

Estimating the Quantitative Importance of Various Sources of Macroeconomic Variability

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I. INTRODUCTION

Providing a reasonable explanation for the business cycle has been the research agenda for many economists since the early 20th century, from Mitchell (1913), Pigou (1927) and Adelman and Adelman (1959) to Lucas (1972), Black (1982) and King and Plosser (1984). For a review, see Zarnowitz (1985). Most attempts to explain the sources of macroeconomic fluctuations attribute the variability in output and prices to only a few sources, sometimes to only one. Kydland and Prescott (1982) and others proposed technology shocks as the main source of aggregate variability; Barro (1977) pointed to unanticipated changes in money stock; Lilien (1982) argued for 'unusual structural shifts' such as changes in the demand for goods relative to services, and Hamilton (1983) concluded in favour of oil price shocks.

As Shiller (1987) noted, various analysts have suggested qualitatively very different exogenous shocks as being important: changes in desired consumption, Hall (1986); breakdowns in the process of borrowing and lending, Bernanke (1981); and breakdowns or establishments of cartels, Rotemberg and Saloner (1986). Moreover, with increased macroeconomic interaction and interdependence, any of these shocks might occur in a foreign country, and be transmitted by trade or financial relations to the domestic country. It seems that there are many possible sources of variability, each of which might, in principle, contribute substantially.

While a significant amount of theoretical research has been undertaken on the business cycle, relatively little empirical work has been conducted that attempts to discriminate among the theories and to measure the quantitative importance of the various sources of macroeconomic variability. Recently, however, Fair (1988)

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has undertaken stochastic simulations using his model of the U.S. economy to estimate the quantitative importance of various sources of variability in U.S. output and prices. Most recently Tahir (1993) adapted and extended Fair's (1988) procedure to quantify the sources of fluctuations in three Canadian macroeconomic variables: output, inflation and the rate of unemployment. Present paper describes Fair's (1988) methodology and provide its extension in a couple of interesting directions. The paper also offers a brief review of some other existing methodologies.

II. REVIEW OF SOME EXISTING METHODOLOGIES

The models in the literature that are used to quantitatively analyse the sources of macroeconomic fluctuations can be classified as follows:

- (i) Structural Econometric Models. (Pioneered by Tinbergen in the 1930s).
- (ii) Vector Autoregression (VAR) Models. (Introduced by Sims (1980)).
- (iii) Index Models. (As used by Sargent and Sims (1980), and Engle and Watson (1981)).
- (iv) Fair's (1988) Technique.¹

The traditional structural econometric models are the oldest, starting with the pioneering work of Tinbergen (1939), and have been used widely during the past few decades. These models vary according to the level of disaggregation with some models comprising only a few equations and others consisting of hundreds of equations. While large scale econometric models have been used to investigate the effects of disaggregated shocks on economic activity for example, Hickman (1972) a summary of the contributions of such shocks has rarely been calculated. These econometric models have been criticised for their use in policy simulation experiments [Lucas (1976)] and for the identifying restrictions used to specify them [Sims (1980)].

An alternative to the traditional structural econometric modelling was introduced in 1980 by Sims. He suggested estimating a set of reduced form equations which treat all variables of interest as endogenous and use an identical number of lags for every variable in every equation. This technique is known as the "Standard Vector Autoregression (VAR)". It has been used widely over the past

¹Although Fair's technique uses a structural econometric model, we have classified his technique separately because it uses these models in a new way.

few years, but has been criticised for its atheoretical nature, in the sense of having no explicit economic structure. An alternative VAR technique, termed as a "structural VAR", has been proposed by Bernanke (1986); Blanchard (1986) and Sims (1986). It retains some of the advantages of the standard VAR approach and makes it possible to explicitly identify and estimate the structural model. However, the structural VAR technique poses complicated computational problems when there are many endogenous variables.

Index models are another method used by some analysts. Sargent and Sims (1980) have used them to study business cycles and Engle and Watson (1981) discussed the general class of index models. These models usually have been designed to explain the behaviour of a vector of time series variables in terms of a small set of unobservable variables and a set of error components which are specific to the particular series. Unlike traditional structural econometric models and the VAR models, index models have not been used a great deal.

III. FAIR'S METHODOLOGY

As Bodkin *et al.* (1991) have noted, Nagar (1969) first applied general stochastic simulation² to the large-scale Brookings model in the late 1960s to analyse the simulation paths of selected endogenous variables. Recently, Fair (1988) has used his econometric model of the U.S. economy to estimate the quantitative importance of various sources of variability by means of stochastic simulation. Having the model specified and estimated using any appropriate estimation technique, stochastic simulation can be used to estimate the variances of endogenous variables in the model. In stochastic simulation, a random shock, drawn from a multivariate distribution which should reflect the stochastic properties of the true model as much as possible, is added to each behavioural equation each time the model is solved. The solution with random shocks can be replicated a number of times for each period in such a way as to produce a distribution of outcomes.

The methodology adopted by fair can be described in general form as follows:

Writing the model as

$$g_i(Y_t, X_t, \alpha_i) = u_{it} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

$$i = 1, 2, \dots, m, \dots, n \quad t = 1, 2, \dots, T$$

²The technique was extended by McCarthy (1972) and adopted by most analysts in stochastic simulation exercises.

Y_t is a vector of endogenous variables at time t , X_t is a vector of lagged endogenous and exogenous variables, and α_i is a vector of the unknown coefficients of the i th equation of the model. There is a total of n equations, m of which are stochastic and $(n-m)$ non-stochastic (model identities). For the m stochastic equations, $u_t = (u_{1t}, u_{2t}, \dots, u_{mt})'$ is the vector of structural disturbances at time t , assumed to be independently and identically distributed as multivariate normal $N(0, \Sigma)$, where Σ is an $m \times m$ symmetric matrix with typical element σ_{ij} being the covariance between the contemporaneous disturbances in the i th and j th equations. It can be estimated as:

$$\Sigma = \frac{1}{T} \sum_{i=1}^T \hat{u}_i \hat{u}_i'$$

where \hat{u}_i is the vector of computed residuals corresponding to U_i .

In order to take into account shocks associated with exogenous variables one can model them with autoregressive equations,³ and then consider the variance covariance matrix to be $(m + k) \times (m + k)$ ⁴ with k being the number of exogenous variables in the model.

Let us now consider the following transformation:

$$D = PR, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

where P is an $(m + k) \times (m + k)$ lower triangular matrix obtained by taking a Choleski decomposition of Σ such that $P/P = \Sigma$, and D and R are described below.

To solve the model for q periods, R is an $(m + k) \times q$ matrix of random numbers where each element is independently drawn from a normal distribution with mean zero and unit variance. D is a $(m + k) \times q$ matrix of shocks to be used in one simulation of the model for q periods. First, the dynamic stochastic simulation solves the model for the first period.⁵ These solved values are then used as

³Another possibility, as Fair (1988) has mentioned, is to assume that exogenous variable shocks are the errors that forecasting services make in their forecasts of exogenous variables.

⁴In estimating the variance covariance matrix, Fair has assumed the errors of structural equations to be uncorrelated with the errors of exogenous variable equations and has taken it to be block diagonal (with $m \times m$ block and $k \times k$ block). We found that, this assumption has significant effects on results.

⁵One must select initial starting values for the lagged variables. These, as in Fair's case, could be the actual values.

lagged right-hand-side variables along with a new column of D to solve the model for the second period, etc. The model can be simulated Z times for q periods by generating R , and hence D , Z times. This gives Z simulated values of all endogenous variables for q periods.

Let y_{it}^z denote the simulated value of variable i for period t when the model is simulated for the z th time. For a total of Z simulations, the estimate of the expected value of variable i for period t , denoted $\hat{\mu}_{it}$, is

$$\hat{\mu}_{it} = \left(\frac{1}{Z}\right) \sum_{z=1}^Z y_{it}^z \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

The estimate of the variance of variable i for period t , denoted by, $\hat{\sigma}_{it}^2$, is

$$\hat{\sigma}_{it}^2 = \left(\frac{1}{Z}\right) \sum_{z=1}^Z (y_{it}^z - \hat{\mu}_{it})^2. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

We will refer to $\hat{\sigma}_{it}^2$ as a "Base Variance".

Now let $\hat{\sigma}_{it}^2(g)$ be the estimated variance of variable i for period t when the error term in the g th⁶ equation is fixed at zero, its expected value. In terms of the above notation, this can be achieved by setting the relevant rows of the matrix P to zero. Let $\hat{\delta}_{it}(g)$ be the difference between the two estimated variances:

$$\hat{\delta}_{it}(g) = \hat{\sigma}_{it}^2 - \hat{\sigma}_{it}^2(g) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

Expressing the contribution of the error in the g th equation to the total variance of variable i for period t in percentage form as:

$$C_{it}^g = 100 [\hat{\delta}_{it}(g) / \hat{\sigma}_{it}^2], \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

then one would expect that

$$\sum_{g=1}^{m+k} C_{it}^g \approx 100, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

i.e. that the base variance should approximately equal the sum of the individual contributions to that variation.

⁶Although g could refer to a subset of equations, we will assume here that g refers to a single equation.

IV. EXTENSION OF FAIR'S TECHNIQUE

As Fair has mentioned, another way of estimating $\hat{\sigma}_i^2(g)$ is to draw just the g th error term and set the rest to zero. Denoting this variance as $\hat{\theta}_i^2(g)$, the contribution of the g th error in the total variance of variable i for period t , in percentage form, denoted as D_{it}^g , is:

$$D_{it}^g = 100 [\hat{\theta}_i^2(g) / \hat{\sigma}_i^2] \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

One would again expect that:

$$\sum_{g=1}^{m+k} D_{it}^g \approx 100 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

Although Fair noticed that these two procedures are not the same if the error term in equation g is correlated with other error terms in the model, he was fortunate that the effect of this correlation was fairly small, inducing him to base his results on the first method. These two methods could produce significantly different results, as long as the correlation among equations is nonzero. An average of the two methods would be more appropriate. i.e.

$$S_{it}^g = [C_{it}^g + D_{it}^g] / 2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

In the appendix we have shown why, instead of choosing one of the two methods, taking the average is more likely to satisfy this approximate "adding up" property. In order to look at the precision of the stochastic-simulation estimates of variances, it is possible to calculate the standard errors of these variances, for a given number of trials. These are explained in the appendix.⁷

From now on, we will refer Equation (6) as **METHOD 1**, Equation (8) as **METHOD 2** and Equation (13) as **METHOD 3**.

Bootstrapping

The bootstrap is a relatively new statistical technique invented by Efron (1979, 1982). It is basically a procedure for estimating standard errors by resampling

⁷ Fair (1988), has shown the calculation of these variances for **METHOD 1**. we have shown the same calculations for only **METHOD 3**.

the data in a suitable way. The idea has been employed by researchers for many applications. Freedman and Peters (1984), for example, applied the bootstrap to an econometric model to attach standard errors to coefficient estimates and forecasts have demonstrated that the usual asymptotic methods seem unsatisfactory. It has also been employed in forecasting and as a tool for verification. See Veall (1989), who emphasises the usefulness of the bootstrap in most applied econometric exercises, and gives a brief review of these applications and other references to the bootstrap literature.

As Veall (1989) has noted, the application of bootstrap-type simulation methods in econometrics is not common.⁸ The bootstrap idea can also be applied for the purpose of stochastic simulation. As compared to Fair's method, in which shocks are drawn from a multivariate normal distribution, bootstrap draws shocks from the empirical distribution of residuals from the real data. In terms of the notation used above, D' in this case is an $q \times (m+k)$ matrix of numbers drawn from actual residuals of the model equations. Each row of D' is a draw of an $m+k$ vector of residuals from a single time period. In this way, the covariances among the errors are captured. D' is redefined, through resampling of actual residuals, each time the model is being simulated. The rest of the procedure, estimating the variances, etc., is similar to the one already described.

V. CONCLUSION

Traditional structural econometric models are typically detailed enough to allow a decomposition of output variability into a variety of constituent shocks. The methodology described in this paper uses structural econometric model and thus allows one to estimate the relative contributions of various shocks to the variance of endogenous variables like real GDP and the GDP price deflator. Index models are suitable only for a small number of variables. The standard VAR technique could also be employed to analyse a system with a large number of variables, but then one has to compromise on the a theoretical nature of the approach. On the other hand, the structural VAR technique, as noted earlier, does not allow one to analyse a system with a large number of variables because of difficult computational problems.

⁸Becoming more common only over the past few years.

A.1. The Sums of all Contributions C_{it}^{ξ} , D_{it}^{ξ} and S_{it}^{ξ}

In this section we will provide some theoretical basis for the violation of Equations (2.4.6) and (2.4.8) and will demonstrate: (i) why the sums of all contributions, C_{it}^{ξ} and D_{it}^{ξ} , can either be less or greater than hundred and (ii) why the average of the two sums, S_{it}^{ξ} , is likely to come closer to satisfying this “adding up” property.

To avoid notational complication, we drop the subscript ‘it’ and assume that the following discussion pertains to variable i for period t .

For demonstration purposes we will consider the example, where $y = U1 + U2 + U3$ with U 's being stochastic variables. The total variance of y when all the error terms are drawn can be expressed as:

$$\begin{aligned} \text{var}(y | y = u1 + u2 + u3) &= \hat{\sigma}^2 \\ &= \text{var}(u1) + \text{var}(u2) + \text{var}(u3) + \\ &\quad 2\text{cov}(u1, u2) + 2\text{cov}(u1, u3) + \\ &\quad 2\text{cov}(u2, u3) \quad \dots \quad \dots \quad \dots \quad \text{(A.1)} \end{aligned}$$

According to Method 1

In method 1, the shocks are set to zero one by one

$$\begin{aligned} \text{var}(y | y = u2 + u3) &= \hat{\sigma}^2(u1) = \text{var}(u2) + \text{var}(u3) + 2\text{cov}(u2, u3) \\ \text{var}(y | y = u1 + u3) &= \hat{\sigma}^2(u2) = \text{var}(u1) + \text{var}(u3) + 2\text{cov}(u1, u3) \\ \text{var}(y | y = u1 + u2) &= \hat{\sigma}^2(u3) = \text{var}(u1) + \text{var}(u2) + 2\text{cov}(u1, u2) \end{aligned}$$

Then the variance difference when $u1 = 0$ would be:

$$\delta(u1) = \hat{\sigma}^2 - \hat{\sigma}^2(u1) = \text{var}(u1) + 2\text{cov}(u1, u2) + 2\text{cov}(u1, u3),$$

the variance difference when $u2 = 0$ would be:

$$\delta(u2) = \hat{\sigma}^2 - \hat{\sigma}^2(u2) = \text{var}(u2) + 2\text{cov}(u1, u2) + 2\text{cov}(u2, u3),$$

and the variance difference when $u3 = 0$ would be:

$$\delta(u3) = \hat{\sigma}^2 - \hat{\sigma}^2(u3) = \text{var}(u3) + 2\text{cov}(u1, u3) + 2\text{cov}(u2, u3).$$

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Comments on
“Estimating the Quantitative Importance of
Various Sources of Macroeconomic Variability”

This paper provides a theoretical framework for estimating quantitative importance of various sources of variability using a macroeconomic model. Suggesting limitations of the existing procedure of Fair (1988) and alternative methodologies such as VAR and Index models, there is an attempt to provide an extension which is thought to be superior.

Let me briefly outline the contribution of Fair's methodology. According to this procedure, output or unemployment variability can be attributed to shocks in stochastic equations or to shocks in exogenous variables such as technology, prices or "other" unanticipated shifts. In Fair's model, the estimated base variance is compared with the "simulated" variance where the error term(s) in a subset of equations are fixed at their expected values. Thus, Fair's methodology captures the contribution of the error term in the exogenous variable equation to the variance of the dependent variable. It may be noted that this contribution is not the same as the multiplier effect of the exogenous variable on the dependent variable.

The paper by Rizwan Tahir provides a simple extension to the above methodology. Instead of fixing error terms of one of the equations to expected values, or fixing error terms of all equations except gth equation to their expected values, the author proposes to use a simple average of these two alternatives. Despite over-simplicity, there is one problem with this technique and that concerns with stochastic simulation error. As noted by Fair, it takes about 1000 trials to make base and simulated variances really small to be acceptable. This may not be a difficult with efficient computers of modern times. What really bothers me is the computation of the difference between the two estimated variances. Even in a simple case it takes a great deal of effort and many "tricks" to make the difference in variances small. I wonder how this riddle has been resolved by the author.

My second concern relates to the use of econometric model for simulations. Even though the stochastic simulation technique is different, it nevertheless utilises parameter estimates in computing covariance matrix. I would like to know how Lucas critique has been avoided.

Third, on different pages the author has thrown statements which could only be made if the model has already been estimated. I know that the paper is drawn from author's Ph. D dissertation where the model must have been estimated, but for a reader it is too difficult to confirm these statements without looking at the actual results.

Finally, I noticed a few errors in the paper which could easily be avoided by a careful revision. For example, the variance difference when $u_2 = 0$ is not correct even though the subsequent derivation is correct. Similarly, it is not clear whether the model is simulated for z th trial or for z th time. Finally, it was difficult for me to locate Equations (2.4.6) and (2.4.8). I understand that the author was referring to Equations 6 and 8.

All in all, the paper is a constructive effort in the right direction which should be complimented.

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