

Non-linear Speculative Bubbles in the Pakistani Stock Market

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Since 1987 many stock markets of the world have experienced volatility. This has been true of many emerging stock markets. Our study of daily stock market data from Pakistan between June 1987 and May 1993 finds the results to be consistent with the impression of great volatility and unpredictability thought to be common in such emerging markets. We used the VAR technique to estimate a "presumed" fundamental on stock indices using lagged first differences of natural logs of daily exchange rates and stock indices. We used the Hamilton switching model and associated Walk test to see if such speculative trends were present. We were significantly unable to rule them out. We then tested for ARCH effects, whose presence we failed to reject. We then used ARCH-generated residuals to apply the BDS test of general non-linear structure. We failed to reject the lack of such non-linear structure quite significantly. Thus, the Pakistani stock market during the period of study seems to have exhibited quite complex dynamics, along with apparently strong trends that may indicate the presence of speculative bubbles. This has many important implications for Pakistani as well as other emerging markets.

I. INTRODUCTION

The possible existence of speculative bubbles in stock markets has been discussed since at least the time of the great Mississippi and South Sea Bubbles in France and England in 1719 and 1720 [Kindleberger (1989)]. That the rapid rise and then decline of the New York stock market in 1929 may have reflected a speculative bubble has also long been argued [Galbraith (1954); Rappoport and White (1993)]. The crash of many stock markets on October 19, 1987 also triggered speculation

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that speculation on bubbles was involved in the run-up to the peculiar events of that day [Hardouvelis (1988)].

More recently, the opening up and development of stock markets in less-developed countries (LDCs) has led to significant run-ups in the value of stocks in many of these markets. In many cases, there has been concern that some of these upward movements may have exceeded what would be justified by rational expectations forecasts of the future paths of fundamentals; in other words, that many of these markets have been experiencing speculative bubbles with the attendant dangers of sudden crashes and their associated instabilities. However, we note that it is almost impossible to state for certain that any particular price movement is a bubble because an observer can posit that market participants may have had a rational expectation of a non-occurring outcome, the problem of the "misspecified fundamental" [Flood and Garber (1980)].

A recently developed technique has been the regime-switching model due to Hamilton (1989), subsequently used by Engel and Hamilton (1990) to study long swings in the value of the US dollar. This approach can be combined with the one developed by Canova and Ito (1991) that examines residuals of vector autoregressions (VARs) of stock prices and exchange rates, using the VARs as imputed estimates of the fundamentals. Such an approach has been used by Ahmed, Ayogu and Rosser (1992) to study non-linear bubbles in exchange rates in major currencies. This is the basis of the approach used in this paper for studying the Pakistani stock market.

A further element of our analysis is to examine the element of non-linear dynamics. One aspect is to test for autoregressive conditional heteroskedasticity (ARCH) [Engle (1982)], that is, volatility induced by volatility. Another is a more general test for non-linearity that may indicate the possible presence of chaotic dynamics known as the BDS statistic [Brock, Dechert and Scheinkman (1987)]. We apply this to the residuals after they have been corrected for ARCH effects. We carry out both of these tests in the prescribed order.

We use a unique set of daily data from mid-1987 to early 1993 for Pakistani exchange rates and two stock market indices. The basic outcome of our estimates is that we fail to reject the presence of persistent trends in residuals of our estimated fundamentals, and we fail to reject the presence of both ARCH effects and the more general non-linearity implied by the BDS statistic for both stock market indices for the data set in question even after ARCH effects have been accounted for. Of course, our study does not overcome the possibility of a "misspecified fundamental" problem as noted above.

We are not especially surprised to find such results for an LDC stock market. However, such a finding has a special significance for the Pakistani stock market because of the long-standing effort within Pakistan to make its financial markets

behave in a manner consistent with the principles of Islamic economics [Kuran (1993); Rosser and Rosser (1995), Chapter 5]. Among those principles is an abjuration of *gharar*, or betting on chance [Siddiqi (1980)]. Although this is subject to interpretation, it can be argued according to a strict Islamic view that speculation on a price that is disconnected from its underlying profit-related fundamental, that is, a speculative bubble, constitutes such *gharar*. Thus, its possible presence in Pakistan's stock market represents a potential difficulty for Pakistani economic policy. The finding of possible chaotic non-linearity beyond ARCH effects simply adds to this potential policy difficulty.

The rest of this paper is organised as follows. The second section is organised in four parts. Part A discusses a theory of speculative bubbles in general and will present a model of non-linear bubbles that could potentially behave chaotically. Part B discusses the regime-switching tests on VAR residuals for bubbles. Part C will explore the presence of ARCH effects, and Part D will handle the generalised non-linearity tests, namely, the BDS statistics. After the discussion of methodology in Section II, the method of gathering data from the Pakistani market will be covered in Section III. Section IV details the results from the four techniques mentioned in Section II. These results are presented in various tables at the end of this paper. Section V of the paper discusses the implications of this study for emerging markets in general and the Pakistani markets in particular.

II. METHODOLOGY

(A) Theory of Non-linear Speculative Bubbles

A bubble, B , exists on an asset if

$$P_t = F_t + B_t \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where P_t is the price of the asset in time t and F_t is the fundamental value of the asset. A general definition of a fundamental is the long-run, steady-state, general equilibrium value of the asset based on tastes and technology. For financial assets, a simpler partial equilibrium version of this is the present discounted value of the rationally expected stream of future net earnings of the asset. That it is extremely difficult to know for certain what that is for any asset is the basis for the "misspecified fundamental" problem. In practice, it is almost impossible to absolutely prove that any particular price movement is definitely a bubble because one does not know absolutely what its fundamental truly is.

A central issue in the theory of speculative bubbles has been the question of circumstances under which they are theoretically possible in a world of fully ratio-

nal and well-informed agents.¹ There is no question that they can happen if people are risk-lovers (*gharar*-following gamblers), if people are caught up in irrational fads [Shiller (1984)] or manias [Kindleberger (1989)], or if some of them are poorly informed (“noise traders” [Black (1986)], or more pathetic yet, “suckers”). The issue is the “hot potato problem”. If agents are fully rational and informed, then they will not buy at the end of the bubble because they would not wish to be stuck with the “hot potato” (the asset about to crash to its fundamental). But if it is not rational to enter at the end, it will not be rational to do so in the period before that, or the one before that, unravelling backwards to the beginning of time. Hence, the bubble should never start. Thus, it is not surprising that the conditions under which fully rational bubbles can come about are somewhat limited.

The strongest version of this argument is due to Tirole (1982), who showed that bubbles were impossible in a discrete-time world with a finite number of fully-informed, risk-averse, infinitely-lived agents trading in a finite number of real assets with real returns. In this world, every asset will equal the present discounted value of the stream of its future real returns. Relaxing those assumptions allows for various bubbles.

Thus, Tirole (1985) showed that if there are an infinite number of finitely-lived agents in an overlapping generations context, bubbles can exist under certain conditions in a growing economy.² Gilles and LeRoy (1992) have shown that in an infinite commodity world, there can exist static, infinite-valued “charges” that they interpret as being equivalent to bubbles.³ Brock (1974, 1975) showed the possibility of hyper-inflationary bubbles in a world of fiat money, an asset with no real return whose true fundamental is actually zero.⁴ Blanchard (1979) and Blanchard and Watson (1982) have shown that if a bubble will crash with some probability, rational investors can be compensated with a risk premium if the bubble rises fast enough. Allen and Gorton (1993) show the possibility of “churning bubbles” with rational agents if there is asymmetric information between investors and portfolio managers.

The latter model fits in with what has increasingly become the trend in the modelling of bubbles, namely, allowing for some kind of heterogeneity among

¹For a more extended discussion of rational versus irrational bubbles, see [Rosser (1991), Chapters 4 and 5]. It can be argued that the first suggestion of the possibility of rational bubbles was the “beauty contest” model of [Keynes (1936), Chapter 12] in which the rational agent guesses the judgments of his fellow judges rather than the actual beauty of the contestants.

²A considerable literature following Tirole suggests that in many cases the bubble will adversely affect growth as savings are drained off into the bubble [Grossman and Yanagawa (1993)].

³If static bubbles exist, then a change in fundamentals can generate over- and under-shooting of prices as non-linear functions, the so-called “intrinsic bubbles” [Froot and Obstfeld (1991)].

⁴In fact, such hyper-inflationary bubbles really amount to the collapse towards the zero fundamental of the otherwise static bubble value of such money.

traders, with at least one agent or group of agents being either less well-informed or just plain irrational [De Long, Shleifer, Summers and Waldman (1990)]. As noted above, bubbles then become much more possible because there is somebody on whom the "hot potato" can be dumped. The movement in this direction has been further stimulated by empirical tests that have not only found asset markets not following fundamentals but also not fulfilling the conditions for a rational bubble.

Frankel (1985) found this for the behaviour of the US dollar in the early 1980s, and Dwyer and Hafer (1990) found it so for stock markets in seven different countries.

A model in this heterogeneous agent tradition that allows for non-linear and even chaotic dynamics is due to Day and Huang (1990). Three groups of traders are rational investors who follow fundamentals, trend-chasing "sheep", and market specialists who set prices.

Rational investors know the constant fundamental, F , and also an upper bound M , and a lower bound m , and they sell when P is above F and buy when P is below F according to the weighting function $f(P)$. Their excess demand is given by

$$a \alpha (P) = \begin{cases} a(F - P) f(P), & P \in [m, M] \\ 0, & P < m, P > M \end{cases} \quad \dots \quad \dots \quad \dots \quad (2)$$

with

$$\alpha (P) > 0, P < F \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

$$\alpha (P) < 0, P > F \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

$$\alpha (P) = 0, P = F, P < m, \text{ or } P > M \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

$$\alpha (P) < 0, m < P < M. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

It is the trend-chasing behaviour of "sheep" (chartists) that is responsible for the emergence of bubbles. Their excess demands are given by

$$b \beta (F) = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

$$\beta' (P) > 0. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

The combined excess demand of these two groups is given by

$$E(P) = a \alpha (P) + b \beta (P). \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

Market specialists impact dynamics through an adjustment coefficient, c , leading to

$$P_{t+1} = P_t + cE(P_t). \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

Equilibrium at the fundamental will be unstable if sheep behaviour is stronger than rational investor behaviour, that is

$$a \alpha' (F) + b \beta' (F) > 0. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

This will generate two temporary equilibria, $P_m < F$ and $P_\mu > F$ which will also be unstable if at both $P = P_m$ and $P = P_\mu$

$$-2 > c[a\alpha' (P) + b\beta' (P)] \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

which holds if at both temporary equilibria

$$a (F - P) f' (P) - af (P) < -2/c - b\beta' (P). \quad \dots \quad \dots \quad \dots \quad (13)$$

Under these conditions there will be both up and down bubbles as the sheep chase the up-trend bull markets and down-trend bear markets.

Day and Huang (1990) show that if the excess demand functions exhibit sufficiently strong degrees of non-linearity, then these unstable bubble dynamics can exhibit sensitive dependence on initial conditions, the standard *sine qua non* for chaotic dynamics. This means that a slight change in a starting position can lead to a completely different trajectory. The dynamics will appear to be highly irregular and will be locally unstable, but bounded. Such a situation can be labelled a "chaotic bubble."⁵

(B) Regime-switching Tests on VAR Residuals for Bubbles

Of course the fundamental for a stock should be related to its dividend, and comparisons of the dynamic behaviour of dividends and stock prices have been a focus of many of the studies of excess volatility noted above. However, given that dividends are issued fairly infrequently, they are not very useful for deriving perceived fundamentals for stocks as information flows through markets on a daily basis. Many studies of the short-run equities behaviour often take a univariate approach. However, we prefer to bring in some other indicator of information about

⁵De Grauwe, Devachter and Embrechts (1993) present models of chaotic bubbles in foreign exchange rates.

the economy that can be related to stock prices in the hope of elucidating an estimated path of the fundamental, even if we cannot fully resolve the “misspecified fundamental” problem.

In many markets some interest rate would be such a useful indicator. However, this is not the case for the Pakistani stock market, given that such rates also vary infrequently and are subject to intense regulation, complicated by Islamic financial rules.

Thus, following the work of Canova and Ito (1991), we have used the foreign exchange rate as a variable that is reasonably flexible and reflective of general confidence in the economy. We note that especially for LDCs, in general, foreign capital often plays an important role in the stock markets. Thus, one would naturally expect a positive relationship between the exchange rate and the stock market. Of course, this is not a perfect relationship as there can be countervailing sectoral effects, notably that a currency depreciation can stimulate the prices of stocks in export industries.

Presuming that bubbly sheep chase trends in a lagged manner, their driving behaviour is not strictly observable. This suggests the use of the Hamilton (1989) regime-switching model to test for trends and switches of trends. Considering the residual series as possibly driven by bubbles gives

$$e_t = n_t + Z_t \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (14)$$

where

$$n_t = \mu_1 + \mu_2 s_t \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

and

$$Z_t - Z_{t-1} = \phi (Z_{t-1} - Z_{t-2}) + \dots + \phi_r (Z_{t-r} - Z_{t-r-1}) + e_t \quad \dots \quad \dots \quad (16)$$

with $s = 1$ being a positive trend, $s = 0$ being a negative trend, and $\mu_i \neq 0$ indicating the existence of any explanatory trend element beyond the VAR and AR processes.

Following Engel and Hamilton (1990), a general test of “no bubbles” is to let the null hypothesis of no trends be given by $p = 1 - q$. This can be tested using a Wald test statistic given by

$$[p - (1-q)] / [\text{var} (p) + \text{var} (1-q) + \text{covar} (p, 1-q)]. \quad \dots \quad \dots \quad (17)$$

The results of Wald test are discussed in Section IV and actual statistics are presented in Table 1.

Table 1

*Wald Test Results Based on Residuals from the
Two-Variable VAR Model
(Exchange Rate and Stock Market Index)*

Index	$H_0^1: \mu_1 = \mu_2$ $\chi^2(1)^*$	$H_0^2: \rho_1 = 1 - \rho_2$ $\chi^2(1)^*$
KSE	20.80	320.23
SBP	0.45	127.82

* Critical Value $\chi^2(1) = 3.84$.

Samples: KSE: Daily Data from June 1, 1987 to May 18, 1993.

Number of observations used = 1,328.

SBP: Daily Data from June 1, 1987 to March 17, 1993.

Number of observations used = 1,268.

(C) ARCH Effects

It was Mandelbrot (1963) who first suggested that volatility in financial markets could beget further volatility. A measure of this idea was formalised by Engle (1982) as autoregressive conditional heteroskedasticity (ARCH), which implies a non-linear dependence in variances. It is given by

$$x_t = \sigma_t u_t \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (18)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^M \alpha_i \chi_i^2 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (19)$$

with u_t being independently and identically distributed (i.i.d.) and the α_i 's representing different lags. We use a three-period lag to test for the presence of ARCH effects. The results are discussed in Section IV and presented in Tables 2A and 2B.

Table 2A

*ARCH Results Based on Residuals from the Two-Variable VAR Model
(Exchange Rate and KSE Index)*

Parameters	Estimates	S. E.	t-Stat
C	-0.000374	0.000196	-1.91
Alpha 0	0.0000382	0.00000069	55.38*
Alpha 1	0.330968	0.039946	8.28*
Alpha 2	0.204338	0.021200	9.64*
Alpha 3	0.138339	0.019221	7.20*

* Significant at 1 percent.

N = 1325.

Dependent Variable = KSE Index.

Table 2B

ARCH Results Based on Residuals from the Two-Variable VAR Model
(Exchange Rate and SBP Index)

Parameters	Estimates	S. E.	t-Stat
C	-0.000262	0.000116	-2.26
Alpha 0	0.0000317	0.00000024	51.35*
Alpha 1	0.316406	0.040874	7.74*
Alpha 2	0.391021	0.030396	12.86*
Alpha 3	0.394100	0.029662	13.29*

*Significant at 1 percent.

N = 1265.

Dependent Variable = SBP Index.

(D) Generalised Non-linearity: The BDS Statistic

For our next step, we take out of the variability of the residuals all that can be attributed to the ARCH effects. These remaining residuals of residuals are what we further examine for the remaining unexplained non-linearity in the basic series. Our technique of investigation is the BDS statistic due to Brock, Dechert and Scheinkman (1987).⁶

The correlation integral for a data series $\{a_t\}$, $t = 1, \dots, T$ comes from forming m -histories such that $(a^m_t, = [a_t, a_{t+1}, \dots, a_{t+m+1}]$ for any embedding dimension m , and is

$$c_{m,T}(\epsilon) = \sum_{t < s} I_{\epsilon}(a^m_t, a^m_s) [2/T_m (T_m - 1)] \dots \dots \dots (20)$$

with ϵ is a tolerance distance,⁷ $I_{\epsilon}(a^m_t, a^m_s)$ is the indicator function which equals 1 if $\|a^m_t - a^m_s\| < \epsilon$ and equals zero otherwise, and $T_m = T - (m - 1)$.

The correlation integral generates the correlation dimension as

$$D_m = \lim_{\epsilon \rightarrow 0} \lim_{T \rightarrow \infty} \ln [C_{m,T}(\epsilon)] / \ln(\epsilon). \dots \dots \dots (21)$$

⁶For a more detailed discussion of the BDS statistic, see Brock, Hsieh and LeBaron (1991).

⁷It is a common practice to select as ϵ the standard deviation divided by the spread of the data, which we follow.

due originally to Grassberger and Procaccia (1983). It can be interpreted as the fractal dimensionality of an underlying strange attractor that can be ordering the dynamics, which could even be chaotic, thus not appearing to possess any structure.

The BDS statistic also comes from the correlation integral as

$$BDS(m, \epsilon) = T^{1/2} \{c_m(\epsilon) - [c_1(\epsilon)]^m\} / b_m \quad \dots \quad \dots \quad \dots \quad (22)$$

where b_m is the standard deviation of the BDS statistic dependent on m . The null hypothesis of i.i.d. data is that for a given ϵ and an $m > 1$, $c_m(\epsilon) - [c_1(\epsilon)]^m$ will equal zero. Thus, sufficiently large values of the BDS statistic indicate non-linear structure in the relationships between the variables rather than white noise.⁸ The results from BDS tests are discussed in Section IV, and statistics are presented in Tables 3A and 3B.

Table 3A

*BDS/SD Statistics Based on Residuals from
Two-Variable VAR Model
(Exchange Rate and KSE Index)*

m	T = No. of Observations	BDS/SD Statistics
2	1329	11.98
3	1329	12.82
4	1329	14.29

Sample: Daily KSE Index from June 1, 1987 to May 18, 1993.

Table 3B

*BDS/SD Statistics Based on Residuals from
Two-Variable VAR Model
(Exchange Rate and SBP Index)*

m	T = No. of Observations	BDS/SD Statistics
2	1269	15.51
3	1269	17.78
4	1269	19.89

Sample: Daily Data from June 1, 1987 to March 17, 1993.

⁸Estimates of the BDS statistics are subject to the small sample bias. However, this is not a problem for samples in excess of a thousand observations, which we have, thanks to our use of daily data.

III. DATA

Although the Karachi Stock Exchange (KSE) in Pakistan has been in business since 1947, like many stock markets in developing countries it had a very thin volume of trade until the mid-1980s. Market capitalisation increased from Rs 31617.1 million in June 1987 to Rs 218357.2 million in June 1992, showing an increase of 590 percent. The State Bank of Pakistan general index of share prices increased by 155 percent between June 1987 and December 1992. According to a recent report (*Economic Survey 1991-92*), the unprecedented rise in market capitalisation as well as the general index during the last few years may have been due to government liberalisation policies to encourage private domestic and foreign investment, transfers of investible funds from the sagging real estate sector, and the bright prospects for joint ventures with countries like Iran, Saudi Arabia, Japan, and South Korea. Another possibility would have been the introduction of profit-loss based "Mudaraba" securities, which are permissible under Islamic guidelines.

There has been very little econometric research done on the Pakistani stock market. However, two recent studies by Khilji (1992) and Uppal (1992) have tried to explore market capitalisation and the movement of share prices for various industries in Pakistan by utilising monthly data. Daily data have been very difficult to obtain. We have had some success in collecting daily data on the Karachi Stock Index (KSE index) and on the State Bank of Pakistan general index of share prices (SBP index) shown in Figure 1. In particular, we have collected daily data for the KSE index from June 1, 1987 to May 18, 1993 and for the SBP index from June 1, 1987 to March 17, 1993.

The period covered in our sample saw a substantial increase in activity in the Pakistani stock market. Although the monthly SBP index has been available for the last twenty years or so, the daily data had to be collected by doing a thorough search of newspapers and government reports. A major source of the SBP index was daily reports submitted to the Corporate Law Authority in Islamabad by the State Bank of Pakistan. The SBP index is based on prices of shares sold at the KSE. However, an alternative series (listed here as the KSE index) was also obtained from *Daily Business Recorder*. However, a lot of missing observations were supplied by the Karachi Stock Exchange research division. Both series have been adjusted for any base-year changes.

The foreign exchange rate data were obtained from various newspapers and State Bank bulletins. The series is an average of official buying and selling rate of the number of rupees per dollar. This is the most plausible representative of exchange rate movements in Pakistan.

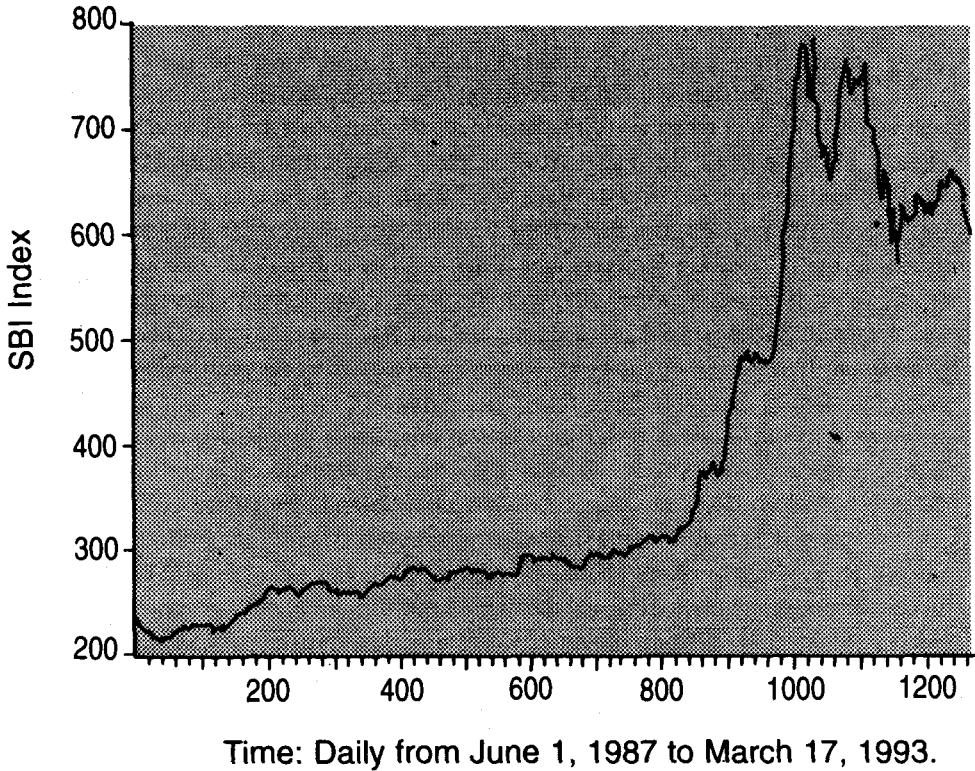


Fig. 1. The State Bank of Pakistan General Index of Share Prices.

IV. RESULTS

We have estimated VAR models using first differences of the natural logarithms of the stock market indices (separate models for each index) and the exchange rate, using an eight-period lag structure.⁹ We estimated residuals for all series, e_t . These series are the objects of our further tests. Table 1 presents the results from the Wald test discussed earlier. The right column of Table 1 contains the estimated values for this statistic. Rejection of the null hypothesis at the 5 percent significance level requires the statistic to exceed 3.84. It does so by about two orders of magnitude for both stock market indices. Thus, trends beyond the estimated fundamental cannot be ruled out.

Also the null that $\mu_1 = \mu_2$ can be tested which implies the series is a stochas-

⁹These models are available on request from the authors. For a precise description on the methodology used, see Ahmed, Rosser and Sheehan (1988, 1989).

tic process with identical means in the two states. The Wald test for this is given by an equation identical to (17) but with μ_1 and μ_2 replacing p and q respectively. Estimated values for this statistic are in the left column of Table 1. The results of this are more mixed, with the null hypothesis rejected at the 5 percent level for the KSE index, but not for the SBP index.

However, the first of these tests is the more general one and has the interpretation of testing for trends, whereas the second really tests for different patterns of up-and-down behaviour. The existence of trends, and thus bubbles, cannot be rejected, whereas it is unclear whether the up-and-down trends behave differently or not.

We have estimated ARCH models on the residuals of the VAR models and presented the results in Tables 2A and 2B. The respective VAR models both use the exchange rate, but vary by use of the different stock market indices. However, this makes no difference as all three lag lengths for both models show up as significant at the 1 percent level. Thus, there is strong evidence of ARCH effects on these residuals, and it is consistent with the findings for most stock markets.

Tables 3A and 3B present the BDS statistics with the accompanying standard errors and t -statistics for the KSE-exchange rate model and the SBP-exchange rate model, respectively, and $m = 3$. The critical value is around 6.¹⁰ As can be readily seen, both models show values well in excess of that indicating a failure to reject the absence of non-linear structure in these series, even beyond ARCH effects. However, we emphasise that this does not prove the presence of chaotic dynamics, although it does not rule it out either.

V. CONCLUSIONS

Our study of daily stock market and exchange rate data from Pakistan between July 1987 and March 1993 finds results consistent with the impression of great volatility and unpredictability thought to be common in such markets in most LDCs. Furthermore, we find evidence suggesting possible presence of speculative bubbles as well as of non-linear structure in the variances, as well as beyond that. However, we cannot say for certain that there were speculative bubbles because of the "misspecified fundamental" problem. Neither have we shown that the non-linearity beyond ARCH effects in the variances is chaotic.

We used VAR techniques to estimate a presumed fundamental on the stock market indices, using the lagged first differences of the natural logs of the exchange rate and the stock market indices. Trends of residuals of this model were then presumed to indicate possible speculative bubble movements. We used the Hamilton regime-switching model and an associated Wald test to see if such trends were

¹⁰This cut-off is somewhat imprecise as it is based on an interpolation from Table E1 in Brock, Dechert and Scheinkman (1991), whose formulation we follow here.

significantly present, and we were significantly unable to rule them out. This test has an interpretation in terms of the bull-bear model of Day and Huang (1990), which is also capable of generating non-linear dynamics, including chaotic dynamics.

We then tested for ARCH effects whose presence we failed to reject for the residuals. We used the ARCH model to remove dynamics attributable to that source and subjected the remaining residuals to the BDS test of general non-linear structure. We failed to reject a lack of such non-linear structure quite significantly. Thus, the Pakistani stock market during the period of study seems to have exhibited quite complex dynamics, along with apparently strong trends that may indicate the presence of speculative bubbles.

Such a possibility raises questions for Islamic economics and its prohibition of *gharar*, or betting on chance, within the context of the self-proclaimed Islamic economic policy of Pakistan. Even if this is not viewed as a serious issue by Pakistani officials, the apparent propensity to unexplained trends and non-linear dynamics suggests that the Pakistani economy may be subject to instabilities and oscillations that may be generated by the erratic and complex dynamics of its stock market. Indeed, that is exactly what we have observed for a number of historical episodes where extreme erraticity of stock markets culminating in grand crashes of great speculative bubbles has eventually rebounded to create difficulties and declines in the broader economies involved.

On the other hand, rapidly rising but volatile stock market prices also reflect optimism regarding future economic development which will hopefully prove to be ultimately warranted. It is precisely the uncertainty surrounding forecasts of future economic development that underlies the "misspecified fundamental" problem. Apparently, unwarranted stock price increases may simply reflect a rationally forecast small probability of astounding increases in economic growth in Pakistan that may or may not actually come to pass.

REFERENCES

- Ahmed, Ehsan, J. Barkley Rosser, Jr. and Richard G. Sheehan (1988) A Global Model of OECD Aggregate Supply and Demand Using Vector Autoregressive Techniques. *European Economic Review* 32: 1711-1729.
- Ahmed, Ehsan, J. Barkley Rosser, Jr. and Richard G. Sheehan (1989) A Comparison of National and International Aggregate Supply and Demand VAR Models: The United States, Japan, and the European Economic Community. *Weltwirtschaftliches Archiv* 125: 252-272.
- Ahmed, Ehsan, Melvin D. Ayogu and J. Barkley Rosser, Jr. (1992) Non-linear Bubbles in Foreign Exchange Rates. Harrisonburg: James Madison University. (Mimeographed.)

- Allen, Franklin, and Gary Gorton (1993) Churning Bubbles. *Review of Economic Studies* 60: 813–836.
- Black, Fischer (1986) Noise. *Journal of Finance* 41: 529–543.
- Blanchard, Olivier J. (1979) Speculative Bubbles, Crashes, and Rational Expectations. *Economics Letters* 3: 387–389.
- Blanchard, Olivier J., and Mark W. Watson (1982) Bubbles, Rational Expectations, and Financial Markets. In P. Wachtel (ed) *Crises in the Economic and Financial Structure*. Lexington, MA: Lexington Books.
- Brock, William A. (1974) Money and Growth: The Case of Long-run Perfect Foresight. *International Economic Review* 15: 750–777.
- Brock, William A. (1975) A Simple Perfect Foresight Monetary Model. *Journal of Monetary Economics* 1: 133–150.
- Brock, William A., W. D. Dechert and José A. Scheinkman (1987) A Test for Independence Based on the Correlation Dimension. Madison, WI: University of Wisconsin-Madison. (SSRI Paper No. 8702.)
- Brock, William A., David A. Hsieh and Black LeBaron (1991) *Non-linear Dynamics, Chaos, and Instability*. Cambridge, MA: MIT Press.
- Canova, Fabio, and Takatoshi Ito (1991) The Time Series Properties of the Risk Premium in the Yen/Dollar Exchange Rate. *Journal of Applied Econometrics* 6: 125–142.
- Day, Richard H., and Weihong Huang (1990) Bulls, Bears, and Market Sheep. *Journal of Economic Behaviour and Organisation* 14: 299–329.
- De Grauwe, Paul, Hans Devachter and Mark Embrechts (1993) *Exchange Rate Theory: Chaotic Models of Foreign Exchange Markets*. Oxford: Blackwell.
- De Long, J., A. Shleifer, L. Summers and R. Waldman (1990) Positive Feedback Investment Strategies and Destabilising Rational Speculation. *Journal of Finance* 45: 379–395.
- Dwyer, Gerald P., Jr., and R. W. Hafer (1990) Do Fundamentals, Bubbles, or Neither Determine Stock Prices? In G. P. Dwyer, Jr. and R. W. Hafer (eds) *The Stock Market: Bubbles, Volatility, and Chaos*. Boston: Kluwer Academic Publishers.
- Engel, Charles, and James D. Hamilton (1990) Long Swings in the Dollar: Are They in the Data and Do Markets Know It? *American Economic Review* 80: 689–713.
- Engle, Robert F. (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* 50: 987–1007.
- Flood, Robert P., and Peter M. Garber (1980) Market Fundamentals Versus Price Level Bubbles: The First Tests. *Journal of Political Economy* 88: 745–776.
- Frankel, Jeffrey A. (1985) The Dazzling Dollar. *Brookings Papers on Economic*

- Activity 1: 199–217.
- Froot, Kenneth A., and Maurice Obstfeld (1991) Intrinsic Bubbles: The Case of Stock Prices. *American Economic Review* 81: 1189–1214.
- Galbraith, John Kenneth (1954) *The Great Crash, 1929*. Boston: Houghton Mifflin.
- Gilles, Christian, and Stephen F. LeRoy (1992) Bubbles and Charges. *International Economic Review* 33: 323–339.
- Grassberger, Peter, and Itamar Procaccia (1983) Measuring the Strangeness of Strange Attractors. *Physica D* 9: 189–208.
- Grossman, Gene M., and Noriyuki Yanagawa (1993) Asset Bubbles and Endogenous Growth. *Journal of Monetary Economics* 31: 3–19.
- Hamilton, James D. (1989) A New Approach to the Economic Analysis of Non-stationary Time Series and the Business Cycle. *Econometrica* 57: 357–384.
- Hardouvelis, Gikas A. (1988) Evidence on Stock Market Speculative Bubbles: Japan, the United States, and Great Britain. *Federal Reserve Bank of New York Quarterly Review* Summer: 4–16.
- Keynes, John Maynard (1936) *The General Theory of Employment Interest and Money*. New York: Harcourt Brace, Jovanovich.
- Khilji, Nasir M. (1993) The Behaviour of Stock Returns in an Emerging Market: A Case Study of Pakistan. *The Pakistan Development Review* 32: 593–604.
- Kindleberger, Charles P. (1989) *Manias, Panics, and Crashes*. 2nd Edition. New York: Basic Books.
- Kuran, Timur (1993) The Economic Impact of Islamic Fundamentalism. In M. E. Marty and R. S. Appleby (eds) *Fundamentalisms and the State: Remarking Politics, Economies, and Militance*. Chicago: University of Chicago Press.
- Mandelbrot, Benoit B. (1963) The Variation of Certain Speculative Prices. *Journal of Business* 36: 394–419.
- Rappoport, Peter, and Eugene N. White (1993) Was There a Bubble in the 1929 Stock Market? *Journal of Economic History* 53: 549–574.
- Rosser, J. Barkley, Jr. (1991) *From Catastrophe to Chaos: A General Theory of Economic Discontinuity*. Boston: Kluwer Academic Publishers.
- Rosser, J. Barkley, Jr., and Marina Vcherashnaya Rosser (Forthcoming) *Comparative Economics in a Transforming World Economy*. Burr Ridge: Richard D. Irwin.
- Shiller, Robert J. (1984) Stock Prices and Social Dynamics. *Brookings Papers on Economic Activity* 2: 457–498.
- Siddiqi, Muhammed Nejatullah (1980) Muslim Economic Thinking: A Survey of Contemporary Literature. In K. Ahmad (ed) *Studies in Islamic Economics*. Leicester: The Islamic Foundation.
- Tirole, Jean (1982) On the Possibility of Speculation under Rational Expectations. *Econometrica* 50: 1163–1181.

- Tirole, Jean (1985) Asset Bubbles and Overlapping Generations. *Econometrica* 53: 1499–1528.
- Uppal, Jamshed Y. (1993) The Internationalisation of the Pakistani Stock Market: An Empirical Investigation. *The Pakistan Development Review* 32:4 605–618.