Switching from the capital-intensive to the relatively more labour-intensive techniques of production in the manufacturing sector is not an easy task, especially in the short- to medium-term. However, a study of the possibilities of substitution between factor inputs is necessary before any attempt can be made to begin altering the relative factor use in industry. If the elasticity of substitution is large, even a minor reduction in the relative price of labour could lead to a substantial change in the employment of labour in that industry. On the other hand, if the elasticity of substitution is low for any industry, then changes in the factor markets will have relatively little effect on changes in factor use. Low elasticities of substitution could, however, induce policy-makers to encourage the development and adoption of new and alternative techniques of production for those industries. Keeping in view these considerations, this paper focuses on estimating the magnitude of the elasticities of substitution for large-scale manufacturing industries in Pakistan. The process of technical change and efficiency in Pakistan's large-scale manufacturing sector is also discussed, and the period covered is from 1960 to 1986.

The following section is a brief review of the literature, mainly related to the estimation of the elasticities of substitution in the Pakistani context. The next section discusses the methodology and the data. The results of the study and their analysis are presented in the following section. The policy implications and certain limitations of the analysis are discussed in the concluding section. The appendices provide the mathematical details of the methodology used, as well as the statistical details of the results.

---

[1] There could be a number of other reasons that may have led to the particular choice of techniques, including trade policies, that led to a bias in favour of manufacturing, thereby leading to inefficiency. However, this paper deals essentially with the issue of capital-labour substitution and does not discuss the reasons for the choice of techniques.

II. REVIEW OF THE LITERATURE

Given the implications of the possibilities of substitution between capital and labour, the number of studies in this area for both developing and developed countries is very large. The estimation of the elasticities of substitution between capital and labour in the manufacturing sector has been done using both the time series and the cross-section data. Although various kinds of production functions have been used, ever since [Minhas *et al.* (1961)] developed the constant elasticity of substitution (CES) production function, this form has been most extensively used in the literature. The number of studies is too extensive to covered by this paper and only a few of relevance to developing countries, and of direct relevance to the Pakistani context, will be discussed.

The pioneering work on Pakistan's manufacturing sector was done by Kazi *et al.* (1976). They used the CES production function to estimate the elasticities of substitution between capital and labour for twelve selected industries, using both the time series and the cross-section data. The main finding of their study was that the time series data yielded comparatively lower elasticity values than the cross-section data, suggesting greater substitution possibilities across industries – which is to be expected.

Kemal (1981), using both the CES the VES (variable elasticity of substitution) functions, and the time series data for sixteen large-scale manufacturing industries and the sector as a whole, estimated the elasticities of substitution between capital and labour. His results show that, despite the fact that for a number of industries the elasticities are quite low, the elasticity of substitution is both high and significant for the manufacturing sector as a whole.

Kemal's work was subjected to severe criticism by Ahmad (1982), who questioned both the data base and the estimation procedure used by Kemal. In his reply, Kemal (1982) admitted that there were some shortcomings in the data base he had used, and also in the estimation procedures employed, but added that these were common problems associated with any set of data relating to developing countries. Kemal further suggested that developing countries should develop their own technologies, guided by their factor endowments, and should be wary of importing a technology that may be ill-suited. Simply assuming that developing countries cannot develop their own technology was not sufficient reason to introduce an inappropriate technology.

In recent years, Battese and Malik (1987) have done work in this area. They have estimated the elasticities of substitution specifying a firm-level stochastic CES production function for both large- and small-scale firms in twelve manufacturing industries in Pakistan. They conclude that there may exist more possibilities for the substitution of labour for capital in manufacturing industries in Pakistan than has been suggested by earlier research.

The most recent work by Battese and Malik (1988) uses data collected during a survey of large-scale firms within the manufacturing sector in Pakistan; and they have estimated the elasticities of substitution for the food-processing industry. They establish that the elasticities of substitution for individual firms within the food processing industry are essentially similar and, therefore, the data can be aggregated to efficiently estimate the elasticity for the entire food-processing industry.

Khan (1989) has estimated the two-level CES production function for the manufacturing sector of Pakistan. This involved using a nested production function for capital and energy within the overall CES function for the sector. This allowed the analysis of substitution possibilities between capital and energy within the manufacturing sector, as well as the substitution between capital and labour. This technique is becoming increasingly popular since it allows a greater insight into the complexities of the manufacturing sector, which a single-level production function approach may not necessarily allow.

In the present paper, gaps and discrepancies in the data base have been remedied by using statistical techniques for both interpolation and extrapolation. The results have been corrected for the resulting loss in the degrees of freedom. Direct estimation procedures for the CES functions, using the latest version of the Gauss program specifically designed for such purposes, have been used to overcome the estimation problems faced in some of the earlier studies.

III. METHODOLOGY AND DATA

The manufacturing sector in Pakistan can be divided into two sub-sectors: the large-scale manufacturing and the small-scale manufacturing. Since there are no time-series data available for the small-scale manufacturing sector, this paper deals only with large-scale manufacturing using data published in the Census of Manufacturing Industries (CMI). The Industries in the manufacturing sector can be broadly classified into three basic groups, namely:

(i) Consumer goods industries;
(ii) Intermediate goods industries; and
(iii) Capital goods industries.

The second group can be further sub-divided into non-investment intermediate goods and investment-related intermediate goods. This grouping of the manufacturing sector helps in understanding the nature of growth in manufacturing and the technical change and possibilities that have taken place and may exist. It also helps with the aggregation of the numerous industries in large-scale manufacturing.

The CMI, the basic source of the data on large-scale manufacturing in Pakistan, contains information at a highly disaggregated level. However, much of the information is aggregated by major industry groups, of which there are a total of thirty-eight (38). For the purposes of this exercise, a further aggregation was done and a total of seventeen (17) industry groups were formed. The list of these seventeen industries divided into three basic groups of industries has been given in Appendix I. The aggregation was done keeping a few relevant factors in mind. First, some of the major well-defined industries (ghee, sugar, cement, fertilizers, etc.) were left as they were. This was not only logical from the point of view of the specification of the production function but also relevant from the policy perspective. Other industry groups were aggregated keeping in mind the structural and economic similarities within the industry groups thus defined. In general, the level of aggregation is quite straightforward and, as far as possible, industries were left as disaggregated as was considered logical. After seventeen major industry groups had been defined, there remained a whole host of relatively less important (in terms of value-added, employment, etc.) industries. These industries, from all three basic groups (consumer goods, intermediate goods, and capital goods) could be termed "all other manufacturing" and have not been covered in the present analysis.

The functional form used for the seventeen industries is the CES production function. This form can accommodate both fixed and variable coefficients and allows for the greatest flexibility in terms of substitution between capital and labour. The dependent variable is the gross value of output. The independent variables are capital and labour. A time-variable is used to take into account any technical change or time trend.

Most industries in the manufacturing sector are characterised by excess capacity. Installed capacity is fixed in the short run and the level of use of labour (and intermediate inputs) will vary. In the general case, the production function will be:

$$Q_i = \eta_i \{\delta_i K_i^{-\rho i} + (1-\delta_i) L_i^{-\rho i}\}^{-1/-\rho i} . e^{\lambda it} \quad \ldots \quad \ldots \quad (1)$$

where:

Q_i = Value of output in *i*th industry;

K_i = Capital in the *i*th industry;

L_i = Labour in the *i*th industry;

t = Time;

η_i = Efficiency parameter in the *i*th industry;

δ_i = Distribution parameter in the *i*th industry;

ρ_i = Substitution parameter in the *i*th industry;

λ_i = Technical change parameter in the *i*th industry;

σ_i = $1/1+\rho_i$ = elasticity of substitution between capital and labour in the *i*th industry; and

K_i is fixed in the short run while labour, L_i, can vary.

In the general case, we have assumed that capital, K_i, is fixed in the short run. Thus $K_i = K$. Labour, L_i, on the other hand, can vary. In such a case, profits can be defined as:

$$\pi_i = P_i\ [Q_i] - w_i\, L_i \qquad \dots \quad \dots \quad \dots \quad \dots \quad \dots \qquad (2)$$

where:

π_i = Profits in the *i*th industry;
P_i = Price of output in the *i*th industry; and
w_i = Wage rate in the *i*th industry.

Maximising profits, we have:

$$\frac{\partial\pi}{\partial L_i} = P_i\,(\partial Q_i/\partial L_i) - w_i = 0 \dots \quad \dots \quad \dots \quad \dots \quad \dots \qquad (3)$$

$$\Rightarrow\ w_i = P_i f_{Li} \qquad \dots \quad \dots \quad \dots \quad \dots \quad \dots \qquad (4)$$

For the simultaneous estimation of the CES production function and the labour demand function, we are allowing for non-constant returns to scale (non-CRTS) and technical change. From (1) we have:

$$Q_i = \eta_i\ \{\delta_i K_i^{-\rho i} + (1 - \delta_i)\, L_i^{-\rho i}\}^{-1/\rho i} \,.\, e^{\lambda i t} \quad \dots \quad \dots \quad \dots \qquad (5)$$

and from (4) : $L_i = g\,(P_i, Q_i, w_i)$ ... ... (6)

Equation (6) is the derived labour demand function, with labour a function of output and prices. (For detailed mathematical derivation of the labour demand function see Appendix II.) The CES production functions, allowing for non-CRTS and the possibility of technical change, were estimated using a simultaneous mixed estimation technique with the conditional labour demand function as given in relation (5) and (6). Using the Gauss technique allowed for the direct estimation of the parameters of the CES function for all seventeen industries. This helps avoid the manipulation of the functional form into a linear equation, while giving at the same time the NLS estimates of both the output of each industry and that industry's demand for labour.

Before the estimation was done, prior values for the parameters, δ, η, and ρ were assigned for each industry. The priors are starting-values for the iterative nonlinear estimates. The choice of the priors was limited because of the lack of any substantial work on production in the manufacturing sector in Pakistan. Independent information from various industries – on technical change, returns to scale, capital-labour ratios, and the possibilities of substitution between capital and labour – was used to supplement the scant information available. The main source of the previous estimates for these parameters was the work of Kemal (1976, 1981). Unfortunately, the classification of industries used by Kemal was not the same as used in our study; thus, certain assumptions and approximations had to be made when assigning the prior values. The prior values, alongwith their variance, used for each of the industries are given in Appendix III. The sensitivity of the estimates to different prior values was tested and the results were found to be robust to different starting-values.

The primary source of information on the manufacturing sector in Pakistan are the CMIs. There are, however, some gaps and the data for various years have not been published. Statistical techniques of interpolation and extrapolation were used to fill in these gaps and the data were completed for the years 1960 to 1986 inclusive. These are the years for which the estimations were made.

IV. RESULTS AND ANALYSIS

The summary of the estimated parameters is given in Table 1. The separate estimated results for each of the seventeen industries, alongwith the relevant statistical information, are given in Appendix IV.

The manufacturing sector in Pakistan appears to be characterised by varying returns to scale. For sugar, rubber products, textiles, fertilizers, and non-ferrous metals, the value of η is fairly close to One and these industries are possibly characterised by constant returns to scale. For the rest of the industries, there are, strictly speaking, decreasing returns to scale, since the efficiency parameters are less than One. For the chemical products industry, the value of η is as low as 0.1138. Transport equipment, on the other hand, appears to have increasing returns to scale, with the value of η much higher than One at 1.4512. This appears to be slightly on the high side and could be due to the specific conditions faced by the industry over the years.

The technological change that has taken place in the transport equipment industry has been endogenous to the industry, and is possibly being captured here rather than in the exogenous technical change parameter, λ.[2] Also, in the early years, this industry was characterised by under-utilisation of capacity, but this has reduced over the years, thereby suggesting increasing returns to scale. For the

---

[2] There is no control for the endogeneity of technological change in the estimates, and it is possible that such change is being reflected in all the estimates. However, for transport equipment, for reasons stated, this effect is very pronounced.

Table 1

Estimated Parameters for the Manufacturing Sector 1960-1986

Industry	Efficiency Parameters [Returns to Scale] Nu (η)	Distribution Parameter Delta (δ)	Technical Change Parameter Lambda (λ)	Substitution Parameter Rho (ρ)	Elasticity of Substitution Sigma (σ) σ = 1/1+ρ
1. Ghee	0.8020	0.7511	0.0790	1.0382	0.49062
2. Sugar	1.0332	0.6315	0.0150	2.5127	0.28468
3. Other Food	0.7696	0.6764	–0.0022	2.2434	0.30832
4. Beverages	0.7768	0.6879	0.0918	1.6359	0.37937
5. Tobacco	0.7814	6.2279	0.0912	1.3977	0.41706
6. Textiles	0.9093	0.7990	0.0792	1.3563	0.42440
7. Drugs and Pharmaceuticals	0.7088	0.0900	0.1092	–0.2456	1.32556
8. Industrial Chemicals	0.5879	0.6465	0.0054	1.8884	0.34620
9. Chemical Products	0.1138	0.7000	0.1695	3.5339	0.22056

Continued –

Table 1 – (*Continued*)

10. Fertilizers	0.9777	0.6469	0.0936	8.3277	0.10721
11. Petroleum Products	0.8255	0.6466	0.1273	3.8850	0.20471
12. Non-ferrous Metals	0.9850	0.7167	0.0625	1.1251	0.47056
13. Rubber Products	1.0427	0.5900	0.1214	0.8696	0.53487
14. Cement	0.8647	0.6095	0.0328	3.0939	0.24426
15. Iron and Steel	0.7260	0.8208	0.0133	1.4922	0.40125
16. Machinery	0.8594	0.7895	0.0754	1.1482	0.46550
17. Transport Equipment	1.4512	0.9589	0.0529	2.3549	0.29807

majority of the industries, however, the value of the returns to scale parameter, η, is between 0.75 and One, suggesting near-constant returns to scale; and, in general, one could say that the manufacturing industries in Pakistan, with a few exceptions, are reasonably efficient. The estimates for the efficiency parameter have very high *t*-statistics. The significance of these estimates is, at least, at the 95 percent level (see Appendix IV).

Over the period of this study, 1960–1986, it would appear that there has been very limited exogenous technical change and the values of the technical change parameter, λ, are generally low though statistically significant. The value of λ is the lowest for industrial chemicals and the highest for chemical products. Petroleum products, rubber products, and drugs and pharmaceuticals are also characterised by more technical change than the other industries, while iron and steel, and sugar display very little exogenous technical change. In general, consumer goods industries appear to have had very little technical change.

Looking at the distribution parameter, δ, it is apparent that industry in Pakistan is generally capital-intensive, with the values higher than 0.5. The exception is the drugs and pharmaceuticals industry, which is significantly labour-intensive ($\delta = 0.09$) and has always been so. The most capital-intensive industry is transport equipment ($\delta = 0.9589$), followed by iron and steel ($\delta = 0.8208$), textiles ($\delta = 0.79$), and machinery ($\delta = 0.7895$). Textiles is the only consumer goods industry with such high capital intensity.

The elasticities of substitution (given in the last column of Table 1) for all the industries are between zero and One. The exception is the drugs and pharmaceuticals industry, where the elasticity is greater than One. For several of the industries, the value of σ is very low (fertilizers = 0.107, petroleum products = 0.205, chemical products = 0.22), suggesting that these industries are displaying a tendency towards fixed input-output coefficients (as $\sigma \rightarrow$ zero, the CES function tends towards a fixed coefficients input-output function). Generally, the substitution is low, and even though the returns to scale are fairly close to One, the degree of substitution is not very high; which is what one would expect if the production function were of a Cobb-Douglas variety with constant returns to scale (as $\sigma \rightarrow$ One, the CES function tends towards a C-D function). The use of the CES function with non-CRTS was made for several reasons. If the industries did display substitution between capital and labour, then the CES function would be more suitable since it could accommodate all possibilities, from a degree of substitution close to zero, or very low, to a high elasticity of substitution. Also, since we are dealing with over 25 years of data, there is every likelihood that, over time, capital-labour substitution has occurred as a result of new machinery, which is more capital-intensive, endogenous technical change, and a move towards lower labour use. Thus, even though technology in the short run may be of a fixed coefficients variety, in the longer run, say 25 years or more, there may be significant substitution possibilities.

The results indicate that these substitution possibilities do exist, even though they are not very high. The exception is clearly the drugs and pharmaceuticals industry, where it appears that substitution possibilities are quite high. In general, this is one industry that is highly dependent on labour, particularly skilled labour. Over time, there may have been an accelerated tendency to reduce this dependency and bring in capital-intensive technology. Clearly, the aggregation of firms within an industry leads to this situation, where technologically different firms, or more capital-intensive firms within an industry, add up to give an impression of substitution taking place. In reality, this may be a reflection of the changing nature of technology within an industry, and not necessarily a result of substitution along a production isoquant.

Industries in Pakistan are generally considered to be characterised by low substitution between capital and labour, near-constant returns to scale, high capital intensity, and low exogenous technical change. The results of this study bear this out with a few exceptions. However, without exception, the *t*-statistics for all the estimates are high and their significance is, at least, at the 95 percent level.

Unfortunately, our estimated elasticities of substitution cannot be easily compared with earlier studies because the classification and aggregation of industries in the earlier studies are not the same as in this study. In any event, Table 2 attempts a comparison with Kemal (1981) and Kazi *et al.* (1976). The different time-period under consideration in each study makes this comparison more difficult.

For food, textiles, and iron and steel, the results compare favourably. For chemical products and rubber, the elasticities appear to have declined over the years; while for tobacco, machinery, and transport equipment, they have increased over time. In general, there appears to be less variation between the industries in the present study than in the previous studies.

V. CONCLUSIONS

The basic limitation of the present analysis is related to the data. Even though various techniques have been used to complete the time series from 1960 to 1986, the year-to-year fluctuations in the CMI data have not been corrected. These fluctuations are a result of under-reporting and non-response by individual firms. Some firms may respond to the CMI questionnaire in some years and not in others, and there is no way of controlling for the selectivity as to which firms respond to the CMI. This leads to irregular fluctuations from one year to the next. The problem with the data base would clearly influence the results. Despite this problem, however, the estimates generally support the existing foreknowledge about the manufacturing sector in Pakistan.

Table 2

Comparison of Elasticities of Substitution with Other Studies

			Elasticity of Substitution		
			Present Study (1960–1986)	Kemal's Study (1960–1970)	Kazi, *et al.* Study (1954–1970)
1.	Ghee	Food	0.491	0.298	0.260
2.	Sugar		0.286		
3.	Other Food		0.304		
4.	Beverages		0.379		
5.	Tobacco		0.380	–7.946	–
6.	Textiles		0.417	0.391	0.460
7.	Drugs and Pharmaceuticals		0.424	–	–
8.	Industrial Chemicals		1.325	–	–
9.	Chemical Products		0.346	0.298	1.590
10.	Fertilizers		0.220	–	–
11.	Petroleum Products		0.107	–	–
12.	Non-ferrous Metals		0.205	–	–
13.	Rubber Products		0.470	0.778	1.350
14.	Cement		0.535	–	–
15.	Iron and Steel		0.244	0.183	–
16.	Machinery		0.401	0.016	–
17.	Transport Equipment		0.465	–0.218	–
1.	Footwear		–	1.706	1.020
2.	Paper		–	0.124	0.320
3.	Leather		–	1.658	0.490
4.	Printing		–	2.021	1.520
5.	Non-metallic Numerials		–	–0.900	–1.760
6.	Electrical Machinery		–	–0.102	0.870
7.	Metal Products		–	0.187	–
8.	Others		–	2.023	–
9.	All Manufacturing		–	0.879	–0.220

From a policy perspective, the results of this study have some interesting implications. Clearly, the level of capital intensity in the manufacturing sector is not commensurate with the relative factor endowments and, thus, there is a need to redirect the industries towards greater use of labour-intensive technology. Exogenous technical change, in the past, has been very limited while substitution possibilities, in general, are low. Thus, in the short term, there appears to be little scope for altering the capital-labour ratios in the manufacturing sector. In the long run, however, given the right incentives and pricing structure, the introduction of endogenous technical change is a distinct possibility. This has happened in several of the industries in the past and could occur in the future, as well. In general, therefore, policy measures designed to have a short-term effect on factor intensity are unlikely to succeed. In the long run, however, measures aimed at the gradual replacement of capital with labour in production techniques may come to fruition.

Appendices

Appendix I

List of Industries in the Manufacturing Sector

A. Consumer Goods Industries

1. Ghee
2. Sugar
3. Other Food
4. Beverages
5. Tobacco
6. Textiles
7. Drugs and Pharmaceuticals

B. Intermediate Goods Industries

(i) ***Non-investment Related***

1. Industrial Chemicals
2. Chemical Products
3. Fertilizers
4. Petroleum Products
5. Non-ferrous Metals
6. Rubber Products

(ii) ***Investment Related***

1. Cement
2. Iron and Steel

C. Capital Goods Industries

1. Machinery
2. Transport Equipment

Appendix II

Mathematical Derivation of the Labour Demand Function

The CES production function is:

$$Q_i = \eta_i \{ \delta_i K_i^{-\rho_i} + (1-\delta_i) L_i^{-\rho_i} \}^{-1/\rho_i} . e^{\lambda it} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\Rightarrow \quad f_{Li} = \frac{\partial Q_i}{\partial L_i} = \eta_i \{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1/\rho_i} .$$

$$(-1/\rho_i)[-(1+\delta_i)\rho_i] L_i^{-\rho_i-1} . e^{\lambda it}$$

$$\Rightarrow \quad f_{Li} = \eta_i \{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1/\rho_i} . e^{\lambda it}$$

$$(1+\delta_i) \{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1} . L_i^{-(1+\rho_i)}$$

$$\Rightarrow \quad f_{Li} = Q_i (1+\delta_i) \{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1} . L_i^{-(1+\rho_i)} \quad \ldots \quad \ldots \quad (2)$$

From (1) we have:

$$\{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1/\rho_i} = \frac{Q_i}{\eta_i} \cdot e^{-\lambda it}$$

$$\Rightarrow \quad \{\delta_i K_i^{-\rho_i} + (1+\delta_i) L_i^{-\rho_i}\}^{-1} = \left(\frac{Q_i}{\eta_i} \cdot e^{-\lambda it} \right)^{\rho_i} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

From (2) and (3) we have:

$$f_{Li} = Q_i \, (1+\delta_i) \left(\frac{Q_i}{\eta_i} \cdot e^{-\lambda it} \right)^{\rho_i} \cdot L_i^{-(1+\rho_i)} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

From (4) and the profit-maximising condition $w_i = pif_{Li}$ we get:

$$L_i^{(1+\rho_i)} = \frac{P_i Q_i}{W_i} (1+\delta_i) \left(\frac{Q_i}{\eta_i} \cdot e^{-\lambda it} \right)^{\rho_i}$$

$$\Rightarrow \quad L_i = \left[\frac{P_i Q_i}{W_i} (1+\delta_i) \right]^{1+\rho_i} . \left(\frac{Q_i}{\eta_i} . e^{-\lambda it} \right)^{\frac{\rho_i}{(1+\rho_i)}}$$

$$\Rightarrow \quad L_i = [\frac{P_iQ_i}{W_i}(1+\delta_i))^{1/\rho_i} \cdot (\frac{Q_i}{\eta_i} . \; e^{-\lambda it})]^{\rho_i/(1+\rho_i)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

In logarithmic form we have:

$$ln(L_i) = ln\,(1+\delta_i)\,(\frac{P_iQ_i}{W_i}) \,.\; 1/\rho_i + (lnQ_i - ln\eta_i - \lambda_{it}) \;.\; \rho_i/(1+\rho_i) \qquad \ldots \qquad (6)$$

Thus, we have a derived labour demand function (5) or, in logarithmic form, (6) which gives labour in each industry as a function of output, Q_i, price of output, P_i, and the wage rate, W_i. For the estimations in this study the CES function for each industry was estimated, allowing for non-CRTS and technical change, simultaneously with a conditional derived labour demand function. The newly developed Gauss estimation technique, which allows for the direct estimation of the parameters, was used as it gives the NLS estimates of both output and labour.

Appendix III

Prior Values and Prior Variances Used in the Estimation

Name of Industry	Parameter	Prior Value	Prior Variance
Ghee	Delta	0.7000	0.0100
	Rho	2.5700	1.0000
	Nu	1.0000	0.0900
Sugar	Delta	0.7000	0.0100
	Rho	1.7000	1.0000
	Nu	0.5940	0.2500
Other Food	Delta	0.7000	0.0400
	Rho	1.7000	4.0000
	Nu	1.0000	0.0064
Beverages	Delta	0.7000	0.0100
	Rho	0.1574	1.0000
	Nu	1.0000	0.0100
Tobacco	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0100
Textiles	Delta	0.7000	0.0100
	Rho	2.8460	1.0000
	Nu	1.0000	0.0100
Drug and Pharmaceuticals	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0100
Industrial Chemicals	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0900
Chemical Products	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0400

Continued–

Appendix III – (*Continued*)

Name of Industry	Parameter	Prior Value	Prior Variance
Fertilizers	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0100
Petroleum Products	Delta	0.7000	0.0100
	Rho	2.3979	1.0000
	Nu	1.3674	0.1600
Non-ferrous Metals	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.0000	0.0100
Rubber	Delta	0.7000	0.0100
	Rho	0.0000	4.0000
	Nu	1.2353	0.0100
Cement	Delta	0.7000	0.0100
	Rho	0.8519	1.0000
	Nu	1.5548	0.0400
Iron and Steel	Delta	0.7000	0.0025
	Rho	6.0870	9.0000
	Nu	0.9318	0.0025
Machinery	Delta	0.7000	0.0400
	Rho	0.0000	4.0000
	Nu	1.3649	0.0900
Transport Equipment	Delta	0.7000	0.0400
	Rho	0.0000	4.0000
	Nu	3.0625	0.4900

Appendix IV

Detailed Estimates of Parameters for All Industries

Name of Industry	Parameter	Est. Coef.	*T*-Stat.	Prob. V.
Ghee	A	–0.1321	–0.1174	0.9070
	Delta	0.7511	7.7691	0.0000
	Rho	1.0382	4.0281	0.0002
	Nu	0.8020	5.1349	0.0000
	Lambda	0.0790	6.5466	0.0000

Iterations: 18.000
R^2 0.734
F-Stat. 33.782

Name of Industry	Parameter	Est. Coef.	*T*-Stat.	Prob. V.
Sugar	A	–0.2434	–0.2567	0.7985
	Delta	0.6315	6.7302	0.0000
	Rho	2.5127	7.6289	0.0000
	Nu	1.0332	8.1723	0.0000
	Lambda	0.0150	1.2414	0.2204

Iterations: 14.000
R^2 0.918
F-Stat. 137.528

Name of Industry	Parameter	Est. Coef.	*T*-Stat.	Prob. V.
Other Food	A	1.9402	3.9470	0.0002
	Delta	0.6764	3.9873	0.0002
	Rho	2.2434	3.7068	0.0005
	Nu	0.7696	12.0944	0.0000
	Lambda	–0.0022	–0.3584	0.7216

Continued –

Appendix IV – (*Continued*)

Name of Industry	Parameter	Est. Coef.	*T*-Stat.	Prob. V.
Iterations: 18.000				
R^2 0.657				
F-Stat. 10.395				
Beverages	A	0.7932	1.8207	0.0748
	Delta	0.6879	7.2184	0.0000
	Rho	1.6359	6.4592	0.0000
	Nu	0.7768	9.5034	0.0000
	Lambda	0.0918	11.1696	0.0000
Iterations: 9.000				
R^2 0.939				
F-Stat. 189.512				
Tobacco	A	1.9157	3.3352	0.0016
	Delta	0.6227	6.5694	0.0000
	Rho	1.3977	8.2342	0.0000
	Nu	0.7814	9.9764	0.0000
	Lambda	0.0912	22.6185	0.0000
Iterations: 8.000				
R^2 0.877				
F-Stat. 87.220				
Textiles	A	–0.4222	–0.4407	0.6614
	Delta	0.7990	8.3823	0.0000
	Rho	1.3563	5.2171	0.0000
	Nu	0.9093	9.9238	0.0000
	Lambda	0.0792	22.1756	0.0000
Iterations: 11.000				
R^2 0.932				
F-Stat. 168.602				

Continued –

Appendix IV – (*Continued*)

Drug and Pharmaceutical	A	1.9496	3.4391	0.0012
	Delta	0.0900	6.0460	0.0000
	Rho	–0.2456	–4.8703	0.0000
	Nu	0.7088	8.2350	0.0000
	Lambda	0.1092	31.0915	0.0000
Iterations:	10.000			
R^2	0.981			
F-Stat.	645.189			
Industrial Chemical	A	3.2997	5.2879	0.0000
	Delta	0.6465	6.8356	0.0000
	Rho	1.8884	5.0392	0.0000
	Nu	0.5879	7.0690	0.0000
	Lambda	0.0054	0.9823	0.3308
Iterations:	23.000			
R^2	0.683			
F-Stat.	11.608			
Chemical Products	A	9.8524	38.0291	0.0000
	Delta	0.7000	6.9999	0.0000
	Rho	3.5339	2.4350	0.0186
	Nu	0.1138	2.9864	0.0044
	Lambda	0.1695	42.4081	0.0000
Iterations:	43.000			
R^2	0.996			
F-Stat.	210.722			
Fertilizers	A	–1.2027	–2.0116	0.0498
	Delta	0.6469	7.1658	0.0000
	Rho	8.3277	6.1614	0.0000
	Nu	0.9777	11.9895	0.0000
	Lambda	0.0936	15.4409	0.0000
Iterations:	28.000			
R^2	0.949			
F-Stat.	230.362			

Continued –

Appendix IV – (*Continued*)

Name of Industry	Parameter	Est. Coef.	*T*-Stat.	Prob. V.
Petroleum Products	A	0.9088	1.6427	0.1068
	Delta	0.6466	6.7735	0.0000
	Rho	3.8850	6.3931	0.0000
	Nu	0.8255	10.6397	0.0000
	Lambda	0.1273	14.2616	0.0000
Iterations: 18.000				
R^2 0.747				
F-Stat. 36.215				
Non-ferrous Metals	A	–0.5622	–1.3714	0.1765
	Delta	0.7167	7.5648	0.0000
	Rho	1.1251	5.5078	0.0000
	Nu	0.9850	11.3136	0.0000
	Lambda	0.0625	6.6146	0.0000
Iterations: 8.000				
R^2 0.672				
F-Stat. 25.092				
Rubber	A	–1.6053	–2.6285	0.0114
	Delta	0.5900	6.4935	0.0000
	Rho	0.8696	6.8711	0.0000
	Nu	1.0427	11.3724	0.0000
	Lambda	0.1214	19.7029	0.0000
Iterations: 12.000				
R^2 0.927				
F-Stat. 155.077				
Cement	A	0.4185	0.5365	0.5940
	Delta	0.6095	6.9388	0.0000
	Rho	3.0939	6.0425	0.0000
	Nu	0.8647	8.6145	0.0000
	Lambda	0.0328	5.9026	0.0000

Continued –

Appendix IV – (*Continued*)

Iterations:	13.000			
R^2	0.748			
F-Stat.	36.438			
	A	2.2807	6.8778	0.0000
	Delta	0.8208	17.2079	0.0000
Iron and Steel	Rho	1.4922	8.6815	0.0000
	Nu	0.7260	16.7488	0.0000
	Lambda	0.0133	1.9661	0.0550
Iterations:	25.000			
R^2	0.760			
F-Stat.	38.565			
	A	–0.5879	–0.4852	0.6297
	Delta	0.7895	4.1942	0.0001
Machinery	Rho	1.1482	2.3066	0.0253
	Nu	0.8594	5.3346	0.0000
	Lambda	0.0754	6.0829	0.0000
[illegible]ations:	21.000			
[illegible]	0.732			
[illegible]at.	33.525			
	A	–5.0377	–4.1499	0.0001
	Delta	0.9589	16.9074	0.0000
Transport Equipment	Rho	2.3549	3.9869	0.0002
	Nu	1.4512	8.2139	0.0000
	Lambda	0.0529	11.4600	0.0000
Iterations:	13.000			
R^2	0.939			
F-Stat.	191.055			

REFERENCES

Ahmad, Meekal A. (1982) Substitution Elasticities in the Large-scale Manufacturing Industries of Pakistan: A Comment. *The Pakistan Development Review* 21:1.

Amjad, Rashid (1982) *Private Industrial Investment in Pakistan, 1960–70.* Cambridge: Cambridge University Press.

Battese, G. E., and S. J. Malik (1987) Estimation of Elasticities of Substitution for CES Production Function Using Data on Selected Manufacturing Industries in Pakistan. *The Pakistan Development Review* 26:2.

Battese, G. E., and S. J. Malik (1988) Estimation of Elasticities of Substitution for CES and VES Production Functions Using Firms-level Data for Food-processing Industries in Pakistan. *The Pakistan Development Review* 27:1.

Kazi, S., Zahira Saleem Khan and Seemin A. Khan (1976) Production Relationships in Pakistan's Manufacturing Industries. *The Pakistan Development Review* 25:4.

Kemal, A. R. (1976) Sectoral Growth Rates and Efficiency of Factor Use in LSM Sector in West Pakistan. *The Pakistan Development Review* 15:4.

Kemal, A. R. (1981) Substitution Elasticities in the Large-Scale Manufacturing Industries of Pakistan. *The Pakistan Development Review* 20:1.

Kemal, A. R. (1982) Substitution Elasticities in the Large-Scale Manufacturing Industries of Pakistan – A Rejoinder. *The Pakistan Development Review* 21:2.

Khan, Ashfaque H. (1989) The Two-level CES Production Function for the Manufacturing Sector of Pakistan. *The Pakistan Development Review* 28:1.

Minhas, B. S., K. J. Arrow, H. B. Chenery and R. M. Solow (1961) Capital Labour Substitution and Economic Efficiency. *The Review of Economics and Statistics* 43:3.