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A Dynamic Model of Milk Production Response for Pakistan

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1. INTRODUCTION

Economists have long been analysing the determinants of milk production while focusing on the relationship between milk price and production. Brandow (1953) used a single equation regression procedure to estimate the supply response function for the United States. Halvorson (1955) also used a single equation regression procedure to analyse the determinants of milk production per cow and found milk production to be highly price inelastic. In another study, Halvorson (1958) used the Nerlovian distributed lag model to estimate both short-run and long-run price elasticities of milk production. Here, he found the long-run price elasticities of United States milk production to be in the range of 0.30 to 0.50. Chen *et al.* (1972) estimated milk production response for both a polynomial and a geometrically declining distributed lag price structure. They found long-run price elasticity to be 2.53. Buckwell (1982) adapted a theory of farm size demonstrated by Kislev and Peterson (1982) to model milk production behaviour in England and Wales. Burton (1984) used a model of the United Kingdom dairy sector to determine simultaneously herd size, number of culls, replacement heifer price, and milk price. In a recent study, Chavas and Kraus (1990) developed a dynamic model of a dairy cow population and milk supply response and applied it to the US Lake States. The authors also calculated dynamic supply elasticities and found the response of supply to market prices to be very inelastic in the short run.

In the case of Pakistan, very few attempts have been made to analyse the determinants of milk production. Anjum *et al.* (1989) estimated a simple two equation simulation model for milk production. The model includes one price equation which is explained by per capita production and per capita income. The other equation aims at explaining changes in milk production with the help of changes in the retail price of milk.

The objective of this paper is to estimate the milk supply response function for Pakistan using a specification which incorporates lags of explanatory variables within the context of the Polynomial Distributed Lag Model and one-period lag of the dependent variable within the context of the Stock Adjustment Model. The paper also aims at estimating dynamic supply elasticities of milk production.

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Section 2 of the paper discusses methodological issues regarding the formulation of the milk supply response function. Section 3 gives the estimation and data details. The results are discussed in Section 4 and Section 5 summarises the study.

THE MODEL

The econometric models which are constructed with a view to explaining milk production usually include among the set of regressors price of milk, prices of inputs used in the production process, prices of competing commodities, capital cost variable, and some measure of risk. The models, generally, utilise some sort of lag structure with a view to incorporating biological lags in the development and reproduction of milk-producing animals. A number of econometric studies of the livestock supply response, for example, introduced lags within the context of the adaptive expectations and partial adjustment models.[1] La France and de Gorter (1985) introduced three lags[2] of dependent variable among the set of explanatory variables but no lags of the independent variables. This type of specification, however, imposes the same pattern of dynamic adjustment to shocks in the explanatory variables.[3] Chen *et al.* (1972) applied two model specifications while estimating the supply response function for milk. The first specification included the lagged dependent variable within the context of the Stock Adjustment Model suggested by Nerlove (1956, 1958). In the other specification, the authors applied the Polynomial Lag Structure suggested initially by Almon (1965) to determine the dynamic adjustment of milk production to changes in the price variable only. Chavas and Kraus (1990) extended the idea of lags and included not only the lags of the dependent variable but also of all the explanatory variables.

The present study aims at applying this generalised specification to analyse complex dynamic adjustments in milk production using data from Pakistan. Consider the following specification:

$$Q_t = \alpha + \sum_{i=1}^{n} \sum_{\tau=0}^{\kappa} \beta_{i\tau} X_{i,\, t-\tau} + \sum_{j=1}^{J} \gamma_j Q_{t-j} + U_t \qquad \ldots \qquad \ldots \qquad (1)$$

where Q_t is quantity of milk produced in year t; X_t is an n-dimensional vector of explanatory variables; κ and J are, respectively, maximum lag lengths of explanatory variables and the lagged dependent variable, α, $\beta_{i\tau}$, and γ_j are the parameters to be estimated; and U_t is the error term which is assumed to be distributed normally with zero mean and constant finite variance.

[1]See for example Klein (1958); Freebairn (1973); Langemeir and Thompson (1974) and Tryfos (1974).

[2]Using annual data.

[3]See [Chavas and Kraus (1990), pp.76].

The specification in (1) may be estimated with various lag lengths both in the explanatory variables and the lagged dependent variable. We choose $J = 1$, because this choice can be interpreted as a partial adjustment model. Following this simplification, Equation (1) becomes:

$$Q_t = \alpha + \sum_{i=1}^{n} \sum_{\tau=0}^{\kappa} \beta_{i\tau} X_{i,\, t-\tau} + \gamma_1 Q_{t-1} + U_t \qquad \ldots \quad \ldots \quad \ldots \quad (2)$$

We do not impose any restriction on κ and propose to consider various values of κ in the empirical section of this study. In order to reduce the number of parameters in (2) we restrict $\beta_{i\tau}$ to lie on a second order polynomial. That is

$$\beta_{i\tau} = a_{o1} + a_{1i}\tau + a_{2i}\tau^2 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Substituting (3) in (2) we get

$$Q_t = \alpha + \sum_{i=1}^{n} \sum_{\tau=0}^{\kappa} (a_{1i} + a_{1i}\tau + a_{2i}\tau^2)]X_{i,\, t-\tau} + \gamma_1 Q_{t-1} + U_t \qquad \ldots \quad \ldots \quad (4)$$

A further restriction is that $\beta_{ik} = 0$, which implies

$$a_{oi} + a_{1i}\kappa + a_{2i}\kappa^2 = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

The restriction makes economic sense because it means that beyond some length of time period κ−1, price changes do not affect current production activity. Solving Equation (5) for a_{oi} and substituting into (4), we get

$$Q_t = \alpha + \sum_{i=1}^{n} \sum_{\tau=0}^{\kappa} [-a_{1i}(\kappa-\tau) - a_{2i}(\kappa^2 - \tau^2)]X_{i,\, t-\tau} + \gamma_1 Q_{t-1} + U_t \qquad \ldots \quad (6)$$

The equation may also be written as

$$Q_t = \alpha - \sum_{i=1}^{n} a_{1i} \sum_{\tau=0}^{\kappa} [(\kappa-\tau)X_{i,\, t-\tau} - \sum_{i=1}^{n} a_{2i}$$

$$\sum_{\tau=0}^{\kappa} (\kappa^2 - \tau^2)X_{i,\, t-\tau} + \gamma_1 Q_{t-1} + U_t \qquad \ldots \quad \ldots \quad (7)$$

3. DATA AND ESTIMATION

This study uses data for the period 1971-72 through 1989-90, 1971-72 being the earliest year for which data on milk production is available. The explanatory variables considered in this study include milk price to consumer price index ratio (PR1C), real credit provided to dairy sector (CRED), and a technology variable proxied by time (TIME).[4] In the price ratio variable, input prices should have been used in the denominator. The number of inputs used in the milk production process is such that we are unable to employ the input prices due mainly to the limited number of observations (only 19 observations are available to estimate the supply response function).

The main sources of data are the Economic Survey, [Government of Pakistan (1991, 1991a, 1991b)]. More specifically, data relating to milk production are taken from the Economic Survey, price information from Pakistan Statistical Year Book, and credit information from Agricultural Credit Indicators. Milk production is in thousands of tonnes, credit is in million rupees, and price is Rs per litre.

The specification given in Equation (7) was estimated using Cooper's (1972) procedure employing the OLS method. The coefficient of the lagged dependent variable caused meaningless results of the delayed parameters. Moreover, its coefficient was found to be insignificant in most of the cases. The response function, as a consequence, was estimated without lagged production. The results in the next section correspond to this specification.

4. RESULTS

OLS estimates of Equation (7) excluding lagged production are reported in Table 1. The estimates correspond to lag lengths of six, seven and eight-year periods. An equation corresponding to a lag length of 9 year period was estimated but results are not reported because most of the parameters were found to be insignificant. Higher lag lengths were not considered because of the limited number of observations. The coefficients, in general, are significant with a desirable degree of precision. The three regressors explain more than 99 percent variation in milk production. Moreover, none of the estimated equations suffer from autocorrelation problem as the value of the Durbin-Watson statistic ranges between 1.82 to 2.09. The equations containing estimates of delayed parameters can be derived from the estimated equation given in Table 1.[5] The three such equations corresponding to ones reported in Table 1 are given in Table 2. The estimates of lagged coefficients are, in general, significantly different from zero at a reasonable degree of confidence.[6] It is interesting to note that the estimates of delayed parameters have a

[4]Credit provided by the Agricultural Development Bank of Pakistan (ADBP) has been used because total credit provided to the dairy sector is not available.

[5]For computation of delayed parameters and their corresponding standard errors see Cooper (1972).

[6]Degrees of freedom in this table are not determined by the coventional formula; $DF = N-k-1$, where k is the number of explanatory variables. It is because delayed parameters are not directly estimated. These are rather estimated from the estimates of the coefficients of constructed price and credit variables. This can be seen from Equation (3). Degrees of freedom for this particular table are determined by the following equation: $DF = N-z-2$, where z is the number of constructed price and credit variables which is four irrespective of the lag length considered (see Equations 8 through 11).

Table 1

Estimated Supply Response Function for Milk

Variables	k=6	k=7	k=8
Constant	4477.900 (3.68)	2496.400* (1.45)	1237.600* (0.33)
PRIC1	863.090 (2.96)	1132.444 (3.88)	1346.383 (2.79)
PRIC2	627.630 (2.52)	881.360 (2.91)	952.140* (1.59)
CRED1	0.477* (0.34)	–0.152* (0.11)	0.020* (0.10)
CRED2	4.926 (11.55)	3.742 (9.67)	3.042 (4.29)
TIME	323.360 (9.01)	393.180 (7.68)	430.390 (3.94)
R^2-Adjusted	0.998	0.999	0.998
F-Statistic	1427.800	1748.270	1141.210
DW-Statistic	1.820	2.040	2.090
Observations	14	13	12

Values in parenthesis are *t*-scores.
* Insignificant at 10 percent level.

$$PRIC_1 = \sum_{\tau=0}^{\kappa} (\kappa - \tau)\, PRIC_{t-\tau} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (8)$$

$$PRIC_2 = \sum_{\tau=0}^{\kappa} (\kappa^2 - \tau^2)\, PRIC_{t-\tau} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (9)$$

$$CRED_1 = \sum_{\tau=0}^{\kappa} (\kappa - \tau)\, CRED_{t-\tau} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (10)$$

$$CRED_2 = \sum_{\tau=0}^{\kappa} (\kappa_2 - \tau_2)\, CRED_{t-\tau} \qquad \ldots \qquad \ldots \qquad .. \qquad (11)$$

Table 2

Derived Supply Response Function for Fresh Milk

Variables	k=6	k=7	k=8
Constant	4477.900	2496.400*	1237.600*
	(3.68)	(1.45)	(0.33)
$PRIC_{t-0}$	863.089	1132.444	1346.383
	(2.96)	(3.88)	(2.79)
$PRIC_{t-1}$	815.964	1118.781	1290.673
	(3.16)	(3.72)	(2.33)
$PRIC_{t-2}$	706.092	1035.087	1189.419
	(2.74)	(3.28)	(1.95)
$PRIC_{t-3}$	533.475	881.362	1042.622*
	(2.34)	(2.91)	(1.69)
$PRIC_{t-4}$	298.110	657.605	850.282*
	(2.05)	(2.64)	(1.51)
$PRIC_{t-5}$	–	363.818	612.398*
		(2.45)	(1.37)
$PRIC_{t-6}$	–	–	328.971*
			(1. 28)
$CRED_{t-0}$	0.477*	–0.152*	0.020*
	(0.34)	(0.11)	(0.01)
$CRED_{t-1}$	3.381	1.995	1.502*
	(5.56)	(2.48)	(1.02)
$CRED_{t-2}$	4.786	3.292	2.489
	(12.83)	(7.06)	(2.37)
$CRED_{t-3}$	4.690	3.742	2.982
	(10.06)	(9.67)	(3.76)
$CRED_{t-4}$	3.095	3.343	2.979
	(8.28)	(8.92)	(4.64)
$CRED_{t-5}$		2.096	2.481
		(7.98)	(4.91)
$CRED_{t-6}$	–	–	1.488
			(4.88)
TIME	323.360	393.180	430.390
	(9.01)	(7.68)	(3.94)
R^2-Adjusted	0.998	0.999	0.998
F-Statistic	1427.800	1748.270	1141.210
DW-Statistic	1.820	2.040	2.090
DF	8	7	6

DF= Degrees of freedom.
Values in parenthesis are *t*-scores.
*Insignificant at 10 percent level.

consistent pattern across equations or lag lengths. A given price change has the highest effect in the current period and in subsequent periods its impact becomes less and less. The pattern of credit variable is different from that of the price variable. Here, a given change in credit availability has little impact on the production of milk in the same period. The impact increases first, peaks at some later stage, and then starts declining. In the first equation credit has its highest impact two periods later and in the remaining two equations the impact peaks three periods later. The estimates of delayed parameters further indicate that the Stock Adjustment Model would not have been an appropriate choice to estimate the dynamic milk production response function because that would have imposed the same pattern (geometrically declining) of dynamic adjustments to changes in milk price and credit availability. The coefficient of the time-trend variable assumes the anticipated sign and is highly significant in all the equations. This result indicates that genetic progress and other structural changes have significantly increased milk production in Pakistan.

Supply Elasticities

From the estimated supply response functions price and credit elasticities can easily be calculated at different points in time. The elasticity of supply associated with a change in real price at any point in time can be calculated as:

$$\eta_{P-\tau} = \frac{\partial Q_t}{\partial P_{t-\tau}} * \frac{\overline{P}}{\overline{Q}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where $P_{t-\tau}$ is short for real price at time $t-\tau$ and P and Q are, respectively, the mean values of real price and quantity over the entire estimation period. The cumulated price elasticity over all κ years then is

$$\eta_{P-\tau} = \sum_{\tau=0}^{\kappa} \frac{\partial Q_t}{\partial P_{t-2}} * \frac{\overline{P}}{\overline{Q}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

The corresponding expressions for credit elasticity can be obtained by replacing credit variable with the price variable in Equations 12 and 13. The estimates of dynamic supply elasticities implied by the three estimated equations are reported in Table 3.

A cursory look at the table reveals that milk production response in Pakistan has been inelastic in relation to changes in milk price and credit availability in the short run as well as the long run. Long-run supply elasticities are reported in the bottom of the table. Estimates of long-run price elasticity range between 0.3 to 0.6.

Table 3

Estimated Supply Elasticities

	k=6		k=7		k=8	
Periods	Price	Credit	Price	Credit	Price	Credit
t+0	0.0800	0.0027	0.1018	–0.0009	0.1180	0.0001
t+1	0.0756	0.0190	0.1006	0.0119	0.1131	0.0095
t+2	0.0655	0.0268	0.0931	0.0196	0.1042	0.0158
t+3	0.0495	0.0263	0.0792	0.0223	0.0914	0.0189
t+4	0.0276	0.0174	0.0591	0.0199	0.0745	0.0189
t+5	–	–	0.0327	0.0125	0.0537	0.0157
t+6	–	–	–	–	0.0288	0.0094
Total Elas.	0.2982	0.0922	0.4665	0.0851	0.5837	0.0883

Chen *et al.* (1972) and Chavas and Kraus (1990) found the long-run response of American milk producers to changes in the milk price to be in the elastic range. The empirical literature about the developing countries, on the other hand, suggests that livestock producers in less-developed economies are not price responsive.[7] Ours' seems to be an in between case where milk producers do respond to changes in milk price but long-run response is in the inelastic range. Long-run credit elasticity estimates are even low (close to 0.1). This implies that by increasing credit supply to the extent of 10 percent milk production can only be increasd to be the extent of 1 percent. Very low credit elasticity is perhaps due to the fact that most of the milk production comes from rural areas where traditional methods are being practiced in milk production. Dairies utilise the credit facilities but they provide only a small fraction of the total milk supply.

5. SUMMARY

Using a dynamic model of milk supply response, this paper estimates the response function for milk production in Pakistan. More specifically, this study introduces lags of explanatory variables within the context of the Polynomial Distributed Lag Model and the one-period lag of the dependent variable within the context of the Stock Adjustment Model. Dynamic supply elasticities implied by the estimated response functions suggest that milk production response in Pakistan has been inelastic to changes in milk price and credit availability in the short as well as the long run.

[7]See Ndzinge, Marsh and Greer (1984).

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Comments on
"A Dynamic Model of Milk Production Response for Pakistan"

The author has addressed the important issue of the determinants of milk production which has so far received very little attention in Pakistan. The author used the Chavas and Kraus (1990) generalised specification to analyse complex dynamic adjustments in milk production using time-series data through 1989-90. Three explanatory variables considered by the author are milk price deflated by the Consumer Price Index, credit availability and technological variable proxies by time.

Results reveal that all explanatory variables are generally significant at the desirable degree of precision. R^2 is more than 99 percent. No autocorrelation problem exists as the D.W. Statistics lie between 1.82 to 2.09. Estimates of elasticities indicate that milk production response in Pakistan has been inelastic to change in milk price and credit availability both in the short as well as the long run. Low credit elasticity may be due to the fact that only a small fraction of total milk production comes from the dairy sector which utilises credit facilities.

The paper is interesting and the author has made a creditable effort. However, I would like to mention a few issues and make a few observations which are not ignored by the author. There is no policy implication or recommendation as milk production is one of the major source of income for subsistence and/or land-less farmers.

The author has already indicated in Section 2 that econometric models which are constructed with a view to explaining milk production usually include a set of variables. Yet he has included only three variables. If more variables are included then the results might be different. Imported dry/canned milk and milk products are very important variables as an alternative to fresh milk. Furthermore, urban milk producers add a substantial amount of dry milk to the fresh milk. There was no clear indication of what deflated price of milk was used in the model because there is a big difference among urban and rural prices. The subsistence and poor producer received very nominal prices.

Beef pricing is also very significant in the milk supply function which was also not considered by the author. It is worth mentioning that both nominal and deflated retail prices of beef have been increasing, reflecting a strong demand for beef. Beef prices are generally lower than mutton prices and are generally more acceptable to a large array of consumers. Beef prices play a role in dairying because the slaughter value of a spent milk animal is a substantial source of farm income.

During the summer season due to high temperatures, there is less production of milk whereas there is higher demand in other seasons. The seasonality aspect of milk production response was overlooked by the author.

Real credit availability needs further explanation about its calculation. The inelastic elasticities both during the short and the long run look astonishing. During the last few years ADBP has been lending a big amount to the dairy industry especially for ultra heat treatment (U.H.T.) plants, although, only 30 percent of their capacity is utilised but the majority of the producer of U.H.T. collect milk from subsistence producers. There is a large spread of U.H.T. milk packed all over the country with a very high price. All these were not considered by the author.

At the first glance all the results look very attractive. R^2 is more than 99 percent which means that the present model explains almost all the variation in milk production which is questionable because the model consists of only three variables.

The author in his model used the Reduced form of Equation after making a number of restrictions. However OLS is not appropriate in estimating this reduced form equation; if it is, certain difficulties will be encountered as follow:

1. The estimates will be inefficient as the disturbance terms in reduced form will probably be serially correlated, irrespective of the correlation of the original disturbance term.
2. the simple least squares estimates will likely be inconsistent in as much as the equation contains the lagged value of the dependent variable; and
3. the equation is over-identified, as the structural parameters cannot be uniquely recovered from the estimated parameters of the reduced form.

There is a strong chance that the model encountered the last difficulty. It is recommended that beside time-series analysis cross-section primary data may be collected and analysed to support and validate the finding.

To conclude, the author has presented an important issue. Hopefully, it will lead to further research. Then model should include substantial data on all direct and indirect variables which affect milk production as milk production is the major source of income for subsistence farmers.

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