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Production Functions for Wheat Farmers in Selected Districts of Pakistan: An Application of a Stochastic Frontier Production Function with Time-varying Inefficiency Effects

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Two models of technical inefficiency with a stochastic production frontier are considered in this paper. In the first model, it is assumed that the frontier itself does not vary with time, while in the second, the frontier is allowed to move. These models are applied to four years of panel data on wheat farmers in four districts of Pakistan: Faisalabad and Attock in the Punjab, Badin in Sindh, and Dir in the NWFP. Using essentially the same stochastic frontier production function in each of the four districts involved, different stochastic specifications for the inefficiency effects are obtained for the different districts. Technical efficiencies of production of the individual farmers are predicted in each year in which they are observed. Varying patterns of technical inefficiency are observed. The null hypothesis (of no technical inefficiency) cannot be rejected in only one district. In the other districts, while inefficiencies appear to be present in all of them, they are declining at a fairly rapid pace in one. The results highlight the importance of analysis at a disaggregated level because it is clear that both the rate of technical change and relative efficiencies vary across regions and explicit cognisance must be taken of this both in research and policy formulation.

1. INTRODUCTION

The modelling and estimation of stochastic frontier production functions has been a subject of considerable interest in econometrics and applied economic analysis during the last two decades. Reviews of frontier production functions are given by Førsund, Lovell and Schmidt (1980, 1986); Bauer (1990) and Battese (1992). Extensive bibliographies of empirical applications of frontier functions and efficiency analysis are given by Ley (1990) and Beck (1991).

This paper applies the stochastic frontier production function model of time-varying technical inefficiencies, proposed by Battese and Coelli (1992), in the anal-

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ysis of agricultural data which have been recently collected in a panel study of Pakistani households by the International Food Policy Research Institute (IFPRI). The structure of the paper is as follows: Section 2 contains a brief discussion of the stochastic frontier production function model which is considered in this paper; Section 3 outlines the panel study conducted by IFPRI in selected districts in Pakistan during 1986–1991; Section 4 presents the frontier production function, in terms of the variables involved; the empirical results are presented and discussed in Section 5; and concluding remarks are made in Section 6.

2. THE STOCHASTIC FRONTIER MODEL

The frontier production function proposed by Battese and Coelli (1992) assumes that a random sample of farms is observed over T periods, such that the production of the N farms over time is a given function of input variables and random variables which involve both traditional random errors and non-negative random variables, which are associated with technical inefficiencies of production. The latter inefficiency effects are assumed to be defined by the product of an exponential function of time and a non-negative farm-specific random variable. The model is defined by

$$Y_{it} = exp\,(X_{it}\beta + V_{it} - U_{it}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

and

$$U_{it} = \{exp\,[-\eta(t - T)]\}U_i, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where

$t \varepsilon I\,(i)$ and $i = 1, 2, \ldots, N$;

Y_{it} denotes the production of the i-th farm at the t-th year of observation;

X_{it} represents a $(1 \times k)$ vector of known functions (for example, logarithms) of factor inputs and other relevant explanatory variables, associated with the production of the i-th farm in the t-th period of observation;

β represents a $(k \times 1)$vector of unknown parameters, associated with the explanatory variables in the production function;

$V_{it}s$ are independent and identically distributed random errors which are assumed to have normal distribution with mean, Zero, and unknown variance, σ_v^2;

$U_i s$ are independent and identically distributed non-negative random variables, which are assumed to arise from the truncation (at Zero) of the normal distribution with unknown mean, μ, and unknown variance, σ^2;

η is an unknown scalar parameter; and

$I(i)$ represents the set of T_i time periods from the T periods, for which production observations are obtained for the i-th form involved (that is, $I(i)$ is a subset of the integers, 1, 2, ..., T, which must contain at least one element).

The U_{it} random variables are non-negative random variables, which arise because of technical inefficiencies of production of the farms producing the particular output involved. The deterministic function of time, $exp[-\eta\ (t - T)]$, is greater or less than One if η is positive or negative, respectively. Thus, if the parameter η is positive, the inefficiency effects for farm i, U_{it} for $t \varepsilon I(i)$ decrease towards U_i as t increases towards the last period in the panel, T. This case would be associated with situations in which the farms producing a particular output are such that the technical inefficiencies of production decline over time (that is, the farms become more technically efficient over time).

The model for the inefficiency effects U_{it} in Equation (2) is such that if the i-th farm is observed in the last period of the panel, T, then the inefficiency effect at time T is U_i. Thus, the assumption that the U_i's, $i = 1, 2, ..., T$, are the non-negative truncation of the $N\ (\mu, \sigma^2)$-distribution specifies the distribution of the inefficiency effects in the stochastic frontier production function for the last period of the panel, T.

The inefficiency model (2) includes the time-invariant model, which has been applied in a number of early studies of frontier production functions involving panel data [*e.g*., Battese and Coelli (1988); Kalirajan and Shand (1989); Schmidt and Sickles (1984)]. Thus, it is possible to test if the technical inefficiencies of production are time invariant, given the particular parametric specification defined by Equation (2).

The technical efficiency of production of the i-th form at the t-the period of observation is defined by

$$TE_{it} = exp\ (-U_{it})\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

which is the ratio of the observed production, Y_{it}, to the corresponding stochastic frontier production, $Y_{it}^* \equiv exp\ (X_{it}\,\beta + V_{it})$, associated with no inefficiency of production. Battese and Coelli (1992) proposed a predictor for this unobservable random variable, which can be evaluated at the maximum-likelihood estimates of the parameters of the frontier model. The maximum-likelihood estimates of the parameters and the technical efficiencies can be obtained by using the programme, FRONTIER, 2.0, written by Tim Coelli.[1] The programme assumes that the stochastic frontier production function is of the form specified by Equation (1) (for example, of the Cobb-Douglas or the translog type). Battese and Coelli (1992) applied their model in the analysis of panel data from a sample of paddy farmers in an Indian village. Additional applications of the model are found in Battese and Tessema (1993); Tran, Coelli and Fleming (1993) and Kanjilal, Zapata and Heagler (1993).

3. IFPRI'S PANEL STUDY IN PAKISTAN

The data used in these analyses are based on IFPRI's ongoing study of household decision-making in Pakistan which aims to assist the Ministry of Food and Agriculture of the Government of Pakistan in achieving its general goal of poverty alleviation. The study was conceived in 1985-86 to provide policy-relevant research and information on planning nutrition and anti-poverty policies based on several long-term studies.

These long-term studies were based on a panel survey, the first of its kind in Pakistan, initiated in July 1986 in selected districts of rural Pakistan. This panel survey of households[2] involved 14 visits to the same household between July 1986 and August 1991.

In order to select the sample, a list of the poorest districts in each of the provinces of Pakistan was drawn up. This list was based on a variety of production and infrastructure indices. The methodology followed was that of Pasha and Hasan (1982), although more recent information and current district boundaries were used wherever possible. From these lists of the poorest districts in each province, the districts selected for obtaining sample data were Attock in the Punjab, Badin in Sindh, Dir in the North West Frontier Province (NWFP), and Kalat in Balochistan.[3] In addition to these districts, one of the more prosperous districts in the Punjab, Faisalabad, was chosen for the purposes of comparison. This choice was based on the recognition that there are poor people even within prosperous districts and it is important to obtain a perspective on such households.

[1]This programme has been written for IBM compatible PCs and can be obtained free of charge by writing to Tim Coelli, Department of Econometrics, University of New England, Armidale, NSW, Australia. FRONTIER, 2.0 can be used to estimate stochastic frontier production functions for cross-sectional data or panel data with time-invariant inefficiency effects. The half-normal distribution can also be assumed for the estimation of the parameters.

[2]For the purposes of this study, a household is defined as a group of individuals who eat at the same hearth, although they are not necessarily directly related. This definition is generally followed in household surveys in Pakistan.

[3]Kalat was subsequently dropped after the fourth round of data collection because of logistic difficulties in continuing fieldwork. The analysis, therefore, is concentrated on districts in three provinces: the Punjab, the NWFP, and Sindh.

The sample villages and households within the selected districts involved were chosen by stratified random sampling. Within each of the selected districts, two *mandis* (markets) were chosen at random. For each *mandi* thus selected, villages were allocated to one of three strata: villages within 5 kilometres of the *mandi*; villages between 5 and 10 kilometres; and those between 10 and 20 kilometres. Villages were then chosen randomly from these strata. Household lists of each of the selected villages were prepared and survey households randomly chosen from this list. Minor variations in this process were made to account for the special conditions in each area; for example, while villages in the Punjab typically cover hundreds of families, villages in the lower part of Sindh are administrative units and may consist of a number of geographically distant *dehs* (settlements). For Sindh, therefore, an additional random sampling was used to select a subset of *dehs* from the village lists.

The interviews were conducted by a team which included three males and three females in each district. A specific male and female household questionnaire was administered to each household. The information thus collected contained detailed information on crop production and input use in addition to information on many other variables.

The survey covers a total of 880 households in the three provinces, of which there are a maximum of 336 wheat-growing households in any of the five wheat-growing seasons. However, in the fourth year, information was only collected on the value of crop production. In the present analysis, the actual quantities of production are used. Therefore, data from the fourth season were not included.

4. THE FRONTIER PRODUCTION FUNCTION

The stochastic frontier production function, which is considered for wheat farmers in the selected districts of Pakistan, is defined by

$$\begin{aligned} log\ Y_{it} = &\ \beta_0 L + \beta_1\ log(Land_{it}) + \beta_2\ log(Labour_{it}) + \beta_3\ (HL_{it}/Labour_{it}) \\ &+ \beta_4 D_{1it} + \beta_5 D_{1it} + log(Fert_{it}) + \beta_6\ log(Prep_{it})\ \beta_7 D_{2it} \\ &+ \beta_8 D_{2it}\ log(Prep_{it}) + \beta_9\ log(Plou_{it}) + \beta_{10}\ log(Seed_{it}) \\ &+ \beta_{11} D_{3it} + V_{it} - U_{it}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \end{aligned} \tag{4}$$

where;

Y_{it} represents the quantity of wheat harvested (in kilograms) in the t-th year of observation on the i-th farm in the particular district involved;

$Land_{it}$ is the total amount of land (in acres) on which wheat is grown in the t-th year on the i-th farm;

$Labour_{it}$ is the total amount of labour (in male equivalent days)[4] by family members and hired labourers in the production of wheat[5] in the t-th year on the i-th farm;

HL_{it} represents the amount of labour (in male equivalent days) which was hired in the t-th year on the i-th farm;

D_{1it} is a dummy variable, which has value One in the t-th year for the i-th farm, if the quantity of fertilizer applied in the production of wheat (as NPK) was positive, and value Zero otherwise;

$Fert_{it}$ is the amount of NPK (in kilograms) applied on land for wheat production in the t-th year on the i-th farm;

$Prep_{it}$ is the total hours of land preparation by use of bullocks or mechanical traction[6] in the t-th year on the i-th farms;

D_{2it} is a dummy variable, which has value One if the i-th farmer used mechanical traction for wheat production in the t-th year, and value Zero otherwise;

$Plou_{it}$ represents the number of ploughings in the t-th year on the i-th farm;

$Seed_{it}$ represents the quantity of wheat seed (in kilograms) sown in the t-th year on the i-th farms; and

D_{3it} is a dummy variable, which has value One if the i-th farmer was an owner/tenant during the t-th year, and value Zero otherwise.

The assumptions about the random variables, V_{it} and U_{it}, are identical to those specified for the frontier model defined in Equations (1) and (2).

The frontier production function of Equation (4) is specified separately for wheat farms in Faisalabad, Attock, and Badin. For wheat farms in Dir, the frontier production function involved has an additional explanatory variable, IL/Land, which is the ratio of acres of wheat which are irrigated (represented by IL) to the total area

[4]Male equivalent days were calculated using the following formula: one adult male working for one day equals one male day; one female or one child working for one day equals 0.75 or 0.50 male days, respectively.

[5]The labour variable involves man-days of labour for all operations involved in the production of wheat upto, and including, harvesting. As stated in the discussion of the empirical results, consideration was also given to the labour variable, involving only pre-harvest labour.

[6]Hours of bullock labour were for bullock pairs. Wheat farmers generally used bullocks or tractors, rather than some of both.

under wheat. The latter variable is included for Dir to account for the expected different productivity of irrigated land from that for rain-fed land under wheat. Wheat is only grown in Faisalabad, Attock, and Dir under rain-fed conditions.

The above production function is of the Cobb-Douglas type, involving eleven explanatory variables for Faisalabad, Attock, and Badin, whereas twelve explanatory variables are involved for wheat farmers in Dir. Although it is possible to estimate a frontier production function of the translog type using FRONTIER, 2.0, the present limitations do not permit the estimation of a translog model with 11 or 12 different explanatory variables.

The inclusion of the ratio of hired labour to the total of hired and family labour in the above production function accounts for the possibility that hired and family labour are not equally productive in the growing of wheat. As explained in Battese and Coelli (1992) and Battese and Tessema (Forthcoming), this model can be considered as a linearised version of a Cobb-Douglas production function involving a labour variable which is a weighted average of the number of hours of hired labour and family labour [cf. footnote 6 of Battese and Coelli (1992), p. 167, in which the discussion involves non-irrigated land and irrigated land]. If the productivities of hired and family labour were equal, then the coefficient β_3 of the labour-ratio variable, HL/Labour, in the production function, would be Zero. Hence, tests of the null hypothesis, H_0: $\beta_3 = 0$, are of interest to indicate whether hired and family labour are equally productive in the growing of wheat in the four districts involved.

The dummy variable D_1 is included in the model to account for different production regimes for farmers who fertilize in the growing of wheat, relative to those who do not. The dummy variable D_2 is included in the production function to permit the response of wheat under conditions involving mechanical traction to be different from that in which bullock labour is involved. Given that the wheat farmers either use mechanical tractors or bullocks in growing wheat in the districts involved, the production functions for farmers with mechanical traction involve the terms $(\beta_0 + \beta_7) + (\beta_6 + \beta_8)$ log(hours of mechanical traction). However, for farmers using only bullock labour, the production function specifies that the relevant terms are $\beta_0 + \beta_6$ log(hours of bullock labour). Thus, β_6 is the elasticity of bullock labour, whereas $\beta_6 + \beta_8$ is the elasticity of mechanical traction.

The dummy variable D_3 is included in the production function to account for the possible different level of production of wheat for owner/tenants from those farmers who are owners only or tenants only. Prior information on the characteristics of farmers operating under different tenancy arrangements indicates that owner/tenants are likely to have a higher level of production than other farmers.

5. EMPIRICAL RESULTS

We consider analysis of the data on wheat production for farmers in the selected districts for which the output was positive; output per unit of labour did not exceed 350 kilograms per male equivalent day; and labour per farm did not exceed 450 male equivalent days. These values were chosen after plots of the data indicated that values which did not meet these specifications were likely to be outliers. These criteria implied that the number of farmers for which at least one observation was obtained during the four years was 109, 138, 113, and 139 for Faisalabad, Attock, Badin, and Dir, respectively. The number of these sample farmers who provided data used in these analyses is presented in Table 1, by crop year and district. This table also shows the acreage under wheat, wheat production, and yield. In 1990-91, there were a total of 317 farmers, of which 85 were from Faisalabad, 111 from Attock, 42 from Badin, and 79 from Dir. The number of observations used in the analysis for a given district is the sum of the observations for the four years involved. The total number of observations is, thus, 330 for Faisalabad, 380 for Attock, 216 for Badin, and 330 for Dir.

The data in Table 1 also show the average yield per acre of wheat by district and crop year. Yields are generally the highest in Faisalabad, being more than twice those in Dir, Attock, and Badin. There has been a steady improvement in yields during the period covered in this study. The district of Attock had a bad crop year in 1987-1988, because there was a severe drought.

A summary of the values of output of wheat and the input variables involved in the frontier production function (4) is presented in Table 2. Sample means and sample standard deviations, together with the minimum and the maximum, are given over the four years involved. The mean acreages under wheat values, in the sample, are generally small and range from 3.0 acres in Badin to 5.7 acres in Attock. This implies that mean yields over the four years vary 306 kilograms per acre for Badin to over 898 kilograms per acre in Faisalabad.

Faisalabad and Badin are canal-irrigated districts, Attock and Dir being largely rain-fed. The proportion of irrigated acreage in Attock is negligible, while about 30 percent of the area in Dir is irrigated.

The sample farmers in Faisalabad have the highest mean amounts of labour in its wheat production, followed by Badin, Attock, and Dir. Given the fact that the mean acreages under wheat in Faisalabad and Attock differ by about one-half of an acre, it may appear surprising that the aggregate labour use in Faisalabad is more than twice that in Attock. It is well-known, however, that as agriculture becomes more commercialised, operational holdings become more skewed and the land is farmed more intensively, thus requiring more labour per acre. Furthermore, Faisalabad is an irrigated district where two crops can easily be grown in one agri-

Table 1

The Number of Farms Growing Wheat, and the Averages of Area, Production, and Yield of Wheat for Farms Involved in this Panel Survey

Year/District	Number of Wheat Farms	Area under Wheat (Acres)	Production of Wheat (Kgs.)	Yield per Acre (Kgs.)
1986-87				
Faisalabad	87	5.30	3688	894
Attock	100	6.52	2367	416
Badin	77	2.94	777	302
Dir	72	5.30	2018	422
All	336	5.12	2271	516
1987-88				
Faisalabad	77	4.68	4274	932
Attock	50	5.74	946	177
Badin	32	2.97	828	305
Dir	102	4.93	1608	418
All	261	4.77	2175	510
1988-89				
Faisalabad	81	5.28	5067	1044
Attock	106	6.10	1506	250
Badin	65	2.98	793	348
Dir	77	5.14	1432	366
All	329	5.05	2235	494
1989-90				
Faisalabad	85	5.22	5295	1010
Attock	111	4.61	2103	453
Badin	42	3.26	1405	473
Dir	79	2.94	1221	500
All	317	4.18	2665	617

Source: IFPRI, Pakistan Panel Survey, 1986–1991.

Table 2

Summary Statistics for Variables in the Stochastic Frontier Production Function for Sample Wheat Farmers in Faisalabad, Attock, Badin, and Dir

Variables/District	Sample Mean	Sample Standard Deviation	Minimum Value	Maximum Value
Output			*(Kgs.)*	
Faisalabad	4,578.5	4,435.0	200.0	31,680.0
Attock	1,865.7	2,390.6	20.0	30,000.0
Badin	919.3	864.4	60.0	8,400.0
Dir	1,565.2	1,466.0	32.0	12,000.0
Land Operated[a]			*(Acres)*	
Faisalabad	5.1	7.1	0.4	105.8
Attock	5.7	4.9	0.5	50.0
Badin	3.0	2.1	0.5	12.0
Dir	4.6	5.6	0.4	42.0
Labour			*(Man Days)*	
Faisalabad	120.2	101.7	3.7	445.0
Attock	49.2	35.6	3.0	215.5
Badin	59.4	49.9	6.0	253.8
Dir	37.3	33.2	1.0	341.6
Ratio of Hired Labour[b]				
Faisalabad	0.19	0.22	0.00	1.00
Attock	0.27	0.25	0.00	1.00
Badin	0.10	0.15	0.00	0.66
Dir	0.22	0.23	0.00	0.93
Land Preparation[c]			*(Tractor Equivalent Hours)*	
Faisalabad	30.9	35.7	0.3	336.8
Attock	20.8	22.3	0.5	236.3
Badin	13.6	12.7	0.2	77.0
Dir	8.5	9.2	0.3	58.0

Continued –

Table 2 – (*Continued*)

Variables/District	Sample Mean	Sample Standard Deviation	Minimum Value	Maximum Value
Fertilizer[d]			*(Kgs. of NPK)*	
Faisalabad	260.7	220.8	0	1,375.0
Attock	127.4	162.5	0	1,650.0
Badin	57.2	74.4	0	495.0
Dir	149.9	138.7	0	1,052.5
Number of Ploughing				
Faisalabad	6.5	2.4	2	16
Attock	6.3	2.2	1	16
Badin	3.1	1.0	1	7
Dir	2.7	0.7	1	8
Seed			*(Kgs.)*	
Faisalabad	197.1	202.3	10.0	2,116.0
Attock	220.3	181.2	15.0	1,752.0
Badin	111.4	70.5	20.0	320.0
Dir	193.9	222.8	10.0	1,600.0
Dummy Variable for Owner/Tenants				
Faisalabad	0.32	0.47	0.00	1.00
Attock	0.28	0.45	0.00	1.00
Badin	0.33	0.47	0.00	1.00
Dir	0.24	0.43	0.00	1.00

Source: IFPRI, Pakistan Panel Survey, 1986–1991.

[a] All land in the districts of Faisalabad and Badin is irrigated, while all land is rain-fed in Attock. In Dir, there is no irrigated land in 54.5 percent of the cases. Of those cases that do have some irrigation, the mean proportion of irrigated land is 63 percent.

[b] The proportion of cases in which no hired labour was used in Faisalabad, Attock, Badin, and Dir was 0.25, 0.26, 0.55, and 0.27, respectively.

[c] The proportion of cases in which tractors were used for land preparation in Faisalabad, Attock, Badin, and Dir was 0.79, 0.91, 0.89, and 0.87, respectively.

[d] The proportion of cases in which no fertilizer was applied in Faisalabad, Attock, Badin, and Dir was 0.021, 0.196, 0.120, and 0.003, respectively.

cultural year, as opposed to Attock where this is not possible. The data bear out this in Faisalabad, which is the district where farming is relatively more commercialised. A simpler possibility is that this reflects the relative labour scarcity in the latter district. On closer inspection, it is family labour that appears to be really scarce, since the ratio of hired labour to total labour is the highest in Attock. The use of mechanical traction is the highest in Faisalabad, followed, in order, by Attock, Badin, and Dir. Badin and Dir have much lower levels of tractor use. Aggregate fertilizer use is the highest in Faisalabad, followed by Dir, Attock, and Badin. The quantity of seed used is the highest in Attock, followed by Faisalabad, Dir, and Badin.

Comparisons of aggregate input use, however, are of little value. It is more meaningful to make comparisons on a per-acre basis. Based on the data in Table 2, one can see that labour use per acre is the highest in Faisalabad and the lowest in Dir. Labour use[7] per acre of wheat is 23.6 in Faisalabad, 19.8 in Badin, 8.6 in Attock, and 8.1 in Dir. Tractor use per acre is the highest in Faisalabad (6.1 hours per acre) and the lowest in Dir (1.8 hours per acre). There is a considerable variation in the amount of fertilizer used per acre. It ranges from 19.1 kilograms per acre in Badin to 51.1 kilograms per acre in Faisalabad. In Attock, the fertilizer use is 22.4 kilograms per acre, while it is 32.6 kilograms per acre in Dir. However, the variation in seed use is much smaller across the districts, ranging from 37.1 kilograms per acre for Badin to 42.2 kilograms per acre for Dir, with farmers in both Faisalabad and Attock reporting 38.6 kilograms per acre.

In our presentation of estimates for the parameters of our frontier production function, we consider three basic models. The first, called Model 1.0, is the traditional response function, in which inefficiency effects U_{it} are not present in the model. This is a special case of the stochastic frontier production function model, defined by Equations (1) and (2), in which the parameters $\gamma \equiv \sigma^2/(\sigma^2 + \sigma_v^2)$, μ and η are identically equal to Zero. Model 1.1 is the general frontier model, defined by Equations (1) and (2). The third case, called Model 1.2, is the special case of the frontier model, in which the parameters μ and η are equal to Zero. This frontier model has time-invariant inefficiency effects, which have half-normal distribution. Whether Model 1.0 or Model 1.2 are an adequate representation, given the specifications of the stochastic frontier model (Model 1.1), can be decided by calculating the generalised likelihood-ratio statistic, which is defined by $\chi^2 \equiv -2\, log[L(\hat{\theta}_0)/L(\hat{\theta})]$, where $L(\hat{\theta}_0)$ and $L(\hat{\theta})$ are the likelihood functions evaluated at the restricted and unrestricted maximum-likelihood estimator for the parameters of the frontier model. In fact, $L(\hat{\theta}_0)$ is the likelihood function for the maximum-likelihood estimators for the parameters of the appropriate sub-model of the general stochastic

[7] In male equivalent days per acre.

frontier (that is, $\gamma = \mu = \eta = 0$ for Model 1.0 and $\mu = \eta = 0$ for Model 1.2). The generalised likelihood-ratio statistic, defined by Equation (5), has approximately chi-square distribution with parameter equal to the number of parameters excluded in the particular restricted model, namely, 3 and 2 for Model 1.0 and Model 1.2 respectively.

The maximum-likelihood estimates for the parameters in the three frontier models for Faisalabad are presented in Table 3. The results obtained indicate that the traditional response function is not an adequate representation of the stochastic frontier model (see footnote 2) and so the inefficiency effects, associated with technical inefficiency of production, are significant for wheat farmers in Faisalabad. Further, the hypothesis that the inefficiency effects are time-invariant and have half-normal distribution (that is, $\hat{\eta} = \mu = 0$) would be rejected for wheat farmers in Faisalabad (see footnote 3 of Table 3).

The maximum-likelihood estimate for the parameter η, associated with the time-varying inefficiency effects, is $\eta = 1.00$, which indicates that there is a decline in th level of technical inefficiency of the Faisalabad wheat farmers over the four years for which observations on production were obtained (that is, technical efficiency increased over time).

The maximum-likelihood estimate for the parameter μ, associated with the distribution of the inefficiency effects during the last period of the panel, indicates that there is somewhat less inefficiency of wheat production in Faisalabad than would be specified under the half-normal distribution (that is, $\mu = 0$).

The maximum-likelihood estimate for the parameter γ is 0.184. This indicates the relative magnitude of the variance associated with the inefficiency effects.[8] It should be noted that σ^2 is not the variance of the inefficiency effects U_i associated with the production in the last year of the panel; hence, σ_u^2 is not recommended notation. Rather, σ^2 is the variance of the $N(\mu, \sigma^2)$-distribution which is truncated at Zero to obtain the inefficiency effects U_i. The variance of the $U_i s$ is a rather complicated expression, involving both μ and σ^2 [see Equation (A.6) of the appendix in Battese and Coelli (1992), p. 163].

A discussion of the individual estimated coefficients of the production frontier is presented later, after presenting the empirical results for the other three districts, in order to facilitate a comparison among the four districts involved.

The maximum-likelihood estimates for the frontier production functions for Attock are presented in Table 4. These results indicate that the traditional response function is an adequate representation of the wheat production technology in Attock, given the assumptions of the more general stochastic frontier production

[8]Although this value is not particularly large, it is significantly greater than Zero and is a key factor in why the frontier model is significantly different from the traditional response function.

Table 3

Maximum-likelihood Estimates for the Parameters of the Stochastic Frontier Production Functions for Wheat Farmers in Faisalabad[a]

Variable	Parameter	Model 1.0[b]	Model 1.1	Model 1.2[c]
Constant	β_0	6.24 (0.50)	6.25 (0.94)	6.34 (0.49)
Log (Land)	β_1	0.239 (0.097)	0.27 (0.18)	0.218 (0.091)
Log (Labour)	β_2	–0.038 (0.032)	0.066 (0.010)	–0.46 (0.031)
HL/Labour	β_3	0.29 (0.11)	0.130 (0.081)	0.23 (0.11)
D_1	β_4	–1.30 (0.26)	–1.29 (0.91)	–1.18 (0.26)
D_1 Log (Fert)	β_5	0.289 (0.042)	0.27 (0.12)	0.268 (0.042)
Log (Prep)	β_6	–0.227 (0.060)	–0.21 (0.12)	–0.222 (0.058)
D_2	β_7	–0.75 (0.19)	–0.69 (0.43)	–0.74 (0.19)
D_2 Log (Prep)	β_8	0.235 (0.059)	0.23 (0.15)	0.231 (0.058)
Log (Plou)	β_9	–0.022 (0.069)	–0.118 (0.070)	–0.021 (0.065)
Log (Seed)	β_{10}	0.44 (0.11)	0.41 (0.17)	0.47 (0.10)
D_3	β_{11}	0.025 (0.050)	0.041 (0.041)	0.034 (0.052)
	σ_s^2	0.153 (0.012)	0.094 (0.016)	0.186 (0.024)
	γ	0	0.184 (0.042)	0.33 (0.10)
	μ	0	–0.80 (0.31)	0
	η	0	1.00 (0.55)	0
Log (Likelihood)		–152.463	–100.039	–148.033
Number of Iterations[d]		–	36	18

[a] The estimators' standard errors for the maximum-likelihood estimators are presented below the corresponding estimates, correct to two significant digits. These values are obtained by use of the computer programme, FRONTIER, 2.0.

[b] This model is the traditional response function involving no inefficiency effects (that is, $\gamma = \mu = \eta = 0$). The generalised likelihood-ratio statistic, $\chi^2 = -2\ log\ [L(\hat{\theta}_0)/L(\hat{\theta})] = -2\ \{log\ [L(\hat{\theta}_0)] - log\ [L(\hat{\theta})]\} = -2\{-152.463 - (-100.039)\} = 104.85$, is highly significant (that is, $\chi_{3,0.95} = 7.81$). Thus, the null hypothesis, H_0: $\gamma = \mu = \eta = 0$, would be rejected, given a test of size, $\alpha = 0.05$.

[c] This model is the stochastic frontier production function with time-invariant inefficiency effects, which have half-normal distribution (that is, $\mu = \eta = 0$). The generalised likelihood-ratio statistic, $\chi^2 = -2\ \{-148.033 - (-100.039) = 95.99$, is highly significant (that is, $\chi^2_{2,0.95} = 5.99$). Thus, the null hypothesis, H_0: $\mu = \eta = 0$, would be rejected, given a test of size, $\alpha = 0.05$.

[d] This gives the number of iterations using FRONTIER, 2.0, when the ordinary least-squares estimates are used to obtain initial values for the parameters.

Table 4

Maximum-likelihood Estimates for the Parameters of the Stochastic Frontier Production Functions for Wheat Farmers in Attock

Variable	Parameter	Model 1.0[a]	Model 1.1	Model 1.2[b]
Constant	β_0	3.89 (0.80)	3.92 (0.99)	3.92 (0.99)
Log (Land)	β_1	0.03 (0.16)	0.03 (0.84)	0.03 (0.94)
Log (Labour)	β_2	0.141 (0.065)	0.14 (0.88)	0.14 (0.83)
HL/Labour	β_3	0.86 (0.16)	0.86 (0.97)	0.87 (1.00)
D_1	β_4	–1.22 (0.35)	–1.22 (0.97)	–1.22 (0.98)
D_1 Log (Fert)	β_5	0.346 (0.075)	0.35 (0.48)	0.34 (0.23)
Log (Prep)	β_6	–0.00 (0.15)	–0.00 (0.87)	–0.00 (0.83)
D_2	β_7	–0.22 (0.43)	–0.22 (0.97)	–0.22 (0.99)
D_2 Log (Prep)	β_8	0.01 (0.16)	0.01 (0.63)	0.01 (0.57)
Log (Plou)	β_9	–0.03 (0.13)	–0.03 (0.91)	–0.03 (0.98)
Log (Seed)	β_{10}	0.45 (0.15)	0.45 (0.80)	0.45 (0.68)
D_3	β_{11}	0.039 (0.091)	0.04 (0.94)	0.04 (0.99)
	σ_s^2	0.534 (0.040)	0.56 (0.49)	0.52 (0.13)
	γ	0	0.07 (0.13)	0.01 (0.27)
	μ	0	–1.51 (0.97)	0
	η	0	0.05 (0.96)	0
Log (Likelihood)		–399.380	–398.643	–399.425
Number of Iterations[a]		–	6	6

[a]The generalised likelihood-ratio statistic, $\chi^2 = -2\{-399.380- (-398.643)\} = 1.47$, is not significant (that is, $\chi^2_{3,0,95} = 7.81$). Thus, the null hypothesis, H_0: $\gamma = \mu = \eta = 0$, would be accepted for Attock.

[b]The generalised likelihood-ratio statistic, $\chi^2 = -2\{-399.425- (-398.643)\} = 1.56$, is not significant. Thus, the null hypothesis, H_0: $\mu = \eta = 0$), would be accepted for Attock.

function. This implies that, for the current level of technology, technical inefficiencies of production are not significant among wheat farmers in Attock.

The maximum-likelihood estimates for the frontier production functions for Badin are presented in Table 5. The tests of the hypotheses given in footnotes (a) and (b) of Table 5 indicate that neither the traditional response function nor the stochastic frontier production function, with time-invariant inefficiency effects having half-normal distribution, are adequate representations for wheat production in Badin; given the specifications of the general stochastic frontier model with time-varying inefficiency effects.[9]

It should be noted that the maximum-likelihood estimates for the parameters η and μ, which specify the time-varying structure of the inefficiency effects, are not large relative to their estimated standard errors which are obtained by use of FRONTIER, 2.0. However, the generalised likelihood-ratio statistic for the joint test of the null hypothesis, H_0: $\mu = \eta = 0$, is highly significant. This demonstrates a result which has frequently been found in using FRONTIER, 2.0, namely, that t-ratios, associated with individual parameter estimates may be quite small and give a false indication of the significance of the coefficient involved. It is recommended that the generalised likelihood-ratio tests be conducted, even for individual coefficients, in order to determine the statistical significance, or otherwise, of estimated parameters in the frontier production function.

The estimate for the parameter η is 0.22, which indicates that technical inefficiencies of production declined over the years of the panel, although the value is smaller than that obtained for wheat farmers in Faisalabad. The estimate for the parameter μ is negative and of similar magnitude to that for Faisalabad. However, the estimate for the parameter γ is considerably larger than the estimate in Faisalabad. This implies that the variance associated with the inefficiency effects is a more significant component of the variability of wheat production in Badin than in Faisalabad.

The maximum-likelihood estimates for the frontier production functions for wheat farmers in Dir are presented in Table 6. These results indicate that there are significant inefficiency effects associated with wheat farmers in Dir, but the hypothesis, that these inefficiency effects are time-invariant and have half-normal distribution, would be accepted at the five percent level of significance.

The estimated standard errors of the maximum-likelihood estimates for the parameters with the variability of wheat production in Dir are unreasonably large. The estimated standard deviation of the maximum-likelihood estimator for γ, name-

[9]It can be shown that individual tests of hypotheses, that the inefficiency effects are time-invariant (*i.e.*, H_0: $\eta = 0$) or that the inefficiency effects in the last year of the panel have half-normal distribution (*i.e.*, H_0: $\mu = 0$), would also be rejected at the five percent level of significance.

Table 5

Maximum-likelihood Estimates for the Parameters of the Stochastic Frontier Production Functions for Wheat Farmers in Badin

Variable	Parameter	Model 1.0[a]	Model 1.1	Model 1.2[b]
Constant	β_0	3.50 (0.81)	3.8 (1.0)	4.34 (0.79)
Log (Land)	β_1	0.26 (0.15)	0.29 (0.23)	0.38 (0.15)
Log (Labour)	β_2	0.166 (0.080)	0.17 (0.19)	0.098 (0.078)
HL/Labour	β_3	0.55 (0.31)	0.45 (0.76)	0.58 (0.28)
D_1	β_4	–0.26 (0.28)	–0.28 (0.79)	–0.17 (0.25)
D_1 Log (Fert)	β_5	0.186 (0.065)	0.17 (0.20)	0.145 (0.059)
Log (Prep)	β_6	–0.25 (0.20)	–0.17 (0.30)	–0.11 (0.18)
D_2	β_7	–0.26 (0.48)	–0.2 (1.1)	–0.08 (0.42)
D_2 Log (Prep)	β_8	0.10 (0.20)	0.04 (0.33)	–0.00 (0.17)
Log (Plou)	β_9	0.37 (0.16)	0.38 (0.22)	0.37 (0.15)
Log (Seed)	β_{10}	0.41 (0.16)	0.40 (0.47)	0.32 (0.16)
D_3	β_{11}	0.090 (0.099)	0.09 (0.17)	0.09 (0.10)
	σ_s^2	0.444 (0.044)	0.72 (0.36)	0.69 (0.12)
	γ	0	0.63 (0.22)	0.59 (0.10)
	μ	0	–1.01 (0.95)	0
	η	0	0.22 (0.79)	0
Log (Likelihood)		–212.659	–200.804	–206.835
Number of Iterations		–	24	18

[a]The generalised likelihood-ratio statistic, $\chi = -2\{-212.659 - (-200.804)\} = 23.71$, is significant at the 5 percent level.

[b]The generalised likelihood-ratio statistic, $\chi = -2\{-206.835 - (-200.804)\} = 12.06$, is significant at the 5 percent level.

Table 6

Maximum-likelihood Estimates for the Parameters of the Stochastic Frontier Production Functions for Wheat Farmers in Dir

Variable	Parameter	Model 1.0[a]	Model 1.1	Model 1.2[b]
Constant	β_0	4.70 (0.52)	4.95 (0.54)	5.01 (0.50)
Log (Land)	β_1	0.356 (0.062)	0.38 (0.27)	0.382 (0.062)
Log (Labour)	β_2	0.108 (0.036)	0.107 (0.091)	0.100 (0.035)
HL/Labour	β_3	0.02 (0.11)	0.01 (0.13)	0.02 (0.10)
D_1	β_4	–0.38 (0.47)	–0.2 (3.9)	–0.29 (0.45)
D_1 Log (Fert)	β_5	0.105 (0.46)	0.09 (0.32)	0.092 (0.046)
Log (Prep)	β_6	0.132 (0.68)	0.12 (0.25)	0.122 (0.067)
D_2	β_7	0.15 (0.14)	0.10 (0.86)	0.11 (0.14)
D_2 Log (Prep)	β_8	–0.021 (0.069)	–0.00 (0.21)	–0.010 (0.068)
Log (Plou)	β_9	0.183 (0.097)	0.14 (0.44)	0.174 (0.093)
Log (Seed)	β_{10}	0.196 (0.057)	0.167 (0.056)	0.176 (0.056)
D_3	β_{11}	0.032 (0.057)	0.027 (0.062)	0.033 (0.059)
IL/Land	β_{12}	0.057 (0.064)	0.055 (0.064)	0.048 (0.064)
	σ_s^2	0.183 (0.015)	0.6 (5.4)	0.216 (0.028)
	γ	0	0.7 (2.2)	0.29 (0.11)
	μ	0	–1.8 (9.6)	0
	η	0	–0.2 (3.0)	0
Log (Likelihood)		–181.739	–176.734	–178.971
Number of Iterations		–	124	18

[a] The generalised likelihood-ratio statistic, $\chi^2 = -2\{-181.739 - (-176.734)\} = 10.01$, significant at the 5 percent level.

[b] The generalised likelihood-ratio statistic, $\chi^2 = -2\{-178.971 - (-176.734)\} = 4.47$, is not significant at the 5 percent level.

ly, 2.2, is obviously an inadmissible value, given that γ is restricted between 0 and 1. These results indicate that the general frontier model is not well-identified in this case. However, the model with time-invariant inefficiency effects which have half-normal distribution (Model 1.2 with $\mu = \eta = 0$) is significantly different from the traditional response function. The estimate for the parameter γ in this time-invariant model is 0.29, which indicates that the variance associated with the inefficiency effects is a moderately large (but significant) component of the total variability of wheat production in the district of Dir.

In the above analysis of stochastic frontier production functions for the four districts in Pakistan, the coefficients of the explanatory variables in the production frontiers are time-invariant. Only the inefficiency effects, which cause the actual production to fall short of the corresponding stochastic frontier production, are permitted to be time-varying. This may be quite restrictive and may lead to biased estimates of the relevant parameters. In this study, we include the year of observation as an explanatory variable in the stochastic frontier production function, in addition to the variables defined in Equation (4). This permits the level of the frontier to change over time (that is, Hicksian-neutral technological change), but the input elasticities remain time-invariant. In Table 7, the maximum-likelihood estimates for the parameters in this modified stochastic frontier function are presented for the preferred models for the inefficiency effects in the districts involved.

From the first column of Table 7, it is evident that, although the year of observation is included in the stochastic frontier, the inefficiency effects are still found to be present for Faisalabad and, in addition, are not time-invariant with half-normal distribution. Further, from the third column of Table 7, it is indicated that, although the inefficiency effects are still significant when the year of observation is included in the frontier function for Badin, the preferred model has time-invariant inefficiency effects which have half-normal distribution. This differs from the case in which the year of observation was not included in the frontier.

Finally, the preferred inefficiency models for Attock and Dir, when the year of observation is included in the frontier function, are the same as those obtained when the year was not included. For Attock, the inefficiency effects are not significant and so the preferred model is the traditional response function. However, for Dir, the inefficiency effects remain significant, but are time-invariant and have half-normal distribution.

The coefficients of the year of observation in the modified frontier functions are negative for Faisalabad and Dir, but positive for Attock and Badin. Tests of the hypothesis that the coefficient of the year of observation is Zero, (H_0: $\beta_{13} = 0$), presented in Table 8, indicate that this null hypothesis would be rejected only in Faisalabad and Badin. This conclusion for Faisalabad is contrary to what might be expected, given that the estimated negative coefficient is small relative to its report-

Table 7

Maximum-likelihood Estimates for the Parameters in the Stochastic Frontier Function including the Year of Observation, Given the Specifications for the Preferred Inefficiency Models for the Selected Districts of Pakistan

Variable	Parameter	Faisalabad	Attock	Badin	Dir
Constant	β_0	6.27 (0.96)	3.82 (0.81)	3.92 (0.77)	5.13 (0.52)
Log (Land)	β_1	0.26 (0.31)	0.03 (0.16)	0.36 (0.15)	0.382 (0.062)
Log (Labour)	β_2	0.024 (0.064)	0.140 (0.065)	0.176 (0.078)	0.107 (0.036)
HL/Labour	β_3	0.13 (0.98)	0.85 (0.16)	0.32 (0.28)	0.01 (0.10)
D_1	β_4	−1.28 (0.98)	−1.21 (0.35)	−0.24 (0.24)	−0.32 (0.46)
D_1 Log (Fert)	β_5	0.26 (0.12)	0.349 (0.075)	0.161 (0.057)	0.102 (0.047)
Log (Prep)	β_6	−0.20 (0.27)	−0.01 (0.15)	−0.00 (0.17)	0.125 (0.067)
D_2	β_7	−0.67 (0.86)	−0.23 (0.43)	0.22 (0.41)	0.11 (0.14)
D_2 Log (Fert)	β_8	0.22 (0.27)	0.01 (0.16)	−0.12 (0.17)	−0.006 (0.068)
Log (Plou)	β_9	−0.07 (0.12)	−0.03 (0.13)	0.31 (0.14)	0.14 (0.10)
Log (Seed)	β_{10}	0.44 (0.29)	0.46 (0.16)	0.26 (0.15)	0.156 (0.060)
D_3	β_{11}	0.031 (0.086)	0.040 (0.091)	0.08 (0.10)	0.031 (0.059)
IL/Land	β_{12}	0	0	0	0.052 (0.064)
Year	β_{13}	−0.06 (0.14)	0.047 (0.082)	0.264 (0.067)	−0.045 (0.051)
	σ_s^2	0.097 (0.010)	0.535 (0.040)	0.65 (0.12)	0.214 (0.028)
	γ	0.145 (0.036)	0	0.60 (0.10)	0.28 (0.11)
	μ	−0.810 (0.085)	0	0	0
	η	1.05 (0.55)	0	0	0
Log (Likelihood)		−95.807	−399.207	−199.242	−178.574

ed estimated standard error. However, on the basis of the generalised likelihood-ratio test, the frontier production function for wheat farmers in Faisalabad has significantly declined in its level over the years involved, whereas that for wheat farmers in Badin has significantly increased over time.

Table 8

Test of Hypotheses of the Coefficients of the Year of Observation and the Hired-Labour Ratio in the Modified Stochastic Frontier Production Functions for Wheat Farmers in Selected Districts of Pakistan

Null Hypothesis	District	χ^2-Statistic	Decision
Coefficient of Year H_0: $\beta_{13} = 0$			
H_0	Faisalabad	8.46*	Reject
H_0	Attock	0.35	Accept
H_0	Badin	3.90*	Reject
H_0	Dir	0.79	Accept
Coefficient of Hired Labour Ratio H_0: $\beta_3 = 0$			
H_0	Faisalabad	12.53*	Reject
H_0	Attock	29.17*	Reject
H_0	Badin	2.12	Accept
H_0	Dir	0.02	Accept

*Significant at least at the 5 percent level.

Also included in Table 8 are tests of the null hypothesis that the coefficient of the hired-labour in the stochastic frontier production function (4) ratio is Zero. The generalised likelihood-ratio statistics presented indicate that the coefficient of the hired-labour ratio is significantly different from Zero in Faisalabad and Attock only. Therefore, we conclude that only in Badin and Dir is it likely that hired and family labour are equally productive in the growing of wheat. Further, the positive estimates for the hired-labour ratio in the frontier models for all districts indicate that the higher the proportion of hired labour in wheat production, the greater will be the level of the production of wheat, other things being equal. Thus, it can be concluded that hired labour has significantly greater productivity than family labour in growing wheat in the districts of Faisalabad and Attock.

The estimated parameters, presented in Table 7, indicate that the elasticities of land vary from 0.03 for Attock to 0.38 for Dir; the elasticities for labour vary from 0.024 for Faisalabad to 0.176 for Badin; the elasticities for fertilizer vary from

0.102 for Dir to 0.349 for Attock; the elasticities for bullock labour (the coefficient of the variable of land preparation, that is, Log (Prep)) vary from –.20 for Faisalabad to 0.125 for Dir; the elasticities for mechanical traction (the sum of the coefficients of Log (Prep) and D_2 Log (Prep)) vary from –0.12 for Badin to 0.119 for Dir; the elasticities of the number of ploughings vary from –0.07 for Faisalabad to 0.31 for Badin; and, finally, the elasticities for seeds vary from 0.156 for Dir to 0.46 for Attock.

The coefficients of the dummy variable, D_3, are all positive, which indicates that wheat farmers who are owner/tenants tend to produce higher yields of wheat than do either the owners only or the tenants only.

It can be shown that the returns-to-scale parameters for farmers, who use fertilizer and mechanical traction in growing their wheat, are estimated to be 0.931, 0.946, 1.151, and 1.004 for the districts of Faisalabad, Attock, Badin, and Dir, respectively.[10]

The technical efficiencies of the wheat farmers in Faisalabad were found to be time-varying. The individual technical efficiencies of the 109 farmers observed at least once over the four years of the panel survey are presented in Table A.1 of the Appendix, for each year in which the observations were obtained. A summary of these technical efficiencies is presented in Table 9, in terms of the percentages of

Table 9

Percentages of the Occurrence of Technical Efficiencies within the Specified Ranges for Wheat Farmers in Faisalabad during the Four Years of Observations

Ranges of Technical Efficiencies	1986-87	1987-88	1988-89	1990-91
		(Percent)		
Over 0.95	3.4	13.0	85.2	97.6
0.90 to 0.95	20.7	50.6	12.3	1.2
0.85 to 0.90	19.5	18.2	1.2	1.2
0.80 to 0.85	25.3	9.1	...	...
0.75 to 0.80	5.7	3.9	...	...
0.70 to 0.75	5.7	3.9	...	...
0.65 to 0.70	3.4	...	1.2	...
0.60 to 0.65	5.7	...	...	...
0.55 to 0.60	...	...	...	...
0.50 to 0.55	10.3	...	...	...
Less than 0.50	...	1.3	...	...
Total Number of Observations	87	77	81	85
Number of Missing Observations	22	32	28	24

[10]The estimated standard deviations of the maximum-likelihood estimators for the returns-to-scale parameters are 0.23, 0.13, 0.16, and 0.098 for Faisalabad, Attock, Badin, and Dir, respectively.

occurrence of technical efficiencies in different ranges in the different years involved.

Because the inefficiency effects for wheat farmers in Attock were not significant, no individual technical efficiencies are presented for Attock farmers. In fact, the technical efficiencies of Attock wheat farmers can be reported to be equal to unity, given the present level of technology of wheat production in the district.

The technical inefficiencies of production of wheat farmers in Badin and Dir were found to be time-invariant. The individual technical efficiencies of the 113 and 139 wheat farmers involved in the panel surveys in Badin and Dir are presented in Tables A.2 and A.3 of the Appendix, respectively. A summary of the technical efficiencies obtained for these farmers is presented in Table 10 for the farmers in Badin and Dir.

Table 10

Percentages of the Occurrence of Technical Efficiencies within the Specified Ranges for Wheat Farmers in Badin and Dir

Ranges of Technical Efficiencies	Percentages of Occurrence	
	Badin	Dir
	(Percent)	
Over 0.95	...	...
0.90 to 0.95	...	7.2
0.85 to 0.90	3.5	33.1
0.80 to 0.85	7.1	38.1
0.75 to 0.80	13.3	14.4
0.70 to 0.75	20.4	5.8
0.65 to 0.70	14.2	0.7
0.60 to 0.65	11.5	...
0.55 to 0.60	8.0	...
0.50 to 0.55	7.1	...
Less than 0.50	15.0	0.7
Number of Farms	113	139

6. CONCLUSIONS

Two models of technical inefficiency with a stochastic production frontier are presented in this paper. In the first, it is assumed that the frontier itself does not vary with time, while in the second, the frontier is allowed to move (that is, Hicks-neutral technical change is allowed). In both these models, it is important to remember that the stochastic frontier is determined within the sample and does not depend on, for example, the state of research and development. Thus, it may be that if the policy environment favoured the development and adoption of the best possible agricultural techniques, the frontier would be higher than that found from within the sample. In effect, these models take the output of the most efficient farmers in a sample as a measure of potential output. The comparison is against the maximum output actually achieved in the sample, and not against the maximum output that can be achieved given the level of input use.

These models were applied to the data on wheat farmers in four districts of Pakistan: Faisalabad and Attock in the Punjab, Badin in Sindh, and Dir in the NWFP. Faisalabad is a prosperous, irrigated district with high wheat yields and intensive use of labour, fertilizer, seeds, and tractors. Of the other districts, Badin is also irrigated but rather poor, with wheat yields which are the lowest among the four districts, and generally lower intensities of use of the other inputs. Attock, which is entirely rain-fed, and Dir, which is partly irrigated, are generally in the middle with regard to these inputs.

The main set of results on technical efficiency for the four districts is as follows:

- In Faisalabad, the hypothesis of no technical inefficiency is rejected, whether or not one allows for shifts in the technology frontier. Furthermore, the inefficiency, that is, the gap between the maximum achievable output and what is actually produced, *declines* over time at a fairly high rate. This is because the frontier itself is declining over time in Faisalabad. Therefore, if the output for a given level of inputs is increasing over time while the potential output for a given level of inputs is falling, the gap between them will close rapidly.
- In Attock, the hypothesis of no technical inefficiency cannot be rejected, in both the models. Moreover, the stochastic production frontier appears to be moving out over time, but this result is not statistically significant.
- In Badin, as in Faisalabad, there appears to be some technical inefficiency, which is declining over time if it is assumed that there is no Hicks-neutral technical change. However, when that is allowed for, the technical inefficiency does not change over time. This is probably because the results show that the frontier is shifting out over time and this result is statistically significant.
- The results for Dir show that inefficiencies are present but are not changing over time, whichever model is chosen. Moreover, the stochastic frontier appears to be moving in over time, but this is not statistically significant.

The results for Faisalabad are intriguing. The first point to note is that some of the early high yielding varieties (HYVs) of wheat–for example, Mexipak–were adopted by a large number of farmers in Faisalabad as early as the mid-1960s, and practically the entire sample had adopted HYVs by the early seventies. Of the varieties currently grown in Faisalabad, most had been adopted by the early eighties, and there has not been much improvement in seed varieties in the eighties. The wheat industry in Pakistan is not well-developed and farmers generally use seeds which may currently be in their third or fourth generation with a resultant loss of vigour. Furthermore, there is considerable evidence that fertilizer is not being used appropriately. It is important that the right ratio between Nitrogen and Phosphorus be maintained, since there is evidence [Saleem *et al.* (1987)] going back to the late fifties that there are widespread deficiencies of Phosphorus throughout the country.[11] For these reasons it is not surprising that the frontier is shifting in. These results also receive confirmation from the elasticity estimates for some critical inputs. Turning to the individual elasticities, the elasticity of output with respect to labour is 0.024, which is the lowest among all four districts. This is as expected, because Faisalabad has the highest level of labour use amongst the districts. The elasticity of fertilizer use, for farmers who used fertilizer, is barely 0.26. The elasticity of output with respect to tractors is Zero for all practical purposes. The critical importance of good seed is clear from the very high elasticity, 0.44, which is almost the highest in the sample.

Analysis of a questionnaire on technical skills given to wheat farmers in the four districts shows that Attock farmers have very high levels of technical skills. Furthermore, they are very well-integrated into the urban economy of the surrounding region with much better access to infrastructure and better information flows. The estimate for the returns-to-scale parameter is 0.95, implying decreasing returns to scale, as in Faisalabad. It is not clear, however, whether this is statistically significantly different from unity. Some of the critical elasticities are as follows: the elasticity for labour, at 0.14, is approximately six times the figure for Faisalabad. The elasticity estimate for fertilizer is 0.01, reflecting the fact that additional fertilizer is not very useful on rain-fed land. Similarly, the elasticity for mechanical traction is essentially Zero. As in Faisalabad, the elasticity for seed is very high, at 0.46. The foregoing analysis suggests that the critical inputs in Attock are labour and seeds.

Turning now to Badin and Dir, the result that the frontier is shifting out in Badin and is probably static in Dir can be explained by the fact that HYVs have been adopted more recently in these two districts and there is still much scope for further adoption. It is interesting to note that the returns-to-scale parameter is greater than unity in both districts. Finally, the elasticity estimates for some critical

[11]Some progress in this direction has been made. A recent study shows that the ratio of nitrogen to phosphorus in fertilizer use has fallen from 3.6:1 to 3.3:1.

inputs are more or less in the same range, except that the elasticities for labour, seed, and fertilizer (for fertilizer users) are higher in Badin than in Dir.

The finding that wheat farmers who are owner-tenants tend to have higher wheat yields than farmers who are either pure owners or pure tenants in all four districts in consistent with the widespread belief that owner-tenants are more progressive farmers, who are likely to regard land as a productive asset and to make the best possible use of it.

The general conclusion that emerges from this analysis is that the development of new and improved seed varieties needs to be a continuous process, and that the various agricultural research institutes cannot afford to rest on their laurels. There is also a clear need for better agricultural extension services[12] to familiarise farmers with new technologies and with the best ways to use them–especially fertilizer and seed.

There are some fairly obvious extensions to this work. For example, the possibility of interactions between the variables in the production equation and the equation for the inefficiency effect can be allowed for. One of the most important restrictions of these models is that they do not take an account of risk in production and the possible responses to it. Thus, the fact that a farmer is inefficient may indicate that he has chosen to trade off expected return against a lower but more certain level of output. The policy implications of this research, therefore, depend on what is assumed about the availability of risk-sharing arrangements, that is, insurance and credit, and the terms on which these are made available.

[12]This conclusion also emerges from another study currently underway on technology adoption and technical skills.

Appendix

Table A.1

Technical Efficiencies of Sample Wheat Farmers in Faisalabad during the Four Years of Observation

Sample Farmer	1986-87	1987-88	1988-89	1990-91	Number of Observations
1	...	0.882	...	...	1
2	...	...	...	0.983	1
3	0.577	0.819	0.932	0.976	4
4	0.852	0.944	0.980	0.993	4
5	...	...	0.958	0.985	2
6	...	...	...	0.982	1
7	0.774	0.911	0.968	0.989	4
8	...	0.909	0.966	0.988	3
9	0.858	0.946	0.981	0.993	4
10	0.786	...	...	...	1
11	0.761	...	...	0.988	2
12	0.487	0.772	0.913	0.969	4
13	0.812	0.927	0.974	0.991	4
14	0.720	0.887	0.958	...	3
15	0.805	0.924	0.973	0.990	4
16	...	...	0.950	0.982	2
17	...	0.869	0.951	0.982	3
18	0.623	0.842	...	0.979	3
19	0.853	0.944	0.980	...	3
20	0.772	0.910	0.967	0.988	4
21	0.857	...	...	...	1
22	...	...	0.952	0.983	2
23	0.765	0.907	0.966	0.988	4
24	0.786	0.916	0.969	0.989	4
25	0.658	0.859	...	0.981	3
26	0.872	...	...	0.994	2
27	0.795	0.920	0.971	0.990	4
28	...	...	0.955	...	1
29	0.807	0.925	0.973	0.990	4
30	0.552	0.807	0.927	0.974	4
31	0.818	0.930	0.974	0.991	4
32	0.770	0.909	0.967	0.988	4
33	0.890	0.959	0.985	0.995	4
34	0.456	0.754	0.905	0.966	4

Continued –

Table A.1 – (*Continued*)

Sample Farmer	1986-87	1987-88	1988-89	1990-91	Number of Observations
35	0.739	0.896	0.962	0.986	4
36	0.873	0.952	0.983	0.994	4
37	0.680	0.869	0.952	0.983	4
38	0.603	0.832	0.937	0.977	4
39	0.771	0.910	...	0.988	3
40	0.811	0.927	0.973	0.991	4
41	0.568	0.815	0.930	0.975	4
42	0.654	0.857	0.947	0.981	4
43	0.835	0.937	0.977	0.992	4
44	0.817	...	...	...	1
45	0.914	0.968	0.989	0.996	4
46	0.848	0.942	0.979	0.993	4
47	0.392	0.715	0.889	0.959	4
48	0.817	0.929	0.974	...	3
49	0.885	0.957	0.985	0.995	4
50	0.894	0.961	0.986	0.995	4
51	0.373	0.703	...	...	2
52	...	0.895	0.961	0.986	3
53	0.620	...	...	...	1
54	0.776	0.912	0.968	0.989	4
55	0.763	...	...	...	1
56	0.782	...	...	0.989	2
57	0.735	0.894	...	...	2
58	...	...	0.957	0.985	2
59	0.831	...	0.977	...	2
60	0.846	0.941	0.979	0.993	4
61	0.811	...	...	...	1
62	0.848	...	...	...	1
63	0.854	...	...	0.993	2
64	0.785	0.916	0.969	0.989	4
65	...	...	0.958	0.985	2
66	0.772	0.910	0.967	...	3
67	...	...	0.951	0.982	2
68	...	0.952	0.983	0.994	3
69	0.272	...	...	...	1
70	0.034	0.306	0.661	0.865	4
71	0.894	0.961	0.986	0.995	4
72	0.863	0.948	0.981	0.993	4

Continued –

Table A.1 – (*Continued*)

73	0.799	0.922	0.971	0.990	4
74	0.844	0.941	0.979	0.992	4
75	0.856	0.945	0.980	...	3
76	0.855	...	...	...	1
77	0.725	0.890	0.959	0.986	4
78	0.812	0.927	0.974	0.991	4
79	0.797	...	0.971	0.990	3
80	0.856	0.945	0.980	0.993	4
81	0.874	0.953	0.983	0.994	4
82	...	0.901	0.963	0.987	3
83	...	0.888	0.958	0.985	3
84	0.800	0.922	0.972	0.990	4
85	...	0.894	0.961	0.986	3
86	0.805	0.924	0.972	0.990	4
87	0.761	0.906	0.965	...	3
88	0.689	...	0.953	0.983	3
89	0.575	0.818	0.931	0.975	4
90	0.909	0.966	0.988	0.996	4
91	0.416	0.730	...	...	2
92	0.770	0.909	0.967	0.988	4
93	0.865	0.949	0.982	0.994	4
94	0.426	...	...	0.963	2
95	...	0.882	0.956	0.984	3
96	0.917	0.970	0.989	0.996	4
97	0.676	0.867	0.951	0.982	4
98	0.771	0.910	0.967	0.988	4
99	...	...	...	0.985	1
100	0.555	0.808	...	...	2
101	0.813	...	...	0.991	2
102	0.857	0.946	0.981	0.993	4
103	...	0.792	0.919	0.971	3
104	0.798	...	0.971	...	2
105	0.758	...	0.965	0.988	3
106	...	0.931	0.975	0.991	3
107	0.315	...	...	0.950	2
108	0.710	...	...	...	1
109	...	0.922	0.971	0.990	3
Number of Missing Observations	22	32	28	24	106

[a] When observations on farmers were not obtained in some years of the panel survey, no technical efficiencies are reported. The missing values are indicated by... .

Table A. 2

Technical Efficiencies of Sample Wheat Farmers in Badin during the Four Years of Observation

Sample Farmer	Technical Efficiency	Number of Observations
1	0.695	4
2	0.493	2
3	0.775	1
4	0.556	2
5	0.491	4
6	0.670	1
7	0.607	1
8	0.857	3
9	0.742	2
10	0.737	4
11	0.758	2
12	0.254	2
13	0.887	3
14	0.689	2
15	0.687	3
16	0.509	3
17	0.576	1
18	0.560	1
19	0.733	2
20	0.719	1
21	0.683	1
22	0.798	3
23	0.615	2
24	0.322	1
25	0.797	1
26	0.692	2
27	0.703	1
28	0.659	1
29	0.299	1
30	0.729	2
31	0.649	1
32	0.759	1
33	0.732	2
34	0.700	2
35	0.474	1
36	0.592	2

Continued –

Table A. 2 – (*Continued*)

37	0.698	1
38	0.717	3
39	0.739	1
40	0.835	2
41	0.836	2
42	0.811	2
43	0.542	2
44	0.603	1
45	0.470	3
46	0.677	1
47	0.527	1
48	0.717	2
49	0.705	1
50	0.656	1
51	0.811	1
52	0.798	2
53	0.694	1
54	0.798	2
55	0.614	2
56	0.448	3
57	0.864	1
58	0.830	2
59	0.550	2
60	0.608	1
61	0.690	1
62	0.359	1
63	0.589	3
64	0.773	1
65	0.240	2
66	0.715	2
67	0.703	1
68	0.514	1
69	0.816	1
70	0.737	1
71	0.705	1
72	0.636	2
73	0.722	2
74	0.829	4
75	0.245	2
76	0.799	2
77	0.764	3

Continued –

Table A.2 – (*Continued*)

Sample Farmer	Technical Efficiency	Number of Observations
78	0.857	2
79	0.622	2
80	0.754	2
81	0.628	1
82	0.724	3
83	0.722	2
84	0.616	4
85	0.460	3
86	0.534	3
87	0.569	4
88	0.567	4
89	0.816	4
90	0.521	3
91	0.765	3
92	0.725	2
93	0.763	4
94	0.798	3
95	0.475	4
96	0.645	1
97	0.585	2
98	0.691	1
99	0.401	1
100	0.440	1
101	0.682	2
102	0.711	1
103	0.749	2
104	0.707	1
105	0.364	2
106	0.716	1
107	0.626	1
108	0.458	2
109	0.670	1
110	0.580	1
111	0.540	2
112	0.636	2
113	0.790	1
Total Number of Observations		216

Table A. 3

Technical Efficiencies of Sample Wheat Farmers in Dir during the Four Years of Observation

Sample Farmer	Technical Efficiency	Number of Observations
1	0.836	3
2	0.799	3
3	0.786	1
4	0.791	1
5	0.700	3
6	0.860	3
7	0.853	1
8	0.464	3
9	0.810	3
10	0.780	1
11	0.824	2
12	0.850	1
13	0.844	3
14	0.816	1
15	0.865	4
16	0.714	4
17	0.865	4
18	0.839	3
19	0.824	1
20	0.836	1
21	0.803	1
22	0.864	1
23	0.824	3
24	0.823	1
25	0.770	1
26	0.866	4
27	0.804	4
28	0.783	4
29	0.720	4
30	0.854	2
31	0.843	3
32	0.895	4
33	0.830	3
34	0.850	2
35	0.877	4
36	0.912	4
37	0.840	4
38	0.899	2
39	0.891	4
40	0.854	1
41	0.812	2
42	0.840	1
43	0.876	2
44	0.873	1

Continued –

Table A.3 – (*Continued*)

Sample Farmer	Technical Efficiency	Number of Observations
45	0.884	2
46	0.789	2
47	0.832	3
48	0.862	2
49	0.804	1
50	0.801	1
51	0.878	4
52	0.857	3
53	0.894	2
54	0.889	4
55	0.828	4
56	0.908	2
57	0.907	3
58	0.799	4
59	0.749	1
60	0.853	3
61	0.914	4
62	0.796	1
63	0.807	1
64	0.811	3
65	0.826	2
66	0.865	2
67	0.877	2
68	0.868	4
69	0.923	4
70	0.861	3
71	0.909	4
72	0.741	2
73	0.863	4
74	0.827	1
75	0.814	3
76	0.831	4
77	0.828	3
78	0.852	2
79	0.910	3
80	0.932	4
81	0.880	3
82	0.913	4
83	0.885	3
84	0.816	4
85	0.896	4
86	0.888	3
87	0.864	1
88	0.890	4
89	0.880	4
90	0.867	2
91	0.830	1
92	0.787	2
93	0.718	1

Continued –

Table A.3 – (*Continued*)

94	0.814	2
95	0.883	1
96	0.809	1
97	0.833	1
98	0.825	2
99	0.843	3
100	0.851	1
101	0.822	1
102	0.798	4
103	0.759	2
104	0.769	1
105	0.876	3
106	0.830	1
107	0.752	2
108	0.816	2
109	0.825	1
110	0.879	4
111	0.749	2
112	0.808	3
113	0.909	4
114	0.879	3
115	0.773	1
116	0.873	4
117	0.797	3
118	0.769	2
119	0.814	2
120	0.835	1
121	0.817	4
122	0.861	1
123	0.831	2
124	0.887	4
125	0.821	1
126	0.791	2
127	0.798	2
128	0.839	1
129	0.774	1
130	0.851	1
131	0.708	1
132	0.833	1
133	0.850	2
134	0.833	1
135	0.875	1
136	0.852	2
137	0.828	1
138	0.809	2
139	0.746	2
Total Number of Observations		330

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