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Pricing Irrigation Water in Pakistan: An Evaluation of Available Options

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Irrigation water shortages have lately been a main area of concern for policy-makers and planners in Pakistan. Current literature on the country's water resources predicts an alarming situation regarding the availability of irrigation water in the future due to declining water tables and serious financial, environmental, and social constraints of developing big storage reservoirs. Since there is little room to augment water supplies by building new dams, the existing supply-driven surface irrigation system needs to be replaced by a demand-based system with special focus on water use efficiency through the introduction of an appropriate water pricing system. The present study aims to evaluate several alternative water pricing systems in the search for choosing one that will ensure efficient use of irrigation water in Pakistan. A related objective is to test the extent of sensitivity of the demand for irrigation water to a change in alternative water prices. A major conclusion that emerges from this research is that irrigation water shortages are the result of the inflexibility of the present irrigation water supply system for agricultural use and have little to do with the existing water pricing practice in the country. Furthermore, the results of our water price simulations exercise confirm the general perception that demand for irrigation water is less sensitive to changes in alternative irrigation water prices. Two findings from the pricing policy perspective are: (i) irrigation water is not available in adequate quantity to farmers in the nine sub-districts surveyed at almost all of the alternative prices in Pakistan's irrigated agriculture sector since the predicted water usage at all prices is greater than the actual usage for all districts; and (ii) our empirical analysis indicates significant inefficiency of resource allocation in respect of irrigation water as shown by its positively large marginal value product to opportunity cost ratio.

1. INTRODUCTION

Agriculture is a major economic sector in Pakistan and 90 percent of its output comes from irrigated farms. Water is a critical input for agricultural productivity but its inadequate and untimely delivery limits the farmers' use of other inputs, thus resulting in considerably lower yields. Irrigation water shortages have lately been a main area of concern for farmers, planners, and policy-makers in Pakistan. Current

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literature on the country's water resources predicts an alarming situation regarding the availability of irrigation water in the future. Prospects for increasing water supplies are considered dim since the development of water resources is approaching its limits. Additional increases in water supply through the construction of new storage reservoirs are also not possible due to financial, environmental, and social constraints. This alarming situation has generated a serious debate among experts as to how to cope effectively with this potentially very serious scenario.

There is also a growing realisation on the part of the government, as well as the donors, that past investment in irrigation has not paid the expected returns mainly due to the sub-optimal and often wasteful utilisation of existing irrigation facilities. Farmers are faced with unreliable and inadequate water supply due to an inflexible and highly inefficient irrigation water delivery system. Surface irrigation is a public sector activity and is heavily subsidised since water rates are abysmally low, thus putting an enormous fiscal drain on the national exchequer. Budgetary constraints do not permit sufficient financial outlays for proper maintenance of the irrigation system. Inadequate attention given to the level and form of water charges, and to the need for an appropriate mechanism for pricing irrigation water, has been a critical policy lapse on the part of water sector planners in Pakistan. Water pricing is an important way of improving water allocation and encouraging users to conserve water resources. Critical issues related to the operation and maintenance costs, the rate of return on investment, and the provision of irrigation services on a sustained basis are all directly and indirectly linked to water pricing policy.

A review of water pricing literature reveals that a variety of methods for pricing water have been developed over time. These methods differ in their implementation, the institutions they require, and the information on which they are based [Tsur and Dinar (1997)]. A wide range of literature addresses irrigation water management in general and water pricing in particular [Rhodes and Sampath (1988); Cummings and Nercissiantz (1992); Le Moigne, *et al.* (1992); Sampath (1992); Small and Carruthers (1991); Shah (1993); Plasquellec, Burt, and Wolter (1994); Tsur and Dinar (1995)]. Several studies [Rhodes and Sampath (1988); Sampath (1992); and Dinar and Subramanian (1997)] focus on water pricing methods practised in various countries. These methods include volumetric, output, input, per unit area, tiered pricing, two-part tariffs, betterment levy and water markets. The best water price is a price that reflects opportunity costs or is marginal cost-based but it is hard to implement. Two-part tariff pricing ensures cost recovery and is a more realistic immediate objective from the point of view of financial viability of water projects [Dinar and Subramanian (1997)].

Dinar and Subramanian (1997) while reviewing and comparing water pricing experiences in 22 selected countries (Algeria, Australia, Botswana, Brazil, Canada, France, India, Israel, Italy, Madagascar, Namibia, New Zealand, Pakistan, Portugal, Spain, Sudan, Taiwan, Tanzania, Tunisia, Uganda, United Kingdom, and the United

States of America) find variations in water pricing methods used by different countries. The most common method used to charge for irrigation water is reported to have been the average cost-based. Marginal costs are more relevant but full marginal cost pricing has never been recommended anywhere in the water sector since the situation of increasing average cost frequently prevails in water development projects [Dinar and Subramanian (1997)]. Use of two-part tariff system of water pricing, even though a better option, is a rare phenomenon. Its different versions are in operation in several countries. In Australia and Brazil, a portion of capital costs is recovered from users [McGovern (1999); Musgrave (1997) and Todt de Azevedo (1997)]. France is the only country where water for irrigation is generally sold on the binomial tariff basis [Dinar, Rosegrant, and Meinzen-Dick (1997)]. The binomial system accounts for off-peak and on-peak costs. In the peak period, long-run marginal capital costs plus marginal operating costs are recovered while in the off-peak period only marginal operating costs are recovered.

The optimal volumetric pricing rule requires that the water price be set equal to the marginal cost of water supply. Different countries/regions use different versions of this method to charge for water. Irrigation water charges consist of a volumetric water charge to cover operation and maintenance costs, and a per hectare water charge to recover the public investment in off-farm irrigation infrastructure [Dinar and Subramanian (1997)]. California uses multi-rate volumetric pricing for publically supplied water according to which prices range between US$ 2 per acre foot to more than US$ 200 per acre foot [Tsur and Dinar (1997)]. Following this method, water rates vary as the amount of water consumed exceeds certain threshold values. In India, a volumetric rate per estimated volume of water consumed is used in areas with pumped irrigation and tubewells [Dinar and Subramanian (1997)]. These estimates are based on crop water requirements. In the Jordan Valley, where most of the agricultural activity is concentrated, water is provided through pipes to more than three quarters of the irrigated land [Tsur and Dinar (1997)]. Water authorities use volumetric pricing, but water is greatly underpriced, and the price does little to induce efficient use of water [Tsur and Dinar (1997)]. In Peru, the existing legislation defines two classes of water tariffs, one for agricultural use and the other for non-agricultural use. In general, tariffs do not reflect the true cost of water. For agriculture, the volumetric water tariff includes three components; (i) a "water users' association" component intended to raise funds to finance operations and maintenance, the conservation and improvement of common irrigation infrastructure, and the administration budget; (ii) a water levy calculated as 10 percent of the first component for financing agricultural development/special irrigation projects; and (iii) an amortisation component to recover the cost of public investments in irrigation storage infrastructure [Dinar and Subramanian (1997)]. Chile and Mexico are the only two countries that have developed water markets for selling and buying irrigation water [Easter, Rosegrant, and Dinar (1998)].

In Pakistan, where agriculture uses 90 percent of irrigation water, water rates charged to farmers have always been minimal. Because of the nature of the irrigation system and because of the administrative structure designed to supervise it, charges for irrigation water have been made on an acreage—not a volume—basis [Lewis (1969); Chaudhry, Majid, and Chaudhry (1993)]. These charges vary widely between crops. This pattern of charges encourages wasteful use of the country's most limited resources. The structure of water rates has long been subject to criticism. It has been alleged that charges for irrigation water discriminated between various crops in such a way as to distort resource allocation. Moreover, it has generally been argued that water was and is being provided by the public sector at appreciably less than its marginal cost [Chaudhry, Majid, and Chaudhry (1993)]. Some awareness of the different amount of water required by different crops has been introduced by applying differential rates per acre, but these differentials have not been fully compensated for the differences in water use [Lewis (1969); Haufbauer and Akhtar (1970)]. Thus the determination of an efficient pricing system for irrigation water has become a serious issue especially in the backdrop of declining water tables in the country.

The main objective of this paper is to evaluate several alternative water pricing systems and choose one that will ensure efficient use of irrigation water in Pakistan. A related objective is to test the extent of sensitivity of the demand for irrigation water to a change in alternative water prices. For this purpose, a single equation production function[1]—Cobb-Douglas (CD)—will be specified and estimated first and then the CD parameter estimates will be used to derive an input demand function for irrigation water. The derived water demand function will serve as a bridge between production function estimates and water demand policy simulations in analysing alternative water pricing systems.

The paper is organised in six sections. Section 1 is introductory and gives an outline of the study. Section 2 discusses data, variables, and the empirical model. The discussion of the empirical results is reported in Section 3. Section 4 reports policy simulations. In Section 5, sensitivity analysis of the policy simulations has been carried out. The last section, Section 6 summarises conclusions and policy implications.

2. DATA, VARIABLES, AND EMPIRICAL MODEL

2.1. Data and Variables

The data used in this study are from a 1998 survey of 601 farmers in Pakistan for the crop year 1997-98, conducted by the author for the purpose of her doctoral research [Sahibzada (2002)]. A four-stage sampling technique was used

[1]Single-equation approach has been criticised in the literature on the ground that its use in estimating a production function results in the simultaneity bias [Marschak and Andrews (1944); Walters (1963); and Nerlove (1965)], but alternative methods of estimation have been proposed by Zellner, Kmenta, and Dreze (1966) to avoid this bias effectively in the estimation of the Cobb-Douglas production function.

for the selection of a representative sample. As a first step, three provinces—the North West Frontier Province (NWFP), Punjab, and Sindh—were selected as major irrigation water users. Balochistan was not included because it does not have a noticeable share of irrigated agriculture *vis-à-vis* other provinces, at the moment. At the second stage, *tehsils* (sub-districts) were selected to represent average conditions in each province. The third stage involved village selection, and the final stage represented the selection of farmers. While the selection of provinces was self-evident, the procedures adopted at the remaining three stages were as follows. The survey was carried out in nine sub-districts selected from the four regions, which were themselves selected on the basis of the characteristic of growing a major crop. Lodhran (Punjab) for cotton; Thatta (Sindh) for rice; Charsaddah (NWFP) for mixed crops and Sugarcane; and Attock, Mianwali (Punjab) and Kulachi (NWFP) for non-irrigated agriculture. Wheat, the staple food crop, was noticed to be grown in almost all regions. The village selection was made using concentric circles drawn on *tehsil* maps. To accomplish this task the latest maps of the sampled sub-districts were collected from the office of the Survey of Pakistan and the respective District Councils. The selection of farmers constituted the last stage of sampling. The farmers were randomly selected from three sub-districts of Punjab, two of Sindh, and four of the NWFP. Thus the total sampled farmers in the selected 9 sub-districts aggregated to 601 respondents. A questionnaire was formulated to obtain information on crop production and price, size of farm, cropped area, irrigation water, labour, and use of tractor and fertiliser. It is important to point out that water usage was measured only as the number of irrigations in the crop year. The conventional practice assumes an average of three acre inches of water per one irrigation; more precise measurement is time-consuming and expensive, and was not performed in this survey. Again, the estimate of irrigation water used relied on the farmer's memory regarding water received during the crop year. The data base from this survey appears to be representative of the Indus Basin with respondents reporting production of a number of irrigated crops including wheat, rice, maize, cotton, sugarcane, tobacco, vegetables, and various fodder crops.

The analysis of this study relies on a single-equation production function. The dependent variable is total aggregated output (Y) in maunds (1 maund = 40 kgs), weighted by revenue shares. The surveyed farmers have provided information on total production of each crop at the individual farm level, and price per maund of each crop. An established procedure[2] has been used to

[2]Aggregate Output per Unit of Land = Sum(w_jQ_j)/Sum A_j for each farm,
where
Q_j = output in maunds of cropj (one maund equals 40 kgs);
P_j = Price of cropj in Pak rupees per 40 kgs (Pak Rs 46 = US\$1 in 1998);
$w_j = P_jQ_j/\Sigma(P_jQ_j)$ = weights based on revenue shares; and
A_j = Area under cultivation of cropj where the summation is across crops grown for each farm.

aggregate output. Data on various inputs (irrigation, fertiliser man-days, tractor hours) have been collected on a per acre basis, and cropped area at the farm level. Data on fertiliser from the surveyed farmers were basically collected by type (Urea, DAP, NP, NPK, etc.), in kilograms on per acre basis. These were later converted into fertiliser nutrients in kilograms following government's guidelines.[3] The fertiliser nutrients have been derived mainly from urea, diammonium phosphate, calcium ammonium, etc. Total fertiliser input (FERT) at the farm level has been obtained by multiplying the per cropped acre fertiliser nutrient in kilograms with total cropped area at the farm level. Data on labour have been collected in man-days generally but also in working hours from several farmer surveyed, by types of activities (Pre-sowing, Sowing, Hoeing, Irrigation, Harvesting, and Threshing) on a per cropped acre basis. One man-day is normally of eight hours. The data in working hours have been converted into man-days by dividing the total by the number 8. Total man-days input (MD) at the farm level has been obtained by multiplying total man-days with total cropped area at the farm level. Data on tractor hours have been collected on a per cropped acre basis. Total tractor input (TH) in operational hours at the farm level has been obtained by multiplying the per acre tractor input with total cropped area at the farm level. Data on irrigation water (IRR) have been collected from the surveyed farmers in number of irrigations per cropped acre. One irrigation equals on average 3 acre inches of water. Total irrigation input (IRR) in acre inches at the farm level has been obtained by first multiplying the number of irrigations per acre with 3, and then multiplying the data in inches with total cropped area at the farm level. Data on total cropped area (TCA) have basically been collected in acres at the farm level.

The dummy variables DFERT, DTRAC, and DIRRI, respectively for zero observations[4] in respect of fertiliser, tractor, and irrigation, are included in the model in order to correct for the presence of some zero observations for these three inputs (see the discussion of the Battese model later in Section 3.2). D1 to D7 are dummy variables for the seven sub-districts which are included in the equation in order to capture variations in soil quality and climatic conditions in different regions. DMULTI is a multiple crop dummy showing the impact of crop diversification, with DMULTI = 1 for farms growing more than one crop, and = 0 for single-crop farms.

Descriptive statistics for all variables used in model estimation are given in Table 1.

[3]*A Pocket Guide for Extension Workers* (Islamabad: National Fertiliser Development Centre, Planning and Development Division, Government of Pakistan, 1997).

[4]The number of zero observations for the three inputs are: Fertiliser (83); Irrigation Water (43); and Tractor Hours (10).

Table1

Descriptive Statistics for Variables Used in OLS Regression Analysis

Variable	Model 1 (n=509)				Model 2 (n=601)			
	Mean	S.Dev	Mini.	Maxi.	Mean	S.Dev	Mini.	Maxi.
Y (Mds)	3.00	6.36	0.04	86.67	2.52	6.08	0.04	86.67
FERT (Kgs)	7.98	15.12	0.18	273.46	7.16	14.22	0.06	273.46
MD (Mdays)	318.38	556.4	3.25	6400	309.09	528.81	1.94	6400
TH (Hrs)	0.25	0.21	0.01	2.08	0.26	0.22	0.01	2.08
IRR (Inches)	0.47	0.54	0.02	5.84	0.42	0.53	0.00	5.84
TCA (Acres)	13.85	22.86	0.50	300	15.05	23.59	0.5	300
DFERT	–	–	–	–	0.86	0.34	0	1
DTRAC	–	–	–	–	0.98	0.13	0	1
DIRRI	–	–	–	–	0.93	0.26	0	1
D1(Mian)	0.12	0.33	0	1	0.11	0.32	0	1
D2 (Kula)	0.002	0.04	0	1	0.13	0.33	0	1
D3 (That)	0.21	0.41	0	1	0.19	0.39	0	1
D4 (Mirp)	0.11	0.31	0	1	0.09	0.29	0	1
D5 (Pesh)	0.07	0.25	0	1	0.06	0.24	0	1
D6(Lodh)	0.25	0.44	0	1	0.22	0.42	0	1
D7 (Atto)	0.06	0.24	0	1	0.05	0.22	0	1
DMULTI	0.87	0.33	0	1	0.82	0.38	0	1

Y = Total aggregated output (weighted by revenue shares) at the farm level divided by total man-days at the farm level;
FERT = Total fertiliser nutrients (in kgs) at the farm level divided by total man-days at the farm level;
MD = Total man-days at the farm level;
TH = Total tractor hours at the farm level divided by total man-days at the farm level;
IRR = Total irrigation (in acre inches) at the farm level divided by total man-days at the farm level;
TCA = Total cropped area (acres) at the farm level;
DFERT = dummy variable for zero observations for fertiliser;
DTRAC = dummy variable for zero observations for tractor hours;
DIRRI = dummy variable for zero observations for irrigation water;
D1, ..., D7 = dummy variables for seven sub-districts (Mianwali, Kulachi, Thatta, Mirpurkhas, Peshawar, Lodhran and Attock respectively); and
DMULTI = dummy variable for multiple crops.

2.2. Empirical Model

The most widely used forms of production functions in the analysis of agriculture are the Cobb-Douglas (CD) and the Transcendental (Translog). In our study, we initially used a Translog production function, which is a flexible functional form and places no *a priori* restrictions on the production technology such as constant returns to scale, homogeniety, separability, and constant elasticity of substitution. This functional form is a second-order Taylor series approximation, and thus requires a larger number of parameters to be estimated. Consequently, multicollinearity is often a problem when estimating the single-equation translog production function. The present study was no exception. The results of the estimated translog model showed some of the production elasticities to be negative, thus resulting in several violations of regularity conditions. To avoid such problems, the present study relied on the Cobb-Douglas functional form, which is very popular in agricultural production studies because of its parsimony in parameters, ease of interpretation, and computational simplicity. Several studies of Pakistan's agricultural sector have used this form primarily because the resulting coefficients make it possible to interpret the elasticities of production with respect to inputs, and because the coefficients also indicate the relative importance of each input with respect to output [Chaudhry and Kemal (1974); Naqvi, *et al.* (1982, 1983, 1986); and Zuberi (1989)].

The Cobb-Douglas production function, which is considered here for the estimation of input elasticities of the surveyed farmers, is defined below for restricted (Model 1) and full (Model 2) data sets, the former excluding and the latter including zero observations for fertiliser, tractor hours, and irrigation water:

$$Ln\ Y = \beta_0 + \beta_1 ln\,(FERT) + \beta_2 ln(MD) + \beta_3 ln(TH) + \beta_4 ln(IRR) + \alpha_1 D_1 + \alpha_2 D_2 + ... + \alpha_7 D_7 + \gamma DMULT + \varepsilon \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$Ln\ Y = \beta_0 + \beta_1 DFERT\ ln(FERT) + \beta_2 ln(MD) + \beta_3 DTH\ ln(TH) + \beta_4 DIRR\ ln(IRR) + \beta_5 DFERT + \beta_6 DTH + \beta_7 DIRR + \alpha_1 D_1 + \alpha_2 D_2 + ... + \alpha_7 D_7 + \gamma DMULT + \varepsilon \quad \ldots \quad \ldots \quad (2)$$

where

Ln = represents natural logarithm; β_k (k = 1, 2,...., 7), α_t (t = 1, 2, ..., 7), and γ are the unknown parameters to be estimated, and ε is the usual random error term, which is assumed to be normally distributed with zero mean and constant variance $N(0, \sigma^2)$.

Y = represents total aggregated output (weighted by revenue shares) at the farm level, divided by total man-days at the farm level;

FERT = total amount of fertiliser nutrients (in kilograms) used at the farm level, divided by total man-days at the farm level;

MD = total amount of labour (in man-days) used at the farm level;

TH = total number of tractor hours used at the farm level, divided by total man-days at the farm level;

IRR = total quantity of irrigation water (in inches) used at the farm level, divided by total man-days at the farm level;

DFERT = dummy variable which has value of one if fertiliser usage was positive, and zero for zero;

DTRAC = dummy variable which has value of one if tractor usage was positive, and zero for zero;

DIRR = dummy variable which has value of one if irrigation usage was positive, and zero for zero;

D_1 = district dummy variable which has value of 1 if Mianwali sub-district, and 0 otherwise;

D_2 = district dummy variable which has value of 1 if Kulachi sub-district, and 0 otherwise;

D_3 = district dummy variable which has value of 1 if Thatta sub-district, and 0 otherwise;

D_4 = district dummy variable which has value of 1 if Mirpurkhas sub-district, and 0 otherwise;

D_5 = district dummy variable which has value of 1 if Peshawar sub-district, and 0 otherwise;

D_6 = district dummy variable which has value of 1 if Lodhran sub-district, and 0 otherwise;

D_7 = district dummy variable which has value of 1 if Attock sub-district, and 0 otherwise;

$DMULT$ = crop dummy variable which has value of 1 if more than 1 crop and 0 for only one crop.

It is a 4-input "per man-day model"[5] with its two specifications (Models 1 and 2) and CRS-imposed. Model 1 is based on the restricted data set in that zero observations for the three inputs, fertiliser, tractor hours and irrigation water are excluded from model estimation, while Model 2 uses the full data set.

Out of the original nine sub-districts, two, i.e., Charsadda and Mardan, were used as the reference districts. Since the observations from Mardan were very few (10 only), and the climatic conditions and land quality of both the districts were almost the same, both were merged and considered as the single reference district called MarCh.

Since we also have reported zero values for irrigation water, fertiliser, and tractor hours for some farms in the survey, in order to correct for the presence of these zero observations, dummy variables for zero observations in respect of the three inputs have been used in the estimation of Model 2. This has been done following Battese, Malik, and Gill (1996) and Battese (1997). The objective of using this approach is to accommodate the users and non-users of these three inputs and still obtain efficient and unbiased estimates using the full data set.

[5]Several alternative specifications of the 5-input model for both CRS-imposed and CRS-not-imposed have been estimated. The estimated elasticities of land (the fifth RHS variable) for all attempted specifications have been negative and insignificant. The implied zero elasticity of land conforms to recent empirical evidence [Ali and Byerlee (2000) and Ahmad (2001)], which indicates strongly towards the prevalence of a land degradation phenomenon in Pakistan. Given the intractability of a zero input coefficient in the CD model, the land input variable has been dropped from the production function for further model estimation. After dropping the land from the production function, a 4-input "totals" model with several specifications have been estimated. Two best specifications ("per man-day" Models 1 and 2) have been selected for further analysis. The selection procedure is based on both statistical testing and economic analysis [Sahibzada (2002)].

The Battese approach given in his 1996 and 1997 papers consists of two proposed models which look different but in fact yield the same results—a shift in the intercept for zero-valued observations. We have adopted the approach used in the Battese, *et al.* (1996) in the current study. Following the latter approach, a dummy variable (for instance DIRR) is defined for each input, which has some zero observations in the sample, as taking on a value of unity when the input has a positive value, and a value of zero when the input has a zero value. The model is then specified with two related terms—the dummy variable by itself and a second term involving the dummy variable multiplied by the natural log of the input; DIRR and DIRR ln(IRR), for instance, where IRR is the amount of irrigation water used. Thus we get two estimated coefficients for each of the two terms when IRR is positive (i.e., get an intercept shift and an estimated coefficient for ln IRR) whereas both terms fall out (i.e., have zero values) when IRR = 0.[6] Finally, since many of the surveyed farmers grow more than one crop in a season, we have used a dummy variable (DMULTI) for measuring the impact of multiple crops.

Since all parameters in the Cobb-Douglas function are elasticities of production, the value of the marginal physical product (MPP) for a specific input is given by:

$$MPP_k = \delta Y_i / \delta X_{ki} = b_k (Y_i / X_{ki}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where

Y_i represents the ith farmer's output, X_{ki} represents the level of inputs of the kth resource at the ith farm, and b_k is the regression coefficient of the kth input in a Cobb-Douglas model. Following the customary practice, a point estimate of marginal physical product (MPP) can be obtained by evaluating Equation (13) at the mean value of each input. The marginal value product (MVP) of each input at the farm level is then computed by multiplying the MPP of each input at the farm level by the aggregate output price.[7]

2.3. Input Demand Function for Irrigation Water and Policy Simulations

As the main objective of the present study is to evaluate alternative water pricing systems and to choose a price that ensures efficient use of irrigation water, a detailed simulation exercise will be carried out to find such a price. An efficient price will be one which gives more efficient predicted water usage as compared to the present actual water usage at the district level. Data on the present actual water usage

[6]Since one uses natural logs to estimate the CD production function, this requires that either ln IRR is set equal to zero or IRR is put equal to one (same thing) when DIRR is zero.

[7]Aggregate output price is calculated as follows:

(i) prices of various crops are weighted by revenue shares;

(ii) weighted crop prices are aggregated; and

(iii) aggregate output price at the mean level is used for calculating MVP of the four inputs.

has been collected from the surveyed farmers and aggregated at the sub-district level. For conducting the simulation exercise, an input demand function for irrigation water, using the Cobb-Douglas (CD) parameter estimates from Models 1 and 2, will be derived and combined with various alternative prices for irrigation water to predict water requirements at the district level.

The CD demand for irrigation water has been derived through constrained cost minimisation, using a CD production function [Varian (1992)]. In this problem, the choice variables are the inputs (the *Xs*) while the input prices (the *Ws*) and output (*Y*) are parametric variables which are assumed to be exogenous or given to the firm. The derived irrigation water demand function is then used as a bridge between production function estimation and water demand policy simulations in analysing alternative water pricing systems. A detailed derivation of the input demand function for irrigation water is given in the Technical Appendix, but the basic equation for irrigation water demand is given as follows.

$$\hat{X}_4 = \frac{A_o Y^{\alpha y} w_1^{\alpha 1} w_2^{\alpha 2} w_3^{\alpha 3}}{w_4^{\alpha 4}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where,

$$A^0 = \frac{b_4^{\frac{(b1+b2+b3)}{\Sigma bi}}}{b0^{\frac{1}{\Sigma bi}} b1^{\frac{b1}{\Sigma bi}} b2^{\frac{b2}{\Sigma bi}} b3^{\frac{b3}{\Sigma bi}}};$$

$$\alpha_y = \frac{1}{\Sigma b_i};\ \alpha_1 = \frac{b_1}{\Sigma b_i};\ \alpha_2 = \frac{b_2}{\Sigma b_i};\ \alpha_3 = \frac{b_3}{\Sigma b_i};\ \alpha_4 = \frac{b_1 + b_2 + b_3}{\Sigma b_i}.$$

$\hat{X}_4$ = Cost minimising demand for irrigation water;
Y = Aggregate farm output at the mean level;
w_1 = Price of fertiliser nutrient per kilogram;
w_2 = Price of labour per man-day of 8 hours;
w_3 = Price of tractor use per hour;
w_4 = Price of irrigation water per acre inch;
b_0 = Estimated coefficient of the constant;
b_1 = Estimated coefficient of fertiliser;
b_2 = Estimated coefficient of labour;
b_3 = Estimated coefficient of tractor; and
b_4 = Estimated coefficient of irrigation water.

2.4. Alternative Water Prices

The five water pricing systems which have been used in the basic simulations exercise are MVP-based, Market-based, Average Cost-based, short-run Marginal

Cost-based, and long-run Marginal Cost-based. The MVP-based price comes out from the analysis of the survey data. The various cost-based prices have been derived in two ways. First, secondary data on provincial expenditures on irrigation water supply delivery have been used to calculate average variable and average marginal cost per acre foot of water [Sahibzada (2002)]; Secondly, estimates of long-run marginal costs, based on the review of feasibility reports of small, medium, and large dams, and the expert opinion of irrigation engineers have been used as an alternative price for irrigation water in the simulation exercise [Sahibzada (2002)]. The summarised discussion of alternative irrigation water prices used in the basic simulations is presented in Table 2.

Table 2

Assumed Irrigation Water Prices Used in Basic Simulations

(1) MVP-based	The MVP-based price is the marginal value product of irrigation water at the mean level which has been calculated by multiplying the marginal physical product of irrigation water with aggregate output price. The MVP-based prices will vary depending on whether we use restricted or full version of Model 4. Using Model 4.11, the MVP-based price comes to Rs 445.23 per acre inch while based on Model 4.22, it is Rs 415.79 per acre inch (Table 6.7).
(2) AC-based	The AC-based price is the average variable cost of water supply delivery and has been calculated using O&M expenditures on the irrigation system [reported in Sahibzada (2002) Table 4.4 in Chapter Four]. This price comes to Rs 7.8 per acre inch.
(3) SRMC-based-using Deflated Prices	Price based on short-run marginal cost using prices net of inflation. GDP deflator [reported in Sahibzada (2002) Appendix to Chapter Four] has been used to convert the intermediate-run estimate of MC in current prices into the same in real prices. This estimate for our study has been calculated to be Rs 5.7 per acre inch.
(4) LRMC-based-using Deflated Prices	Price based on long-run marginal cost which is the intermediate-run MC plus 1 percent of construction costs on developing irrigation water system in current prices comes to Rs 169 per irrigation [Sahibzada (2002) Chapter Four]. This price has been converted into real price through the use of GDP deflator, resulting in Rs 17.8 per acre inch.
(5) Market Price	It is the price farmers charge for selling (informally) surplus irrigation water in excess of their requirements to friends and relatives. This informal market price is Rs 600 per irrigation.

Source: Sahibzada (2002).

3. RESULTS AND ANALYSIS

Equations 1 (Model 1) and 2 (Model 2) have been estimated using the computer software package EViews 3.1. Equation 1 is a 4-input "per man-day" model in which both the LHS and the RHS variables have been divided by man-days before taking their logs for model estimation, and zero observations of the three inputs, fertiliser, tractor hours, and fertiliser excluded. Thus, this model assumes constant returns to scale. Since graphical analysis of the models' residuals and White's test [White (1980)] have pointed towards the presence of heteroskedasticity, which is a normal phenomenon in the analysis of cross-sectional data, White's estimation procedure has been used to correct for heteroskedasticity. The regression results of the two specifications of this 4-input "totals" model are reported in Table 3.

Table 3

Regression Results: Cobb-Douglas Production Function, Using OLS Techniques

	Model 1(n=509)		Model 2 (n=601)	
	Est. Coeffs.	*t*-value	Est. Coeffs.	*t*-value
Constant	1.259	4.06[a]	0.706	1.33
LFERT	0.308	3.25[a]	0.265	2.97[a]
LMD	0.172[*]	1.76[c]	0.134[*]	1.43
LTH	0.019	0.22	0.117	1.45
LIRR	0.501	4.81[a]	0.484	4.96[a]
DFERT	–	–	–0.158	–0.33
DTRAC	–	–	–0.131	–0.84
DIRRI	–	–	1.113	4.33[a]
D1 (Mian)	–0.321	1.84[c]	–0.408	–2.38[a]
D2 (Kula)	–1.696	–6.93[a]	–0.962	–1.89[c]
D3 (That)	–1.235	–6.59[a]	–1.296	–7.14[a]
D4 (Mirp)	0.169	0.80	0.140	0.65
D5 (Pesh)	0.728	3.72[a]	0.727	3.84[a]
D6 (Lodh)	–1.64	–10.2[a]	–1.734	–11.3[a]
D7 (Atto)	0.553	1.94[c]	0.462	1.65[c]
DMULTI	–0.322	–2.05[b]	–0.347	–2.92[a]
R^2	0.399	–	0.432	–
R^2-adj	0.387	–	0.418	–

[a] Significant at 1 percent confidence level.
[b] Significant at 5 percent confidence level.
[c] Significant at 10 percent confidence level.
[*] These results were computed from estimated results, and represent the elasticity of output with respect to labour.
n = Number of observations.

The regression results for the two specifications of the "per man-day" model, with CRS-imposed, are discussed as follows. The values of R-squared are 0.399 and 0.432 respectively for Models 1 and 2, indicating that 40 to 43 percent of the variations in total aggregated output per man-day is explained by the variables included in the models. Most of the parameter estimates in Models 1 and 2 are statistically significant at least at the 90 percent confidence level and most have the expected signs. The sum of the parameter estimates of the four traditional inputs (labour, fertiliser, water, and tractor) is equal to unity in both the restricted as well as the full data sets, implying that a one-percent increase in the four model inputs results in a one-percent increase in aggregated output per man-day, since constant-returns-to-scale has been imposed. The estimated coefficients of irrigation (LIRR) are large and highly significant in both Models 1 and 2, i.e., 0.501 and 0.484, which means that a one-percent increase in irrigation water input increases aggregate output per man-day by 0.5 and 0.48 percent respectively in the former and latter cases. The estimated coefficient for fertiliser (LFERT) follows the irrigation coefficient in size and significance. It is 0.308 in Model 1 and 0.265 in Model 2, which says that a one-percent increase in fertiliser input increases aggregate output per man-day by 0.31 percent and 0.26 percent respectively in the former and latter cases, with both coefficients again being highly significant. Labour (LMD) is third in line in the input coefficient size, ranking with computed coefficient estimates of 0.172 and 0.134, implying that a one-percent increase in the labour input will bring about 0.17 and 0.13 increase in aggregated output per man-day respectively in Model 1 and Model 2, Tractor hours (LTH) have a comparatively smaller estimated coefficients—0.117 and 0.019—in the two specifications, and both are statistically insignificant.

Results regarding the district-specific dummy variables in the two specifications show Thatta and Lodhran to be significantly less productive and Peshawar to be more productive than the reference districts of Mardan and Charsadda in both the restricted as well as the full data sets. Kulachi and Mianwali are significantly less productive than the reference districts, the former in Model 1, the latter in Model 2.

The multi-crop dummy variable coefficient is negative, significant at 95 percent confidence level in the restricted data set and significant at 99 percent confidence level in the full data. The negative sign means that multiple crop farmers will have slightly less aggregated output for given amounts of all four inputs as compared to single crop farmers. This negative difference reflects the opportunity cost of hedging against crop risk by planting multiple crops—the related benefit is of course the reduced risk of monocrop failure through a more diversified "crop portfolio".

Regression results of Model 2 include zero observations of fertiliser, tractor, and irrigation. This requires that a procedure—the Battese model—be used in order to combine both positive and zero input observations in the model estimation.[8] Following Battese, *et al.* (1996), dummy variables for zero observations of the three inputs are defined to equal 1 for positive or non-zero observations, and equal to zero

[8]Battese, Malik, and Gill (1996): discussion of the model is given in Section 3.2.

for zero observations. Based on this definition, the signs of the estimated coefficients for the three dummy variables (DFERT, DTRAC, and DIRRI) are expected to be positive. The positive sign implies a positive relationship between the intercept of the production function of the users and the use of the inputs. When an input, for instance, irrigation water is used, the intercept moves upwards, and vice versa. In Model 2, it appears that the estimated coefficients for DFERT and DTRAC are negative. The negative sign of the estimated coefficient in the case of fertiliser may imply that owners of fertile land may not use fertiliser because the use may result in lodging of the crops, since excessive use of fertiliser damages the crops. Such cases are rare but can be found in actual life. The negative sign of the estimated coefficient for DTRAC seems simple to explain here. Since there are only 10 zero observations out of 601 for tractor, variations in the dependent variable due to them will be very negligible, even non-existent. Moreover, since the estimated coefficients for both DFERT and DTRAC are not significantly different from zero, the negativity problem in their context becomes meaningless.

The coefficient estimate for DIRRI is positive and highly significant. DIRRI's positive and significant estimated coefficient means that the intercept of the production function for irrigated farms is higher than that of the unirrigated farms, implying that there is a positive relationship between the shift in the intercept and the use of irrigation water. When a farmer uses irrigation, the intercept of his production function moves upwards implying an increase in productivity; when he stops using irrigation, the intercept moves downwards, causing a decline in productivity.

Using the regression results of Models 1 and 2, the marginal physical product (MPP) of irrigation water is calculated using Equation 3. Table 4 reports MPP and MVP of irrigation water. Marginal Value Product (MVP) of irrigation water has been calculated by multiplying its MPP with the aggregated output price.[9] The aggregated output price has been calculated to be Rs 358.48 per maund (40 kgs) for the restricted data set and Rs 350.59 per maund for the full data set. The MPP of irrigation water varies between 49.69 kgs (1.242 maunds) per acre inch for the restricted data set and 47.55 kgs (1.186 maunds) per acre inch under the full data set. The MVP of irrigation water per acre inch comes to Rs 445.23 and Rs 415.79, respectively, under restricted and full data sets. These MVP estimates will be used as one of the several alternative water prices in the simulation exercise. Table 4 also reports the MVP to opportunity cost (OC) ratio, which is a measure of use efficiency. Market price of irrigation water—Rs 200 per acre inch—has been used as an approximation to the OC of irrigation water. An MVP/OC ratio equal to one indicates efficient use of a resource and a ratio greater/less than one indicates its under- and over-usage respectively.

[9]See footnote 7.

Table 4

MPP, MVP, and MVP/OC Ratio of Irrigation Water

	MPP (Per Acre Inch)		MVP (Pak Rs) (Per Acre Inch)	MVP/OC
	Kgs[a]	Mds[b]	Mds	(Pak Rs)
Model 1 (n=509)	49.69	1.242	445.23 (59.28)	2.23 (0.2964)
Model 2 (n=601)	47.45	1.186	415.79 (54.24)	2.08 (0.2713)

Figures in the parentheses indicate estimated standard errors for the MVP and MVP/OC ratio of irrigation water.
[a]Kilograms.
[b]Mds = Maunds (One maund = 40 kilograms).

Since the calculation of input demand for irrigation water using CD parameter estimates requires the use of input prices, and in our survey no information on the prices of inputs was collected, hence average prices charged for the services of these inputs have been used. These average prices are quite consistent with those reported in Pakistan (1998) which vary between Rs 100–150 per tractor ploughing (one ploughing is completed in one tractor hour), Rs 202–210 per hour of tubewell water (in one hour one acre inch of water is delivered), and Rs 70–80 per man-day of hired labour. The price of fertiliser per nutrient kilogram is about Rs 15.00 [Pakistan (2001)]. Farm labour is in man-days. One man-day is assumed to be eight hours of work and the mean wage rate charged these days for agricultural labour is Rs 80.00 per man-day [Pakistan (1998)]. This is a minimum norm although there is some variation in the wage rate from place to place. The open market price for one operational hour of tractor on average is Rs 120.00 [Pakistan (1998)], even though variations do exist in the rates across geographical divisions. As for irrigation water, it is not a common practice to sell canal water, but its trading does take place in the Punjab region of Pakistan. Tubewell water is frequently sold or exchanged in Punjab since the market for tubewell water in Punjab is more developed *vis-à-vis* other provinces. Variations in irrigation water prices exist due to variations in soil, topography, season, the nature of crops, the quality of water, and the availability of alternative sources of water for irrigation. The price paid by farmers for tubewell water in Punjab currently varies between Rs 100 and Rs 150 per hour, and for canal water it varies between Rs 200 and Rs 250 per hour. In one hour, about one acre inch of irrigation water is used by farmers. It is generally believed that in the absence of formal water markets, Rs 200 per acre inch for canal water is a good approximation of the opportunity cost of irrigation water.

4. POLICY SIMULATIONS

The derived demand function for irrigation water (Equation 4) has been computed using coefficient estimates of Models 1 and 2 combined with prices of the four inputs and aggregated output in order to calculate the predicted water usage at the sub-district level. The average prices for the three inputs—fertiliser, tractor hours, and man-days—and aggregate output used in the CD production function estimation have been discussed in Section 3.1. The summarised discussion of alternative prices for irrigation water used in the basic simulations is given in Table 2.

These simulations are called simulations at the base-line prices and are reported in Table 6. These will be used as reference simulations in the sensitivity analysis exercise.

Table 5 reports information on the various variables aggregated at the district level for district level analysis. These variables include the number of observations, farm output, the number of irrigations, total cropped area (TCA), and total farm area (TFA), all aggregated at the district level since simulations are carried out at the district level. Aggregate output per irrigation (Output/Irri) and aggregate irrigations per acre (Irrig/acre) are also included in Table 5.

Table 5

Variables Aggregated at the District Level

Variable	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1									
Obs.[a]	31	1	129	64	56	35	109	84	509
Agg. Farm Output[b]	8316	240	33642	29355	73570	23382	91364	39446	299278
Output/Irri	16.1	2.7	1.2	7.4	18	11.5	5	8.4	4.9
Agg. IRR[c]	517	90	26863	3938	4090	2026	18342	4669	60535
Irrig/Acre	3.4	3	7.2	5.7	7.8	7.7	15	11.3	8.6
TCA[d]	151	30	3757	686	524	263	1222	413	7046
TFA[e]	481	30	5847	1988	1354	558	3160	1076	14495
TCA as a % of TFA	31.1	100	64.2	34.5	38.7	47.1	38.7	38.4	48.6
(B) Model 2									
Obs.[a]	31	77	133	69	56	36	113	86	601
Agg. Farm Output[b]	8316	9530	34017	29792	73570	23442	92130	39994	310791
Output/Irri	16.1	8.7	1.2	7.4	18	11.5	4.9	8.5	4.9
Agg. IRR[c]	517	1091	27196	4003	4090	2030	18762	4718	62407
Irrig/Acre	3.4	0.6	7.1	5.6	7.8	7.7	15	11.3	6.9
TCA[d]	151	1920	3813	710	524	264	1243	417	9044
TFA[e]	481	5506	5952	2028	1354	559	3184	1086	20150
TCA as a % of TFA	31.8	34.9	64.1	35	38.7	47.2	39	38.4	44.9

[a]Number of observations at the district level;

[b]Farm output in maunds (1 maund = 40kgs), aggregated at the district level;

[c]Irrigation water in number of irrigations aggregated at the district level; one irrigation equals three acre inches of water; and

[d, e] Total cropped area and size of the farm, both in acres, and both aggregated at the district level.

Table 6

Predicted Water Usage (Number of Irrigations[a]) based on Alternative Water Pricing Systems

Alternative Prices	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1(n = 509)									
Current Actual Water Usage[b]	517 (3.4)	90 (3)	26863 (7.2)	3938 (5.7)	4090 (7.8)	2026 (7.7)	18342 (15.0)	4669 (8.6)	60535
MVP-based: Rs 445.23 Per Acre Inch	616	132	23647	5404	8422	1506	41366	5126	86219
AC-based: Rs 7.8 Per Acre Inch	4635	992	177934	40663	63374	11336	311266	38570	648770
SRMC-based: Rs 5.7 Per Acre Inch	5420	1160	208081	47552	74112	13257	364004	24105	758691
LRMC-based: Rs 17.8 Per Acre Inch	3071	657	117884	26940	41986	7510	206218	25553	429819
Market-based: Rs 200 Per Ace Inch	918	196	35253	8056	12556	2246	61670	7642	128537
(B) Model 2 (n = 601)									
Current Actual Water Usage	517 (3.4)	1091 (0.6)	27196 (7.1)	4003 (5.6)	4090 (7.8)	2030 (7.7)	18762 (15.0)	4718 (11.3)	62407
MVP-based: Rs 415.79 Per Acre Inch	670	4616	26103	5877	8655	1489	27540	11797	86747
AC-based: Rs 7.8 Per Acre Inch	5216	35916	203101	45726	67345	11587	214283	91791	674965
SRMC-based: Rs 5.7 Per Acre Inch	6133	42225	238782	53759	79177	13623	251929	107918	793546
LRMC-based: Rs 17.8 Per Acre Inch	3408	23463	132683	29872	43996	7570	139988	59966	440946
Market-based: Rs 200 Per Acre Inch	978	6734	38080	8573	12627	2172	40177	17210	126551

Note: Figures in parentheses are the number of irrigations per acre at the district level.
[a]One irrigation equals three acre inches of water; and
[b]Number of irrigations at the district level based on survey data;

For the computation of the predicted water usage at the sub-district level, zero values for the dummy variables of other districts are assumed, while that for the district for which water usage is simulated is set equal to one. Other dummy variables, such as the dummy variable for multiple crops (DMULTI) in Models 1 and 2, and dummy variables for zero observations for fertiliser (DFERT), tractor hours (DTRAC), and irrigation water (DIRRI) in Model 2 take on values equal to the sub-district sample mean, i.e., averaged across all sample farms in the sub-district.

Using the irrigation water demand function and the production function coefficient estimates from Models 1 and 2, water usage in terms of number of irrigations at the sub-district level is predicted. As mentioned in an earlier section, the results of the base-line simulations are reported in Table 6, which shows actual and predicted water usage (in the number of irrigations) under alternative water pricing systems at the sub-district level.

The first row in the table presents actual water usage in the number of total irrigations aggregated at the sub-district level. Data on actual water usage obtained from surveyed farmers were basically in the number of irrigations per acre for individual farms. These have been converted into total irrigations by multiplying by total cropped area at the farm level for the estimation of several alternative regression models. Figures in the parentheses in Table 6 are the number of irrigations per acre which have been calculated by dividing the aggregated irrigations at the sub-district level by the total cropped area, also aggregated at the sub-district level (Table 5).

Looking at the base-line simulations in Table 6, two important findings from the water pricing policy perspective are noted:

(i) Irrigation water is not available in adequate quantity to farmers in almost all districts at all the alternative prices in Pakistan's irrigated agriculture sector, as the predicted water usage at all prices is greater than the actual usage for all districts. The last column under Total in the same table shows this fact distinctly since the total actual usage for all sub-districts is 60535 irrigations as compared to the predicted usage of 86219 irrigations even at the MVP-based price, which is the highest among the five alternate prices. In percentage terms, total actual current water usage is 70-72 percent, 9 percent, 8 percent, 14 percent, and 47-49 percent of the predicted water usage respectively at the MVP-based, AC-based, SRMC-based, LRMC-based, and market-based prices. In other words, the inadequacy of the current water usage can be seen from the fact that if irrigation water is charged according to, say, the market-based price, even then water requirements of the farmers in all sub-districts will be much more than the current actual usage. This finding points towards the overall general scarcity of irrigation water available to farmers.

(ii) Discussing the water requirements of the individual districts and defining water use efficiency in terms of the highest agricultural produce per unit of irrigation water, the table reports that Mirpur Khas stands out as the most efficient user of irrigation water, with 7.8 irrigations per acre and producing the maximum aggregated farm output; its aggregate farm output per irrigation is the highest—18 maunds per irrigation—followed only by Attock and Peshawar, with 16 and 11.5 maunds per irrigation (Table 5). At the same time,

> Mirpur Khas again is the only district whose predicted water usage at the market-based price for instance is 207 percent more than its current actual usage. In the case of Attock, Kulachi, Lodhran, Mianwali, Peshawar and MarCh, this percentage increase in the predicted water usage *vis-à-vis* the actual usage is 78 percent for Attock, 118 percent for Kulachi, 31 percent for Lodhran, 104 percent for Mianwali, 11 percent for Peshawar, and 64 percent for MarCh. Peshawar turns out to need much less water at the market-based price specifically than the two districts (Attock and Mirpur Khas) with which it competes on the basis of productivity per irrigation. Its predicted water usage at this price registers an increase of about 11 percent as compared to 78 percent for Attock and 207 percent for Mirpur Khas. The case of Thatta is unique. With low productivity per irrigation (5 maund) and using the highest number of irrigations (15) per acre, its predicted water usage at the market-based price is the highest—236 percent more than its current usage. This means that Thatta could be considered as a classic case of an inefficient user of scarce water resource.

In the full data set, the nature of the change in predicted water usage is the same for all eight districts but the magnitude of the percentage change in the predicted water usage for two districts has substantially increased. In the case of MarCh the percentage increases from 64 percent to 265 percent, and for Kulachi it has increased from 118 percent to 517 percent. It may be mentioned here that almost all of the zero observations for irrigations are found in Kulachi as it has mainly rain-fed agriculture, and farmers in Kulachi get very little irrigation water.

Tables 7 and 8 report the differences in the predicted water usage from the base-line simulations reported in Table 6, both in absolute terms and in percentage terms, as a result of a 10 percent increase and a 10 percent decrease respectively in alternative water prices at the sub-district level. Table 7 shows the outcome of an assumed 10 percent increase in the alternative water prices on predicted water usage. A look at the figures in the parentheses in both the restricted as well as in the full data sets reveals that when water price is increased by 10 percent, the predicted water usage in all sub-districts decreases by less than 10 percent, i.e., demand for water decreases by only 5 percent, which implies a price-inelastic demand for water usage. Table 8 presents the results of an assumed 10 percent decrease in alternative prices on predicted water usage at the sub-district level. It shows almost the same magnitude of price elasticity of demand in the restricted data set, but in the full data set the degree of elasticity is slightly more than that of the restricted set. In the case of 10 percent decrease in water prices, again the price elasticity of demand for water is less than unity since the predicted water usage increases by about 5 percent in Model 1 and 6 percent in Model 2 in response to a 10 percent decrease in alternative water prices.

Table 7

Predicted Water Usage Assuming 10 Percent Increase in Alternative Water Prices

Alternative Prices	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1 (n = 509)									
Current Actual Water Usage[b]	517	90	26863	3938	4090	2026	18342	4669	60535
MVP-based:	587	126	22547	5153	8031	1436	39443	4887	82210
Rs 489.8/ Acre Inch	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)
AC-based:	4415	945	169471	38729	60360	10797	296464	36736	617917
Rs 8.6/ Acre Inch	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)
SRMC-based:	5156	1103	197942	45236	70501	12611	346271	42907	721727
Rs 6.3/ Acre Inch	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)
LRMC-based:	2927	626	112352	25675	40016	7158	196540	24354	409646
Rs 19.6/ Acre Inch	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)	(–4.7)
Market-based:	876	328	33616	7682	11973	2142	58806	7286	122568
Rs 220/ Acre Inch	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)	(–4.6)
(B) Model 2 (n = 601)									
Current Actual Water Usage[b]	517	1091	27196	4003	4090	2030	18762	4718	62407
MVP-based:	638	4394	24850	5594	8240	1418	26218	11231	82583
Rs 457.4/ Acre Inch	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)
AC-based:	4960	34151	193122	43479	64036	11018	203755	87284	641805
Rs 8.6/ Acre Inch	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)	(–4.9)
SRMC-based:	5824	40100	226764	51053	75192	12937	23948	102488	753606
Rs 6.3/ Acre Inch	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)
LRMC-based:	3242	22325	126249	28423	41862	7203	133200	57060	419564
Rs 19.6/Acre Inch	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)
Market-based:	931	6411	36253	8162	12021	2068	38249	16385	120480
Rs 220/ Acre Inch	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)	(–4.8)

Note: Figures in parentheses indicate percent change in the predicted water usage as a result of 10 percent increase in alternative water prices.

[a]One irrigation equals three acre inches of water.

[b]Number of irrigations at the district level based on survey data.

Table 8

Predicted Water Usage Assuming 10 Percent Decrease in Alternative Water Prices

Alternate Prices	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1 (n = 509)									
Current Actual Water Usage[b]	517	90	26863	3938	4090	2026	18342	4669	60535
MVP-based:	649	139	24923	5696	8877	1588	43599	5402	9087
Rs 400.7/ Acre Inch	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)
AC-based:	4885	1045	187539	42858	66795	11948	328069	40652	683791
Rs 7.02/ Acre Inch	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)
SRMC-based:	5730	1226	219957	50266	78341	14013	384777	47679	801989
Rs 5.1/ Acre Inch	(5.7)	(5.7)	(5.7)	(5.7)	(5.7)	(5.7)	(5.7)	(5.7)	(5.7)
LRMC-based:	3237	693	124248	28394	44253	7916	217350	26932	453023
Rs 16.02/ Acre Inch	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)
Market-based:	968	207	37156	8491	13234	2367	64999	8054	135476
Rs 180/ Acre Inch	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)	(5.4)
(B) Model 2 (n = 601)									
Current Actual Water Usage[b]	517	1091	27196	4003	4090	2030	18762	4718	62407
MVP-based:	708	4874	27562	6205	9139	1572	29080	12457	91597
Rs 374.2/ Acre Inch	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)
AC-based:	5508	37922	214449	48280	71108	12234	226255	96922	712678
Rs 7.02 / Acre Inch	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)
SRMC-based:	6495	44720	252887	56934	83854	14427	266810	114295	840422
Rs 5.1/ Acre Inch	(5.9)	(5.9)	(5.9)	(5.9)	(5.9)	(5.9)	(5.9)	(5.9)	(5.9)
LRMC-based:	3598	24774	140097	31541	46454	7993	147810	63318	465585
Rs 16.02 / Acre Inch	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)
Market-based:	1033	7110	40208	9052	13332	2294	42422	18172	133623
Rs 180/ Acre Inch	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)	(5.6)

Note: Figures in parentheses indicate percent change in the predicted water usage as a result of a 10 percent decrease in alternate water prices.

[a]One irrigation equals three acre inches of water.

[b]Number of irrigations at the district level based on survey data.

5. SENSITIVITY ANALYSIS

Sensitivity analysis is a method of exploring the effects of using alternative values of the estimated parameters of a model/project in order to determine as to which parameters the project is most sensitive. This is usually done by varying each parameter one at a time, keeping the other parameters constant and calculating the consequent effect on the baseline scenario. In the present study, two types of economic analysis have been carried out. The first type is changing alternative irrigation water prices by 10 percent upwards and downwards, holding other variables constant, and the second type is changing alternative parameter estimates, especially the input elasticity for irrigation water, by 10 percent, keeping the base-line water prices constant. The first type, which is called price policy simulations exercise, has already been discussed as a part of the simulation exercise in Tables 7 and 8. The second type, called sensitivity analysis, is undertaken by using a 10 percent increase/decrease in the elasticity for irrigation water. Since CRS is imposed on both models, a 10 percent increase/decrease in the input elasticity for irrigation water has been accompanied with a simultaneous decrease/increase in the input elasticities for fertiliser, tractor hour, and man-day, so that the condition of the CRS remains imposed. The results of this exercise are reported in Tables 9 and 10.

Table 9

Predicted Water Usage Assuming 10 Percent Increase in the Elasticity for IRR and 10 Percent Decrease in the Elasticities for FERT, MD, and TH at the Base-line Alternative Water Prices

Alternative Prices	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1 (CRS-imposed)									
Current Actual Water Usage[b]	517	90	26863	3938	4090	2026	18342	4979	60535
MVP-based:	722	154	27697	6330	9865	1765	48451	6004	100988
Rs 445.23/Acre Inch	(17.1)	(17.1)	(17.1)	(17.1)	(17.1)	(17.1)	(17.1)	(17.1)	(17.1)
AC-based:	4453	953	170944	39065	60884	10891	299038	37054	623282
Rs 7.8/Acre Inch	(–3.9)	(–3.9	(–3.9)	(–3.9)	(–3.9)	(–3.9)	(–3.9)	(–3.9)	(–3.9)
SRMC-based:	5128	1097	196858	44987	70114	12542	344370	42672	717768
Rs 5.7/Acre Inch	(–5.4)	(–5.4)	(–5.4)	(–5.4)	(–5.4)	(–5.4)	(–5.4)	(-5.4)	(–5.4)
LRMC-based:	3072	657	117925	26949	42001	7513	206291	25562	429970
Rs 17.8/Acre Inch	(.0004)	(0)	(.0004)	(.0004)	(.0004)	(.0004)	(.0004)	(.0004)	(.0004)
Market-based:	1034)	221	39704	9073	14141	2530	69455	8606	144764
Rs 200/Acre Inch	(12.6)	(12.6)	(12.6)	(12.6)	(12.6)	(12.6)	(12.6)	(12.6)	(12.6)
(B) Model 2 (CRS-imposed)									
Current Actual Water Usage[b]	517	1091	27196	4003	4090	2030	18762	4718	62407
MVP-based:	785	5408	30581	6885	10140	1745	32265	13821	101630
Rs 415.79/Acre Inch	(17.2)	(17.2)	(17.2)	(17.2)	(17.2)	(17.2)	(17.2)	(17.2)	(17.2)
AC-based:	5090	35044	198172	44616	65711	11306	209083	89564	658586
Rs 7.8/Acre Inch	(–2.4)	(–2.4)	(–2.4)	(–2.4)	(–2.4)	(–2.4)	(–2.4)	(–2.4)	(–2.4)
SRMC-based:	5898	40610	229650	51703	76149	13102	242294	103790	763196
Rs 5.7/Acre Inch	(–3.8)	(–3.8)	(–3.8)	(–3.8)	(–3.8)	(–3.8)	(–3.8)	(–3.8)	(–3.8)
LRMC-based:	3454	23779	134471	30274	44589	7672	141875	60774	446888
Rs 17.8/Acre Inch	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)	(1.3)
Market-based:	1108	7628	43136	9712	14303	2461	45511	19495	143354
Rs 200/Acre Inch	(13.3)	(13.3)	(13.3)	(13.3)	(13.3)	(13.3)	(13.3)	(13.3)	(13.3)

Note: Figures in parentheses indicate percent change in predicted water usage relative to the base-line case as a result of a 10 percent increase in water elasticity along with a simultaneous 10 percent decrease in other inputs' elasticities.

[a]One irrigation equals three acre inches of water.

[b]Number of irrigations at the district level based on survey data.

Table 10

Predicted Water Usage Assuming 10 Percent Decrease in the Elasticity for IRR and 10 Percent Increase in the Elasticities for FERT, MD, and TH at the Base-line Alternative Water Prices

Alternative Prices	Atto	Kula	Lodh	Mian	Mirp	Pesh	That	MarCh	Total
(A) Model 1 (CRS-imposed)									
Current Actual									
Water Usage[b]	517	90	26863	3938	4090	2026	18342	4669	60535
MVP-based:									
Rs 445.23 / Acre	497	106	19068	4358	6791	1215	33356	4133	69524
Inch	(–19.4)	(–19.4)	(–19.4)	(–19.4)	(–19.4)	(–19.5)	(–19.4)	(–19.4)	(–19.4)
AC-based:	4594	983	176350	40301	62810	11235	308494	38226	642993
Rs 7.8 / Acre Inch	(–0.01)	(–0.01	(–2.6)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)
SRMC-based:	5459	1168	209554	47889	74636	13351	366579	45424	764060
Rs 5.7 / Acre Inch	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(202.1)	(0.01)	(0.01)	
LRMC-based:	2918	624	112020	25600	39898	7137	195960	24282	408439
Rs 17.8 / Acre Inch	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)	(–5.0)
Market-based:	771	165	29612	6767	10547	1886	51800	6419	107967
Rs 200 / Acre Inch	(–16)	(–16.0)	(–16.0)	(–16.0)	(–16.0	(–16.0)	(–16.0)	(–16.0)	(–16.0)
(B) Model 2 (CRS-imposed)									
Current Actual									
Water Usage[b]	517	1091	27196	4003	4090	2030	18762	4718	62407
MVP-based:	567	3905	22084	4972	7323	1260	23300	9981	73392
Rs 415.79 / Acre Inch	(–15.4)	(–15.4)	(–15.4)	(–15.4)	(–15.4)	(–15.4)	(–15.4)	(–15.4)	(–15.4)
AC-based:	5257	36194	204676	46080	67868	11677	215945	92505	680202
Rs 7.8 / Acre Inch	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)	(–0.01)
SRMC-based:	6266	43144	243978	54928	80900	13919	257411	110268	810814
Rs 5.7 / Acre Inch	(2.0)	(2.0)	(2.0)	(2.0)	(2.0)	(2.0)	(2.0)	(2.0)	(2.0)
LRMC-based:	3312	22802	128945	29030	42756	7356	136045	58278	428524
Rs 17.8 / Acre Inch	(–2.8)	(–2.8)	(–2.8)	(–2.8)	(–2.8)	(–2.8)	(–2.8)	(–2.8)	(–2.8)
Market-based:	854	5883	33271	7490	11032	1898	35103	15037	110568
Rs 200 / Acre Inch	(–12.6)	(–12.6)	(–12.6)	(–12.6)	(–12.6)	(–12.6)	(–12.6)	(–12.6)	(–12.6)

Note: Figures in parentheses indicate percent change in predicted water usage relative to the base-line case as a result of a 10 percent decrease in water elasticity along with a simultaneous 10 percent increase in other inputs' elasticities.
[a]One irrigation equals three acre inches of water.
[b]Number of irrigations at the district level based on survey data.

Tables 9 and 10 report differences in the predicted water usage from the base-line simulations reported in Table 6, both in absolute terms and in percentage terms, as a result of a 10 percent increase/decrease in input elasticity for irrigation water, along with a simultaneous 10 percent decrease/increase in input elasticities for fertiliser, man-day, and tractor hour, using base-line water prices. Table 9 presents changes in predicted water usage as a result of a 10 percent increase in the input elasticity for irrigation water and a simultaneous 10 percent decrease in other input elasticities. Table 10 records the impact of a 10 percent decrease in the input elasticity for irrigation water and a simultaneous 10 percent increase in input elasticities for the other three inputs on predicted water usage.

A review of Table 9 highlights two points:

(i) A 10 percent increase in the water's estimated coefficient along with a 10 percent decrease in the estimated coefficients of other inputs shows a 17 percent and 13 percent increase in predicted water usage, using two alternative prices—the MVP-based and the Market-based respectively—wherein the change in the demand for water is more than the change in the parameter estimates. Moreover, the impact of this change in the estimated parameters has very little impact on predicted water usage when the three cost-based (AC-based, SRMC-based, and LRMC-based) base-line prices are used.

(ii) A 10 percent increase in the input elasticity for irrigation water along with a simultaneous decrease in input elasticities for other inputs brings two types of changes in the predicted water usage. First, in the case of MVP-based and market-based prices, the relationship between the change in the estimated parameters and the change in the predicted water usage is positive, meaning that a 10 percent increase in the estimated coefficient for irrigation water along with a 10 percent decrease in the estimated coefficients for other inputs increases predicted water usage by 17 percent using MVP-based prices, and by13 percent using market-based prices. Secondly, the same 10 percent increase in the water coefficient along with a 10 percent decrease in other inputs' estimated coefficients decreases predicted water usage when the three cost-based prices are used, though by a lesser percentage (2 to 4 percent).

Table 10 reports the impact of a 10 percent decrease in the input elasticity for irrigation water accompanied by a 10 percent increase in input elasticities for fertiliser, man-day, and tractor hour. As would be expected, the trend noticed in Table 9 is repeated in Table 10 but in the opposite direction. Using the base-line MVP-based and market-based prices, the predicted water usage decreases by more than 10 percent, i.e., by 15 to 19 percent in the former and 13 to 15 percent in the latter case respectively in the restricted and full data sets. The only difference noticed in Table 10 is that changes in the predicted water usage as a result of the change in the parameter estimates are in the same direction at the four water prices (MVP-based, market-based, AC-based, and LRMC-based), while in Table 9 this was not so. It means that like the MVP-based and market-based price cases, a decrease in predicted water usage is registered using the two cost-based prices in response to a 10 percent decrease in the input elasticity for irrigation water and a simultaneous10 percent increase in input elasticities of other inputs.

The analysis of the results of changes in various variables and parameters reported in Tables 5 to 10 allows one to arrive at two conclusions: (i) the demand for irrigation water is less sensitive to changes in alternative irrigation prices at the district level in both specifications of the 4-input "per man-day" model; and (ii) a 10 percent increase in the input elasticity for irrigation water along with a simultaneous 10 percent decrease in the input elasticities for fertiliser, man-day, and tractor hour increases water usage by 13 to 17 percent respectively at market-based and MVP-based prices in both data sets. At the cost-based prices the change is in the opposite direction and is less than 10 percent. A 10 percent decrease in the input elasticity for irrigation water, along with a simultaneous 10 percent increase in input elasticities for other three inputs, decreases predicted water usage by 16 to 19 percent respectively at the market- and MVP-based prices for the restricted data set, and by 13 and 15 percent for the full data set.

6. CONCLUSIONS AND POLICY IMPLICATIONS

Two major conclusions that emerge from this study may be summarised as follow:

(1) Policy simulations results, presented in Tables 7 and 8, report severe irrigation water shortages in all sample districts. At almost all alternative prices, predicted water usage exceeds actual water usage in all districts, which may mean that delivery systems do not deliver enough water for the price to ration. Hence, at all prices of water used in the simulations, the optimal amount of water is far more than is delivered.

(2) The reported results also speak loudly about the price less elastic of demand for irrigation water. A 10 percent increase/decrease in all prices decreases/increases predicted water usage by less than 10 percent (5 percent only), which means that any increase in water price will not reduce the demand for water.

The conclusions go against the general perception in Pakistan that water is used inefficiently due to low water rates charged from farmers, and that raising water rates would ensure water use efficiency. The present analysis makes a strong case for increasing water supplies to farmers.

Given the conclusions of the study, the policy-maker has two options for increasing water supplies to farmers: (i) building new water storage reservoirs, and (ii) improving the management of the water delivery system. Since additional increases in water supply through the construction of new reservoirs are not possible (at least in the short run) due to financial, environmental, social, and political constraints, the most cost-effective and feasible solution appears to be the improvement of the management of the water delivery system. Surface water supply deliveries are very inefficient because of losses through seepage and evaporation. Recent investigations have confirmed that water losses in the tertiary water distribution system below the outlet are very significant. While it may be difficult to avoid the losses in the main canals, there is considerable scope for conserving the large losses below the outlets [Sahibzada (2002)]. The on-going programme of On-farm Water Management, involving the improvement of the main watercourses and their partial lining is a move in this direction. However, its efficiency needs to be established especially since these improvements are not extended to the farmers' distribution channels [Sahibzada (2002)].

Irrigation water shortages are also the result of the inflexibility of the irrigation water delivery system for agricultural use. Basically, the irrigation system was designed a century ago—crop water requirements were based on the then designed cropping intensity of 50–75 percent. This intensity has almost doubled over the years which requires modernisation of the system to cope with the emerging scarcity problems. As canals have not yet been remodelled, the existing capacity can not provide adequate water to meet the current enhanced cropping intensity requirements. Hence water availability is far below the needed level.

With regard to the price inelasticity of the demand for water, existing irrigation water shortages are the root cause for that. If the minimum crop water requirements are not adequately met, the demand for water will not respond to a price change.

Two more findings are highlighted below.

(i) Subject to various limitations of data and modelling, our empirical analysis indicates significant inefficiency of resource allocation for irrigation water, as shown by its positively large MVP/OC ratio (2.23 and 2.08) reported in Table 4, implying its under-usage. This may also be the result of the scarcity of water supplies in response to the crop water requirements of the farmers.

MVP of irrigation water seems to be considerably above its costs (OC) under both specifications. In fact, irrigation water is not entirely within the capacity of individual farmers to supply. For example, the supply of irrigation water from tubewells is largely within the capacity of farmers as individuals. Wells can not, however, provide the required irrigation at economical costs in every part of the country. For most areas, canals are the only effective means of irrigation. It is possible, therefore, that the exploitation of irrigation water has been held down by the lack of large-scale canal irrigation facilities. Specifically, as mentioned earlier, the inflexibility of the irrigation system may be a major reason for the under-usage of irrigation water.

(ii) In terms of the water requirements of the individual districts and defining water use efficiency in terms of the highest agricultural produce per unit of irrigation water, Mirpur Khas stands out as the most efficient user of irrigation water, with 7.8 irrigations per acre and producing the maximum aggregated farm output—18 maunds per irrigation, followed by Attock and Peshawar with 16 and 11.5 maunds respectively per irrigation (Table 2).

The purpose of this study has been to evaluate various water pricing systems and choose one for application that ensures water use efficiency in the country. Contrary to what was assumed, the outcome of the empirical analysis goes against the general perception that the existing lower water rates lead to the inefficient use of irrigation water. In view of the major conclusions of the study, the case for introducing an appropriate water pricing system takes the second place on the priority scale. Before any recommendation is made to charge farmers a higher water price, it is essential that farmers are ensured adequate and reliable water supplies at their farm gate.

Technical Appendix

IRRIGATION DEMAND FUNCTION

The input demand function for irrigation water is a function of farm output and the four input prices (including irrigation water) under cost minimisation. Assuming cost minimisation and using the first-order conditions (FOC), the cost minimising demand for irrigation water is derived as follows.

Our Objective Function

$$\min C = w_1 X_1 + w_2 X_2 + w_3 X_3 + w_4 X_4$$

Subject to:

$$Y = b_0 X_1^{b_1} X_2{}^{b2} X_3^{b3} X_4^{b4} e^{\sum riDi + \sum piRi + \sum miCii}$$

where,

X_1 = Fertiliser; X_2 = Labour; X_3 = Tractor; and X_4 =Irrigation. *wi* are respective input prices. *Dis* are dummy variables for zero observations for the three inputs (fertiliser, tractor, and irrigation water): $D1 = 1$ when fertiliser use is positive, $D1 = 0$ for zero fertiliser use; $D2 = 1$ when tractor use is positive, $D2 = 0$ for zero tractor use; and $D3 = 1$ when irrigation use is positive, and $D3 = 0$ for zero irrigation use. *Ris* and *Cis* are respectively regional and multi-crop dummies, *e* represents exponent, *bis*, *ris*, *pis*, and *mis* are parameters to be estimated.

Cost minimisation problem for a firm can be written as a constraint optimisation equation, as:

$$L = w_1 X_2 + w_2 X_2 + w_3 X_3 + w_4 X_4 + \lambda(Y - b_0 X_1^{b_1} X_2^{b2} X_3^{b3} X_4^{b4} e^{\sum riDi + \sum piRi + \sum miCi}$$

where λ is the laGrangian multiplier. The first-order conditions for cost minimisation are:

$$(1)\quad \frac{\partial L}{\partial X_1} = w1 - \lambda b_1 bo X_1^{b_1-1} X_2^{b2} X_3^{b3} X_4^{b4} e^{\sum riDi + \sum piRi + \sum miCi} = 0$$

$$(2)\quad \frac{\partial L}{\partial X_2} = w2 - \lambda b_2 b_0 X_1^{b_1} X_2^{b2-1} X_3^{b3} X_4^{b4} e^{\sum riDi + \sum piRi + \sum miCi} = 0$$

$$(3)\quad \frac{\partial L}{\partial X_3} = w3 - \lambda b_3 b_0 X_1^{b_1} X_2^{2}\ X_3^{b3-1} X_4{}^{b4} e^{\sum riDi + \sum piRi + \sum miCi} = 0$$

$$(4)\quad \frac{\partial L}{\partial X_4} = w4 - \lambda b_4 b_0 X_1^{b_1} X_2^{b2} X_3^{b3} X_4^{b4-1} e^{\sum riDi + \sum piRi + \sum miCi} = 0$$

Dividing Equations 1, 2, and 3 by Equation 4 and taking the second term to the right-hand side of the equations, we get,

$$\frac{w_1}{w_4} = \frac{b_1}{b_4} x \frac{X_4}{X_1};\quad \frac{w_2}{w_4} = \frac{b_2}{b_4} x \frac{X_4}{X_2};\quad \frac{w_3}{w_4} = \frac{b_3}{b_4} x \frac{X_4}{X_3}$$

After rearranging the terms,

$$(5)\quad X_1 = \frac{b_1}{b_4}\frac{w_4}{w_1} \bullet X_4$$

$$(6)\quad X_2 = \frac{b_2}{b_4}\frac{w_4}{w_1} \bullet X_4$$

$$(7)\quad X_3 = \frac{b_3}{b_4}\frac{w_4}{w_3} \bullet X_4$$

Substituting 5, 6, and 7 in production function,

$$(8)\quad Y = b_0 \left(\frac{b_1}{b_4}\right)^{b_1} \left(\frac{w_4}{w_1}\right)^{b1} X_4^{b1} \left(\frac{b_2}{b_4}\right)^{b2} \left(\frac{w_4}{w_2}\right)^{b2} X_4^{b2} \left(\frac{b_3}{b_4}\right)^{b2} \left(\frac{w_4}{w_3}\right)^{b3} X_4^{b3} e^{\sum riDi + \sum\ piRi + \sum\ miCi}$$

$$(9)\quad Y = b_0 \left(\frac{b_1}{b_4}\right)^{b_1} \left(\frac{b_2}{b_4}\right)^{b2} \left(\frac{b_3}{b_4}\right)^{b3} \left(\frac{w_4}{w_1}\right)^{b_1} \left(\frac{w_4}{w_2}\right)^{b2} \left(\frac{w_4}{w_3}\right)^{b3} X_4^{(b1+b2+b3+b4)} e^{\sum riDi + \sum\ piRi + \sum\ miCi}$$

solving for X_4

$$(10)\quad X_4^{b1+b2+b3+b4_i} = \frac{Y}{b_0 b_1^{b_1} b_2^{b_2} b_3^{b_3}\left(\frac{1}{b_4^{b_1+b_2+b_3}}\right) w_4^{b_1+b_2+b_3}\left(\frac{1}{w_1^{b_1} w_2^{b_2} w_2^{b_3}}\right) e^{\sum riDi+\sum piRi+\sum miCi}}$$

Equation 10 can be written as:

$$(11)\quad X_4^{\Sigma bi} = \frac{y b_4^{(b1+b2+b3)} w_1^{b1} w_2^{b2} w_3^{b3}}{b_o b_1^{b1} b_2^{b2} b_3^{b3} w_4^{b1+b2+b3} e^{\sum riDi+\sum piRi+\sum miCi}}$$

Equation 11 can be written as:

$$(12)\quad X_4^{\Sigma bi} = \frac{Y^{\frac{1}{\sum bi}} b_4^{\frac{(b1+b2+b3)}{bi}} w_1^{\frac{b1}{\sum bi}} w_2^{\frac{b2}{\sum bi}} w_3^{\frac{b3}{\sum bi}}}{b_0^{\frac{1}{\sum bi}} b_1^{\frac{b1}{\sum bi}} b_{b2}^{\frac{b2}{\sum bi}} b_3^{\frac{b3}{\sum bi}} w_4^{\frac{(b1+b2+b3)}{\sum bi}} e^{\sum riDi+\sum piRi+\sum miCi}}$$

$$(13)\quad \hat{X}_4 = \frac{A_o Y^{\alpha y} w_1^{\alpha 1} w_2^{\alpha 2} w_3^{\alpha 3}}{w_4^{\alpha 4} e^{\sum riDi+\sum piRi+\sum miCi}}$$

where,

$$A_0 = \frac{b_4^{(b1+b2+b3)/\sum b_i}}{b_0^{1/\sum b_i} b_1^{b1/\sum b_i} b_2^{b2/\sum b_i} b_3^{b3/\sum b_i}}$$

$$\alpha_y = \frac{1}{\sum b_i};\ \alpha_1 = \frac{b_1}{\sum b_i};\ \alpha_2 = \frac{b_2}{\sum b_i};\ \alpha_3 = \frac{b_3}{\sum b_i};\ \alpha_4 = \frac{b_4}{\sum b_i}$$

Y = aggregate farm output at the mean level;
w_1 = the price of fertiliser nutrient per kilogram;
w_2 = the price of labour per man-day of 8 hours;
w_3 = the price of tractor use per hour;
w_4 = the price of irrigation water per acre inch;
b_0 = estimated coefficient of the constant;
b_1 = estimated coefficient of fertiliser;
b_2 = estimated coefficient of labour;
b_3 = estimated coefficient of tractor;
b_4 = estimated coefficient of irrigation water; and

the power in the exponent can be written as:

$$\left(\frac{r_1}{\sum b_i}\right)D_1\left(\frac{r_2}{\sum b_i}\right)D_2\left(\frac{r_3}{\sum b_i}\right)D_3 + \left(\frac{P_1}{\sum b_i}\right)R_1 + \left(\frac{P_2}{\sum b_i}\right)R_2 + --- + \left(\frac{P_7}{\sum b_i}\right)R_7$$

REFERENCES

Ahmad, Munir (2001) Agricultural Productivity Growth Differentials in Punjab, Pakistan: A District-level Analysis. *The Pakistan Development Review* 40:1, 1–25.

Ali, M., and D. Byerlee (2000) Productivity Growth, and Resource Degradation in Pakistan's Punjab: A Decomposition Analysis. A Draft for the World Bank. Washington, D.C.: The World Bank.

Battese, G. E., S. J. Malik, and M. A. Gill (1996) An Investigation of Technical Inefficiencies of Production of Wheat Farmers in Four Districts of Pakistan. *Journal of Agricultural Economics* 47:1, 37–49.

Battese, G. E. (1997) A Note on the Estimation of Cobb-Douglas Production Function When Some Explanatory Variables Have Zero Values. *Journal of Agricultural Economics* 48:2.

Chaudhry, M. G., S. A. Majid, and G. M Chaudhry (1993) The Policy of Irrigation Water Pricing in Pakistan: Aims, Assessment and Needed Redirections. *The Pakistan Development Review* 32:4, 809–821.

Chaudhry, M. G., and A. R. Kemal (1974) Wheat Production Under Alternative Production Functions. *The Pakistan Development Review* 13:2, 222–226.

Cummings, R. G., and Vahram Nercissiantz (1992) The Use of Water Pricing as a Means for Enhancing Water Use Efficiency in Irrigation: Case Studies in Mexico and the United States. *Natural Resources Journal* 32 (Fall), 731–55.

Dinar, A., and Ashok Subramanian (1997) Water Pricing Experiences: An International Perspective. Washington, D. C.: World Bank. (A World Bank Technical Paper No. 386.)

Dinar, A., M. W. Rosegrant, and Meinzen-Dick (1997) Water Allocation Mechanisms: Principles and Examples. World Bank and IFPRI. (Policy Research Working Paper 1779.)

Easter, K. William, Mark W. Rosegrant, and Ariel Dinar (1998) The Future of Water Markets: A Realistic Perspective. In K. William Easter, Mark W. Rosegrant and Arial Dinar (eds.) *Markets for Water: Potential and Performance.* Norwell MA: Kluwaer Academic Publishers.

Hufbauer, G. C., and M. M. Akhtar (1970) *The Determination of Water Rates*. Lahore: Planning and Development Board.

Le Moigne, Guy, Shawki Barghouti, Gershon Feder, Lisa Garbus, and Mei Xie, (eds.) (1992) Country Experiences with Water Resources Management: Economic, Institutional, Technological, and Environmental Issues. World Bank, Washington, D. C. (Technical Paper 175.)

Lewis, J. N. (1969) *Criteria for Determination of Water Rates in West Pakistan*. Lahore: Planning and Development Board.

Marschak, Jacob, and William H. Andrews (1944) Random Simultaneous Equations and the Theory of Production. *Econometrica* 12:3, 143–205.

McGovern S. (1999) Two Part Pricing. Feature Article, AECgroup, Brisbane Regional Economy: September (through internet).

Musgrave, Warren F. (1997) Australia. In Ariel Dinar and Ashok Subramanium (eds.) Water Pricing Experiences: An International Perspective. World Bank, Washington, D.C. (Technical Paper No. 386.)

Narlove, Mark (1965) *Estimation and Identification of Cobb-Douglas Production Function.* Chicago.

Naqvi, S. N. H., A. R. Kemal, Rashid Aziz, and Associates (1982) *The P.I.D.E Econometrics Model of Pakistan's Economy (1959-60 to 1978-79).* Islamabad: Pakistan Institute of Development Economics.

Naqvi, S. N. H., A. H. Khan, N. Khilji, and A. M. Ahmed (1983) *The P.I.D.E. Macro-econometrics Model of Pakistan's Economy*, Volume I. Islamabad: Pakistan Institute of Development Economics.

Naqvi, S. N. H., and A. M. Ahmed (1986) *The P.I.D.E. Macro-econometrics Model of Pakistan's Economy (Preliminary Results Revised).* Islamabad: Pakistan Institute of Development Economics.

Pakistan, Government of (1998) *Prices of Major Agricultural Crops.* Islamabad: Agricultural Prices Commission of Pakistan.

Pakistan, Government of (2001) *Pakistan Economic Survey 2000-2001.* Islamabad: Finance Division.

Plasquellec, H., Charles Burt, and H. W. Wolter (1994) Modern Water Control in Irrigation: Concepts, Issues, and Applications. World Bank, Washington, D.C. (Technical Paper 246.)

Rabemanambola, Flex (1997) Madagascar. In A. Dinar and A. Subramanium (eds.) Water Pricing Experiences: An International Perspective. Washington, D. C., World Bank. (Technical Paper No. 386.)

Rhodes, G. F., and R. K. Sampath (1988) Efficiency, Equity, and Cost Recovery Implications of Water Pricing and Allocation Schemes in Developing Countries. *Canadian Journal of Agricultural Economics* 36, 103–17.

Sahibzada, Shamim A. (2002) Efficient Irrigation Water Development in Pakistan: Pricing Issues and Options. Unpublished Ph.D. Dissertation. Binghamton University, State University of New York.

Sampath, R. K. (1992) Issues in Irrigation Pricing in Developing Countries. *World Development* 20:7, 967–77.

Shah, Tushaar (1993) *Groundwater Markets and Irrigation Development: Political Economy and Practical Policy.* Bombay: Oxford University Press.

Small, Leslie E., and Ian Carruthers (1991) *Farmer Financed Irrigation: The Economics of Reform.* Cambridge: Cambridge University Press.

Todt de Azevedo, Luiz Gabriel (1997) Brazil. In Ariel Dinar and Ashok Subramanium (eds.) Water Pricing Experiences: An International Perspective. World Bank, Washington, D. C. (Technical Paper No. 386.)

Tsur, Yacuv, and Ariel Dinar (1995) Efficiency and Equity Considerations in Pricing and Allocating Irrigation Water. World Bank, Washington, D. C. (Policy Research Paper No. 1460.)

Tsur, Yacuv, and Ariel Dinar (1997) The Relative Efficiency and Implementation Costs of Alternative Methods for Pricing Irrigation Water. *The World Bank Economic Review* 2:2.

Varian, Hal R. (1992) *Macroeconomic Analysis.* New York. London: W.W. Norton & Company.

Walters, A. A. (1963) Production and Cost Functions: An Econometrics Survey. *Econometrica* 31:1-2.

White, H. (1980) A heteroskedasticity-consistent Covariance Matrix Estimator and a Direct Test for heteroskedasticity. *Econometrica* 48, 817–38.

Zellner, A., J. Kmenta, and J. Dreze (1966) Specification and Estimation of Cobb-Douglas Production Function Models. *Econometrica* 34, January.

Zuberi, Habib A. (1989) Production Function, Institutional Credit and Agricultural Development in Pakistan. *The Pakistan Development Review* 28:1, 43–56.