

Estimating Standard Error of Inflation Rate in Pakistan: A Stochastic Approach

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“The answer to the question what is the mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as many purposes; and we may almost say in the matter of prices as many purposes as many writers.” Edgeworth (1888).

We estimate standard errors (S.Es.) of month on month and year on year inflation in Pakistan based on data for the period of July 2001 to June 2010 using the stochastic approach as well as extended stochastic approach to index numbers. We develop a mechanism to estimate S.E. of period average headline inflation (rate) using these approaches. This mechanism is then applied to estimate S.Es. of 12-month average rate of inflation in Pakistan for July 2003 to June 2010. The systematic changes in the relative prices of different groups in the CPI basket for Pakistan are also estimated. The highest (positive) relative price inflation occurred in ‘food, beverages and tobacco’ group and the lowest (negative) for ‘recreation and entertainment’ group, during fiscal years 2001 to 2010.

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1. INTRODUCTION

The stochastic approach to index numbers has recently attracted renewed attention of researchers as it provides the standard error of index number (and its growth). One of the most important uses of index number is in the case of measurement of the general price level in an economy (and then inflation, of course). This approach has been applied to measure the rate of inflation¹ in studies like Clement and Izan (1987), Selvanathan

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¹In rest of the paper we would prefer to use the term ‘inflation’ instead of the ‘rate of inflation’ or ‘inflation rate,’ for brevity.

(1989), Crompton (2000), Selvanathan (2003), Selvanathan and Selvanathan (2004), and Clement and Selvanathan (2007).

Historically, there are two main approaches to measure the index number: the functional approach and the stochastic approach. In the functional approach, prices and quantities of various goods and services are considered as connected by certain typical observable relationship [Frisch (1936)]. The stochastic approach is less well known although it has a long history dating back to Jevons (1865) and Edgeworth (1888). In the stochastic approach prices and quantities are considered as two sets of independent variables. It assumes that (ideally) individual prices ought to change in the same proportion from one point of time to the other. This assumption is based upon the quantity theory of money—as the quantity of money increases all prices should increase proportionally. Any deviation of individual prices from such proportionality is seen as ‘errors of observation’ and/or may be the result of non-monetary factors’ effect on prices. Thus the rate of inflation can be calculated by averaging over the proportionate changes in the prices of all individual goods and services. Keynes (1930) criticised the equiproportionate assumption as being ‘root and cause erroneous’. In the functional approach the deviations from proportionality are taken as expressions for those economic relations that serve to give economic meaning to index numbers [Frisch (1936)].

The recent interest of researchers in the stochastic approach to index number theory is led by Balk (1980), Clements and Izan (1981, 1987), Bryan and Cecchetti (1993) and Selvanathan and Rao (1994). Clements and Izan (1987) recognised the Keynes (1930) criticism on the assumption of identical systematic changes in prices and viewed the underlying² rate of inflation as an unknown parameter to be estimated from the individual price changes by linking the index number theory to regression analysis.

Again, using the functional approach to index numbers we obtain an estimate of the inflation rate without knowing its distribution. Thus, we have no basis to make any statistical comment, say about ‘efficiency,’ of the estimated inflation rate for which we shall also need the standard error of the estimated rate. The stochastic approach leads to familiar index number formulae such as Divisia and Laspeyres. As uncertainty plays a vital role in this approach, the foundations differ markedly from those of the functional approach; linking the index number theory to regression analysis we not only get an estimate of the rate of inflation but also its sampling variance. With the relaxation of the assumption that prices of goods and services change equiproportionately, individual prices in the basket of price index move disproportionately (which usually happens) and thus the overall rate of inflation may become less well defined [Selvanathan and Selvanathan (2006b)]. In such situations the ability of the stochastic approach becomes important as it allows us to construct confidence interval around the estimated rate of inflation with the help of standard error (of inflation). Confidence interval can be used for some practical purposes such as wage negotiations, wage indexation, inflation targeting (in interval), etc.

² This ‘underlying’ rate of inflation need not be confused with (a completely different) concept of ‘core inflation’ being used by some central banks to see long-term trend in change in general price level (sans temporary, short term, non-monetary, and supply side related changes in inflation). Core inflation is altogether a different measure of change in prices of a shrunk set of commodities which are more linked with demand side compared to supply side (exclusion based measures like non-food non energy inflation rate) or trimmed set of commodities prices of which are not highly volatile (like 20 percent trimming—those commodities which show extreme price changes—based measures of inflation).

One of the criticisms on this new stochastic approach of Clements and Izan (1987) is on the restriction of homoscedasticity on the variance of the error term in the OLS regression [Diewert (1995)]. Crompton (2000) also pointed out this deficiency and extended the new stochastic approach to derive robust standard errors for the rate of inflation by relaxing the earlier restriction on the variance of the error term by considering an unknown form of heteroscedasticity. Selvanathan (2003) presented some comments and corrections on Crompton's work. Selvanathan and Selvanathan (2004) showed how recent developments in the stochastic approach to index number can be used to model commodity prices in OECD countries. Selvanathan and Selvanathan (2006) calculated the annual rate of inflation for Australia, UK and US using the stochastic approach.³ These studies provided a mechanism for calculating the standard error for inflation. Rather than targeting the headline (year on year or YoY) inflation, some countries track 12 month moving average inflation as the goal of monetary policy. However, there is no work in the literature to estimate the standard error of period average inflation. We contribute by developing a mechanism to estimate the standard error of period average inflation.⁴

In this study we estimate standard errors of month on month (MoM) and YoY inflation in Pakistan using the stochastic approach, following Selvanathan and Selvanathan (2006). Since the State Bank of Pakistan (the central bank) targets 12-month average of YoY inflation, we contribute by applying our mechanism to estimate the standard error of 12-month average inflation in Pakistan.

The criticism on the assumption that when prices change they change equally proportionally, has been responded to by Clements and Izan (1987) who extend the stochastic approach by considering the underlying rate of inflation separate from the changes in relative prices. We also estimate the standard errors of (MoM, and YoY) inflation in Pakistan using the same approach. We contribute by developing a mechanism, and applying this to Pakistan, to estimate the standard error of period average inflation also using the same approach. Furthermore, by applying this approach we also estimate the systematic (MoM, and YoY, and 12-month average) change in relative prices based upon individual prices of 374 commodities in the CPI basket of Pakistan for the period July 2001-June 2010. However, in this paper we present only the average systematic change in relative prices of different groups in the CPI basket.

In the following section we provide the details of the existing mechanisms of stochastic approach and their applications in the index number theory in the context of price index. We then further build upon this approach to estimate YoY inflation, period (12-month) average inflation and their standard errors. In section 3 we present the results of the application of the stochastic and the extended stochastic approach for estimating MoM inflation, YoY inflation, and period (12-month) average inflation along with their standard errors. In section 4 we present the estimated average systematic change in relative prices of different groups in the CPI basket of Pakistan. Concluding remarks follow in the last Section.

³Clements, Izan and Selvanathan (2006) presented a review on the stochastic approach to index number theory.

⁴As mentioned earlier, we have not estimated the standard error of the core inflation rate measures like those exclusion-based or trimming-based. However, our working on the 12 month average inflation rate may be viewed as an exercise on the core inflation (because 12 month average inflation rate can be used as core inflation measure because it smoothes out fluctuations).

2. UNFOLDING THE STOCHASTIC APPROACH TO INDEX NUMBERS

The different ways to apply the stochastic approach to index numbers give various forms of index numbers like Divisia, Laspeyres etc. Since the Federal Bureau of Statistics (Pakistan’s official statistical agency) uses Laspeyres index formula for measuring inflation in period t over the base period, we would like to confine the following analysis to derive the Laspeyres index.

2.1. Derivation of Laspeyres Index Number⁵

Following conventional notations let p represents price and q represent the quantity. We subscript these notations by it where $i(i = 1, 2, \dots, n)$ represents commodity and $t(t = 1, 2, \dots, T)$ the time (which is month, in this study). Under stochastic approach any observed price change is a reading on the ‘underlying’ rate of inflation and a random component (ϵ_{it}). If γ_t is the price index, relating expenditures in period t to expenditures in the base period, then following the stochastic approach we can write

$$p_{it}q_{it} = \gamma_t p_{i0}q_{i0} + \epsilon_{it} \quad t(t = 1, 2, \dots, T) \quad \dots \quad \dots \quad \dots \quad (1)$$

We assume

$$E(\epsilon_{it}) = 0, \text{Cov}(\epsilon_{it}, \epsilon_{jt}) = \sigma_t^2 p_{i0}q_{i0} \delta_{ij} \quad (\delta_{ij} \text{ is the Kronecker delta}) \quad (2)$$

In this way the index number theory has been related to regression analysis as now we can estimate the rate of inflation in period t by estimating the unknown parameter γ_t in (1). Rearranging (1) we get

$$\begin{aligned} \frac{p_{it}}{p_{i0}} &= \gamma_t + \frac{\epsilon_{it}}{p_{i0}q_{i0}} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3) \\ \Rightarrow E\left(\frac{p_{it}}{p_{i0}}\right) &= \gamma_t, \end{aligned}$$

To remove heteroscedasticity in the error term we transform equation (1) into new form which gives homoscedastic variances in the error term across all the n commodities in any particular time period t . For this purpose we divide Equation (1) by $\sqrt{p_{i0} q_{i0}}$ and obtain

$$y_{it} = \gamma_t x_{i0} + \eta_{it} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

Where $y_{it} = \frac{p_{it} q_{i0}}{\sqrt{p_{i0} q_{i0}}}$; $x_{i0} = \sqrt{p_{i0} q_{i0}}$ and $\eta_{it} = \frac{\epsilon_{it}}{\sqrt{p_{i0} q_{i0}}}$

Now assumptions in (2) after above transformation are

$$E(\eta_{it}) = 0 \text{ and } \text{Cov}(\eta_{it}, \eta_{jt}) = \sigma_t^2 \delta_{ij}$$

Now we can apply, say, the least squares to (4) to have an estimator for inflation as below:

$$\hat{\gamma}_t = \frac{\sum_{i=1}^n x_{i0} y_{it}}{\sum_{i=1}^n x_{i0}^2}$$

⁵This sub-section (2.1) is mostly based upon Selvanathan and Selvanathan (2006b).

The base period can be somewhere in the distant past (say five year back) and at any point in time we define headline (or, YoY) inflation as percentage change in price index over the corresponding month last year then

$$\pi_t^H = \frac{\gamma_t - \gamma_{t-12}}{\gamma_{t-12}}$$

From (11) we can get estimate of γ_t only. For estimate of γ_{t-12} we write (11) as

$$p_{it-12}q_{i0} = \gamma_{t-12} p_{i0}q_{i0} + \epsilon_{it-12} \quad t(t = 13,14,15 \dots, T) \quad \dots \quad (12)$$

Here again $E\left(\frac{p_{it-12}}{p_{i0}}\right) = \gamma_{t-12}$, under similar assumptions as in (2)

By subtracting (12) from (11) we have

$$p_{it}q_{i0} - p_{it-12}q_{i0} = (\gamma_t - \gamma_{t-12})p_{i0}q_{i0} + \epsilon_{it} - \epsilon_{it-12} \quad \dots \quad \dots \quad (13)$$

Dividing (13) by $E\left(\frac{p_{it-12}}{p_{i0}}\right)$ and substituting $E\left(\frac{p_{it-12}}{p_{i0}}\right) = \gamma_{t-12}$ on right hand side, we get

$$\left[\frac{p_{it}q_{i0} - p_{it-12}q_{i0}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)} \right] = \left(\frac{\gamma_t - \gamma_{t-12}}{\gamma_{t-12}} \right) p_{i0}q_{i0} + \frac{\epsilon_{it} - \epsilon_{it-12}}{\gamma_{t-12}} = \pi_t^H p_{i0}q_{i0} + e_{it} \quad \dots \quad (14)$$

Where $e_{it} = \frac{\epsilon_{it} - \epsilon_{it-12}}{\gamma_{t-12}}$. Again assuming that

$$E(e_{it}) = 0 \text{ and } Cov(e_{it}, e_{jt}) = \rho_t^2 p_{i0}q_{i0} \delta_{ij} \quad \dots \quad \dots \quad \dots \quad (15)$$

and proceeding as in subsection 2.1 we divide (14) by $\sqrt{p_{i0}q_{i0}}$ and get

$$\left[\frac{\frac{p_{it}}{p_{i0}} \frac{p_{it-12}}{p_{i0}}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)} \right] \sqrt{p_{i0}q_{i0}} = \pi_t^H \sqrt{p_{i0}q_{i0}} + \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}} \quad \dots \quad \dots \quad \dots \quad (16)$$

From (5) and (12) we can write $\gamma_{t-12} = E\left(\frac{p_{it-12}}{p_{i0}}\right) = \sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}$, Thus (16) becomes

$$\left[\frac{\frac{p_{it}}{p_{i0}} \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}} \right] \sqrt{p_{i0}q_{i0}} = \pi_t^H \sqrt{p_{i0}q_{i0}} + \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}}$$

If we take $Y_{it} = \left(\frac{\frac{p_{it}}{p_{i0}} \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}} \right) \sqrt{p_{i0}q_{i0}}$, $X_{i0} = \sqrt{p_{i0}q_{i0}}$ and $\varphi_{it} = \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}}$. Under assumptions that $E(\varphi_{it})=0$ and $Cov(\varphi_{it}, \varphi_{jt}) = \rho_t^2 \delta_{ij}$, for equation

$$Y_{it} = \pi_t^H X_{i0} + \varphi_{it} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (17)$$

Least square estimator of π_t^H is

$$\begin{aligned} \hat{\pi}_t^H &= \frac{\sum_{i=1}^n X_{i0} Y_{it}}{\sum_{i=1}^n X_{i0}^2} = \frac{\sum_{i=1}^n \sqrt{p_{i0}q_{i0}} \left[\frac{\frac{p_{it}}{p_{i0}} \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}} \right] \sqrt{p_{i0}q_{i0}}}{\sum_{i=1}^n p_{i0}q_{i0}} \\ &= \sum_{i=1}^n w_{i0} \left[\frac{\frac{p_{it}}{p_{i0}} \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}} \right] = \frac{\hat{\gamma}_t - \hat{\gamma}_{t-12}}{\hat{\gamma}_{t-12}} \quad \dots \quad \dots \quad \dots \quad (18) \end{aligned}$$

We knew this result from (7). The only benefit of the above process is that now we can have an estimate of the standard error of headline inflation as below:

$$Var(\hat{\pi}_t^H) = \frac{q_t^2}{\sum_{i=1}^n x_{i0}^2} \dots \dots \dots \dots \dots \dots \dots \dots (19)$$

The parameter q_t^2 can be estimated as

$$\hat{q}_t^2 = \frac{1}{n-1} \sum_{i=1}^n (Y_{it} - \hat{\pi}_t^H X_{i0})^2$$

By substitution of the estimated parameter q_t^2 in (19) and rearranging we get

$$Var(\hat{\pi}_t^H) = \frac{1}{n-1} \sum_{i=1}^n w_{i0} \left[\frac{\frac{p_{it}}{p_{i0}} - \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}}{p_{i0}}} - \hat{\pi}_t^H \right]^2 \dots \dots \dots (20)$$

Equation (20) shows that the variance of $\hat{\pi}_t^H$ increases with the degree of relative inflation variability.⁶ Now we move towards estimating the period average inflation and its standard error.

2.3. Application of Stochastic Approach to Estimate Period Average Inflation and Its S.E.

We know that period average (say 12 month average) inflation can be calculated either by averaging the last 12 YoY inflation numbers or by taking YoY inflation of the last 12-month (moving) averaged index number. We would like to use the above result in subsection 2.1 for estimating the 12-month average inflation, and those in subsection 2.2 for the standard error of period average inflation.

We have price index series as p_{it} . If the 12-month averaged price index series is denoted by p_{it}^A then following the results in subsection 2.1, the estimate of YoY inflation of p_{it}^A series will be

$$\hat{\gamma}_t^A = \sum_{i=1}^n w_{i0} \frac{p_{it}^A}{p_{i0}} \dots \dots \dots \dots \dots \dots \dots (21)$$

And thus

$$\hat{\pi}_t^A = \left(\frac{\hat{\gamma}_t^A}{\hat{\gamma}_{t-12}^A} - 1 \right) * 100 \dots \dots \dots \dots \dots \dots \dots (22)$$

Now for estimating the variance of the average inflation we use the result in subsection 2.2 where we derived the standard error of YoY inflation. If we replace the index with the average index in (20) we will get the standard error of average YoY inflation, that is

$$Var(\hat{\pi}_t^A) = \frac{1}{n-1} \sum_{i=1}^n w_{i0} \left[\frac{\frac{p_{it}^A}{p_{i0}} - \frac{p_{it-12}^A}{p_{i0}}}{\sum_{i=1}^n w_{i0} \frac{p_{it-12}^A}{p_{i0}}} - \hat{\pi}_t^A \right]^2 \dots \dots \dots (23)$$

3. MEASURING STANDARD ERRORS OF INFLATION IN PAKISTAN

In this section we present an application of the results described and derived in the previous section by using the monthly data of prices of 374 commodities covering the

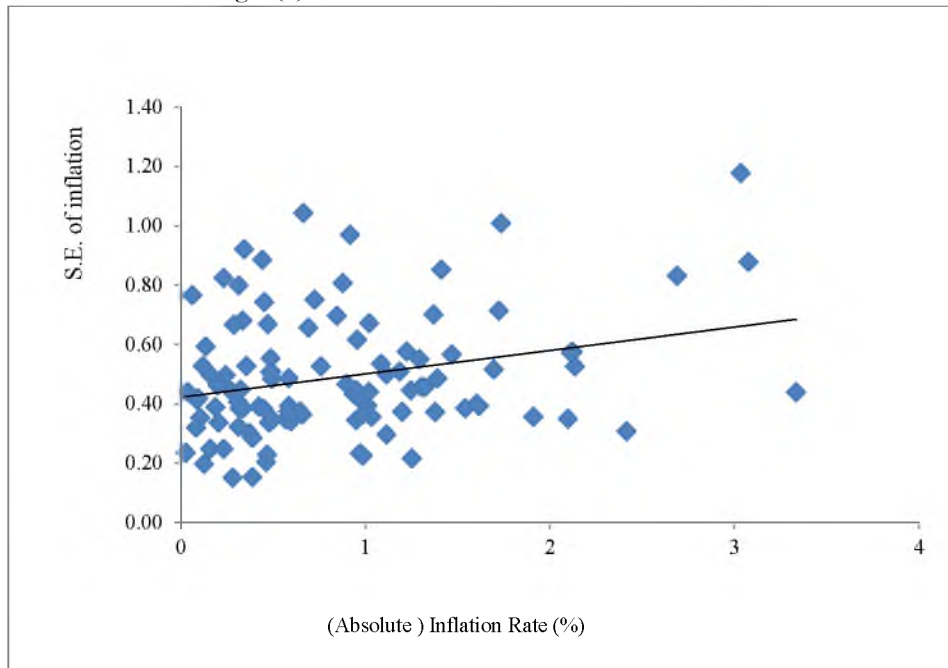
⁶Above procedure can be used to estimate the MoM inflation and its standard error.

period July 2001–June 2010 for Pakistan.⁷ We present the estimated MoM inflation, YoY inflation, along with their standard errors for Pakistan. As discussed above, there are different ways to apply the stochastic approach to index numbers and each culminates in different form of index numbers like Divisia, Laspeyres etc. Just to compare our estimated results of inflation with those from the Federal Bureau of Statistics we have used such application of the stochastic approach which produces Laspeyres index formula for measuring inflation in the current period over the base period. Since the State Bank of Pakistan targets (12-month) average inflation, particular attention has been paid to estimate period (12-month) average inflation rate and its standard error, which is the first empirical application of its type.

Table A1 in the Appendix presents the official rate of (monthly, YoY and 12-month average) inflation and the estimated rate of (monthly, YoY and 12-month average) inflation based on the stochastic approach along with standard error of the estimate of inflation for Pakistan economy based on the data for July 2001 to June 2010.⁸

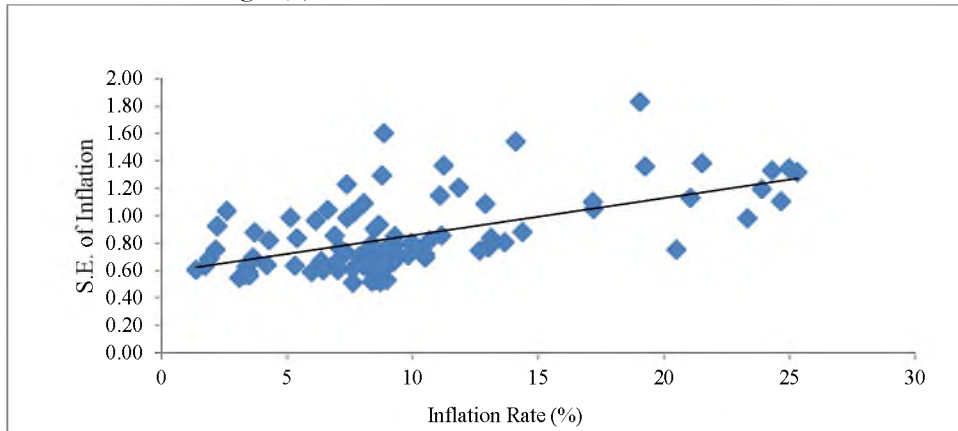
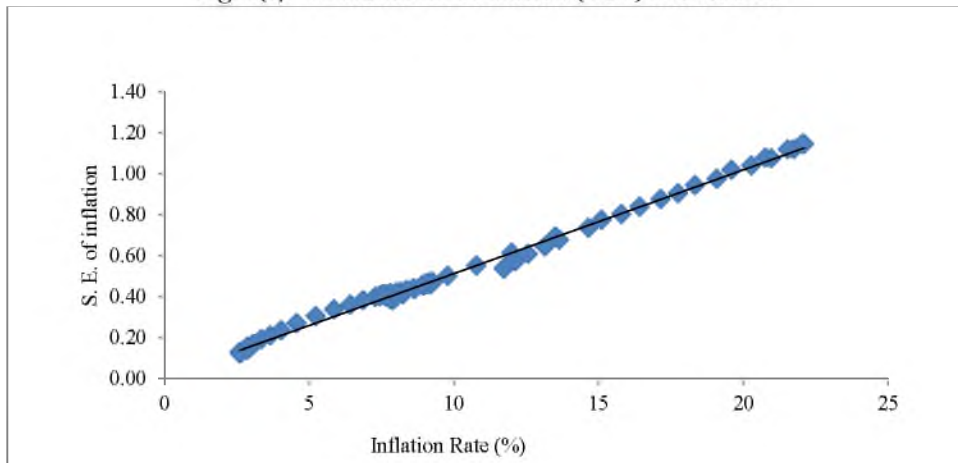
Figures 1(a) to 1(c) present a scatter plot of inflation versus the corresponding standard error for MoM, YoY and 12-month average inflation; the solid line is the linear trend line.

Fig. 1(a). MoM Inflation in Pakistan and its S.E.



⁷Prices, for construction of consumer price index (CPI), are collected by Pakistan Bureau of Statistics (the central statistical agency of the Government of Pakistan) on monthly basis. In August 2011, while changing the base year for CPI from FY 2001 to FY 2008, PBS also expanded the coverage of goods/services in the CPI basket by increasing the number of commodities from 374 to 487. In this study, we have used the previous base (FY 2001) dataset.

⁸First 12 observations are lost in the YoY inflation calculation and next 12 are consumed in calculating the 12 month average. Thus, the results in the Table 1 start from July 2003 instead of July 2001.

Fig. 1(b). YoY infaltion in Paksitan and its S.E.**Fig. 1(c). 12 month MA inflation (YoY) and its S.E.**

From Figures 1(a) to 1(c) we can see that the standard error increases with increasing inflation as depicted by the positive slope of the trend line. This can be interpreted to mean that when inflation is higher it becomes difficult to predict it. This agrees with the intuitive notion that when the individual prices move very disproportionately, the overall rate of inflation is less well defined. These observations are in line with the past literature on the rate and variability of inflation.⁹ Figures 1(b) and 1(c) also answer the question, “Why some central banks pursue (target) 12-month moving average (YoY) inflation rate rather than monthly headline (YoY) rate of inflation?” Some central banks use the 12-month average as the core inflation. The answer is simple: the average (YoY) inflation is less volatile than the headline inflation. This is evident from the lower S.E. of 12-month moving average (YoY) inflation as shown in the figure 1(c) and compared to S.E. of monthly headline (YoY) inflation presented in 1(b).

⁹The inflation may become less predictable at higher inflation rate if government aims stabilising prices rather than stabilising expectations [Logue and Willet (1976)].

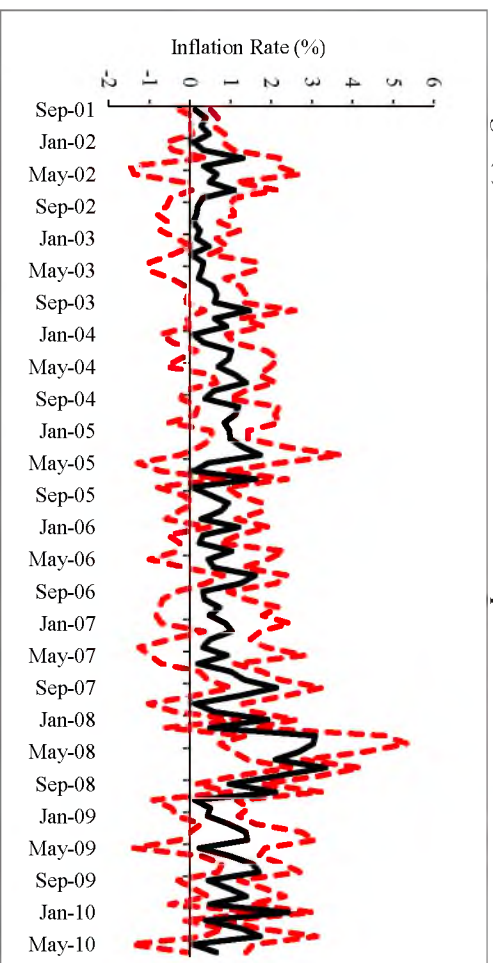


Fig. 2(a). MoM Inflation in Pakistan and 95 percent Confidence Band

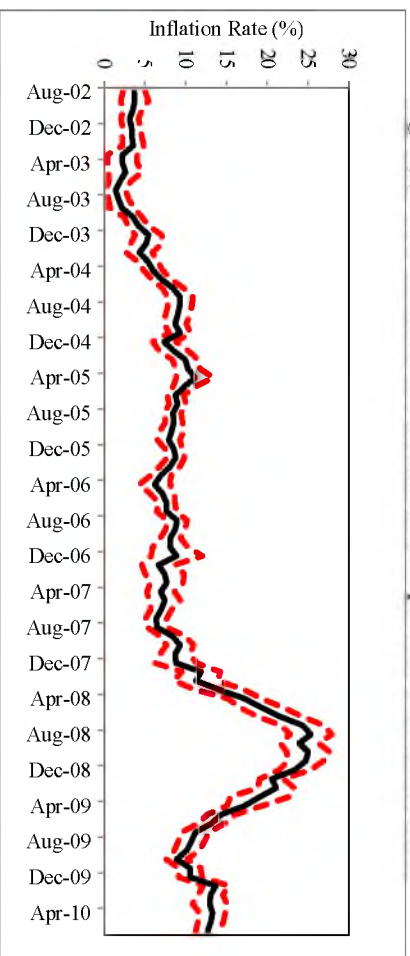


Fig. 2(b): YoY Inflation in Pakistan and 95 percent Confidence Band

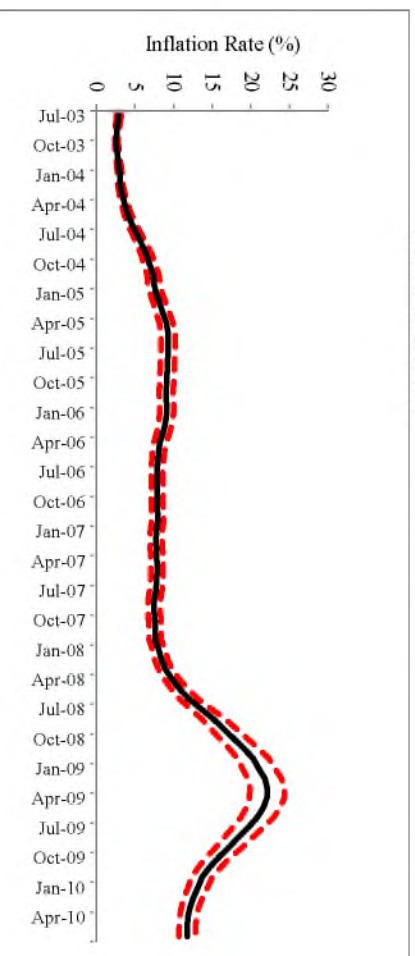


Fig. 2(c). 12 month MA Inflation (YoY) in Pakistan and 95 percent Confidence Band

Figure 2 (a) to Figure 2(c) present the graph of all the three types of inflation along with the respective 95 percent confidence band. From Figure 2 (a) it is clear that the time when there is a jump in inflation, as in April 2005 and May 2008, there is an increase in the width of the confidence bands. Similarly we can note that in other figures where the inflation is high, the width of confidence band is also increased.

4. EXTENDED STOCHASTIC APPROACH AND THE SYSTEMATIC CHANGE IN RELATIVE PRICES

As we discussed in Section 2, the various applications of the stochastic approach to index numbers yield different forms of index numbers like the Laspeyres etc. However, as criticised by Keynes (1930), following this approach it is assumed that when prices change they change equiproportionately and thus relative prices remain the same. Clements and Izan (1987) responded to Keynes’ criticism by considering common trend change in all prices underlying the rate of inflation separate from the systematic change in relative prices. Following Clements and Izan (1987), if we take p_{it} as the price of commodity i ($i = 1, 2, \dots, n$) at time t ($t = 1, 2, \dots, T$) then price log change $Dp_{it} = \log p_{it} - \log p_{it-1}$ can be considered as

$$Dp_{it} = \alpha_t + \beta_i + \xi_{it} \quad i = 1, 2, \dots, n; \text{ and } t = 1, 2, \dots, T \quad \dots \quad (24)$$

Where α_t is the common trend change in all prices (the underlying rate of inflation) and β_i is the change in relative prices of commodity i . Assuming the random component of change in prices, ξ_{it} , to be independent over commodities and time, and the variances $[Var(\xi_{it})]$ inversely proportional to corresponding arithmetic averages of budget shares, Clements and Izan (1987) showed that the least squares estimates of α_t and β_i are subject to budget constraint¹⁰ as given below:

$$\hat{\alpha}_t = \sum_{i=1}^n \bar{w}_i Dp_{it} \quad \dots \quad (25)$$

$$\hat{\beta}_i = \frac{1}{T} \sum_{t=1}^T \left[\frac{1}{\theta_t^2} / \sum_{t=1}^T \frac{1}{\theta_t^2} \right] (Dp_{it} - \hat{\alpha}_t) \quad \dots \quad (26)$$

With respective variances of these estimators as below:

$$Var(\hat{\alpha}_t) = \frac{\theta_t^2}{(n-1)} \quad \dots \quad (27)$$

$$Var(\hat{\beta}_i) = \frac{1}{(n-1) \sum_{t=1}^T \frac{1}{\theta_t^2}} \left(\frac{1}{\bar{w}_i} - 1 \right) \quad \dots \quad (28)$$

Where θ_t^2 is the sum (over commodities) of squares of estimated random component of price changes, that is

$$\theta_t^2 = \sum_{i=1}^n (\xi_{it})^2 = \sum_{i=1}^n \bar{w}_i (Dp_{it} - \hat{\alpha}_t)^2 + \sum_{i=1}^n \bar{w}_i (\overline{Dp}_i - \bar{\alpha})^2 - 2 \sum_{i=1}^n \bar{w}_i (Dp_{it} - \hat{\alpha}_t) \bar{w}_i (\overline{Dp}_i - \bar{\alpha}) \quad \dots \quad (29)$$

While it is obvious that $\overline{Dp}_i = \frac{1}{T} \sum_{t=1}^T Dp_{it}$ and $\bar{\alpha} = \frac{1}{T} \sum_{t=1}^T \hat{\alpha}_t$

¹⁰Budget share weighted average of the systematic component of relative price change is zero.

Our contribution in this section of the study is the application of the Clements and Izan (1987) extended stochastic approach to index numbers to Pakistan's monthly data of prices of 374 commodities covering the period July 2001–June 2010. The extended stochastic approach to index number is closer to Divisia price index. As in the above Section 3, this approach also gives us the (trend) inflation rate and its standard errors which are presented in Table A1 of the Appendix. We can see that the estimated standard errors of the inflation rate based upon extended stochastic approach to inflation are lower than those based upon the stochastic approach for the MoM and YoY inflation. It needs not be true in the case of period average inflation (because of averaging effects).

In addition to inflation and its standard error, the Clements and Izan (1987) extended stochastic approach also gives us the systematic change in relative prices of each commodity in the basket. We have applied this approach to prices of 374 commodities in the CPI basket of Pakistan for the period of FY01 to FY 2010 to investigate the systematic relative price changes. It may be difficult to extract any meaningful result from the detailed presentation of systematic (MoM, and YoY, and 12-month average) change in relative prices of each of the 374 commodities.¹¹ However, it will be useful if we present the systematic (MoM, and YoY, and 12-month average) change in relative prices for various groups in the CPI basket as in Table A2 of the Appendix. There are ten groups in the CPI basket of Pakistan as shown in Table A2. It is clear from the table that coefficients of relative prices of all groups are significantly different from zero. For comparison we have also given the observed relative price changes as measured from FBS price data for all the three cases: MoM, YoY and 12-month moving average.¹²

The estimated relative prices of 'Food Beverages & Tobacco' group are increased by highest percentage point for MoM changes (0.18 percent). In case of YoY changes, we find 'Food Beverages and Tobacco (FBT)' and 'Fuel and Lighting' groups to exhibit increase in relative prices by 1.72 percent and 0.37 percent respectively. In the case of 12-month moving averages, we find 'FBT' and 'House Rent' groups to depict increase in relative prices by 1.72 percent and 0.04 percent respectively. In all the three cases, since the 'FBT' group turned out to have the highest change in relative prices, we can say that inflation during most of the FY01 to FY10 was FBT price change driven.

Interestingly, for each of the three cases of MoM, YoY and 12-month moving average, the change in relative prices is found to be highest (positive) for 'FBT' group and lowest (negative) for 'Recreation and Entertainment (RE)' group during FY01 to FY10. The supply side factor(s) and/or elasticities of demand may be behind this observed phenomenon as commodities in the FBT group are more prone to supply shocks and tend to be less price elastic compared to those in RE group.

Table A1 in the Appendix presents the official rate of (monthly, YoY and 12-month average) inflation and the estimated rate of (monthly, YoY and 12-month average) inflation based on stochastic as well as extended stochastic approaches along with standard error of the estimates of inflation for Pakistan based on the data for July 2001 to June 2010. Numerically, the official and estimated inflation rates seem different. But when we apply t-test we could not find official inflation rate to be statistically different from any of the estimated inflation rates based on stochastic as well as extended

¹¹Detailed results can be obtained from the authors, if desired.

¹²We can see from the Table A2 in the Appendix that the weighted average of the relative prices is zero in each of the observed and estimated case, which should be.

stochastic approaches.¹³ Which approach for measuring inflation is better? Obviously the stochastic approach has an advantage as it estimates the standard errors along with the inflation rate and is therefore preferable. Furthermore, in the case of using extended stochastic approach we also get estimates of systematic change in relative prices and their standard errors. The confidence interval can be built around the estimated rate of inflation for different useful purposes like wage bargaining.

5. CONCLUSION

In this study we estimate the standard errors of month on month and year on year inflation rate using the stochastic approach of Selvanathan and Selvanathan (2006) and the extended stochastic approach of Clements and Izan (1987) based on individual prices of 374 commodities in CPI basket of Pakistan for the period July 2001 to June 2010. We also contribute to the literature by employing the stochastic approach to index numbers by developing a mechanism to estimate the inflation rate and its standard error for period average CPI. Based on this mechanism, we estimate the standard error of 12-month moving average YoY inflation rate for Pakistan for the period July 2003 to June 2010. We find that the standard error of inflation increases with inflation rate in Pakistan. Notwithstanding the fact that the ‘higher the standard error the higher the inflation rate’, the estimated standard errors of inflation rate based on extended stochastic approach are lower than those based on the stochastic approach. Furthermore, for each of the three cases of MoM, YoY and 12-month average, the change in relative prices is found to be highest for ‘Food Beverages and Tobacco’ group and lowest for ‘Recreation and Entertainment’ group during FY01 to FY10.

APPENDIX

Table A1

Official Inflation Rate, Stochastic (Extended Stochastic) Approach Related Estimates of Inflation and Respective Standard Errors

Month	Month on Month					Headline (Year on Year)					12-month moving average				
	Official Inflation Rate	Stochastic Approach Estimates of	Extended Stochastic Approach Estimates of	Official Rate of Inflation	Stochastic Approach Estimates of	Extended Stochastic Approach Estimates of	Official Rate of Inflation	Stochastic Approach Estimate of	Extended Stochastic Approach Estimates of	Official Rate of Inflation	Stochastic Approach Estimate of	Extended Stochastic Approach Estimates of	Official Rate of Inflation	Stochastic Approach Estimate of	Extended Stochastic Approach Estimates of
		inflation	S.E.	Inflation	S.E.	Inflation	S.E.	Inflation	S.E.	Inflation	S.E.	Inflation	S.E.	Inflation	S.E.
Jul-03	0.57	1.01	0.35	0.89	0.24	1.41	1.72	0.60	1.80	0.54	2.89	3.03	0.16	1.54	0.48
Aug-03	0.66	0.76	0.36	0.62	0.15	1.76	2.03	0.63	2.04	0.52	2.73	2.68	0.14	1.52	0.48
Sep-03	0.60	0.53	0.35	0.32	0.18	2.18	2.34	0.75	2.21	0.54	2.60	2.50	0.13	1.50	0.50
Oct-03	1.47	1.57	0.57	1.20	0.33	3.51	4.94	0.56	3.47	0.50	2.60	2.37	0.12	1.64	0.50
Nov-03	0.60	0.69	0.34	0.74	0.14	4.22	5.08	0.64	4.29	0.57	2.70	2.52	0.13	1.89	0.49
Dec-03	0.90	1.17	0.47	1.12	0.17	5.41	6.92	0.83	5.43	0.56	2.87	2.68	0.14	2.22	0.46
Jan-04	-0.09	0.11	0.41	-0.07	0.27	5.15	6.67	0.98	5.26	0.58	3.02	3.06	0.16	2.54	0.42
Feb-04	-0.34	-0.48	0.38	-0.23	0.31	4.31	5.43	0.82	4.62	0.59	3.09	3.29	0.17	2.79	0.40
Mar-04	1.02	0.81	0.44	0.74	0.26	5.34	6.19	0.63	5.52	0.51	3.35	3.73	0.19	3.21	0.38
Apr-04	0.96	0.81	0.62	0.42	0.35	5.99	6.39	0.59	6.02	0.48	3.66	4.07	0.21	3.56	0.37
May-04	0.69	0.78	0.65	1.02	0.43	7.04	7.57	0.60	7.03	0.50	4.03	4.56	0.23	4.08	0.36
Jun-04	1.12	1.00	0.29	1.12	0.20	8.45	9.10	0.67	7.89	0.53	4.57	5.26	0.27	4.70	0.36
Jul-04	1.38	1.58	0.37	1.20	0.25	9.33	9.72	0.78	8.21	0.56	5.23	5.94	0.30	5.23	0.36
Aug-04	0.59	0.64	0.39	0.62	0.14	9.25	9.59	0.75	8.20	0.52	5.86	6.58	0.34	5.75	0.37
Sep-04	0.37	0.27	0.30	0.28	0.12	9.00	9.30	0.76	8.16	0.55	6.43	7.03	0.36	6.27	0.40
Oct-04	1.19	1.19	0.51	0.76	0.14	8.70	8.89	0.69	7.72	0.49	6.86	7.41	0.38	6.62	0.41
Nov-04	1.12	0.99	0.50	1.05	0.21	9.26	9.22	0.66	8.04	0.51	7.29	7.77	0.40	6.94	0.42

Continued—

¹³The results of t-test are not reported in the paper to save the space. However, those can be obtained from the authors, if required.

Table A1—(Continued)

Month	Month on Month					Headline (Year on Year)					12-month moving average				
	Official Inflation Rate	Stochastic Approach Estimates of		Extended Stochastic Approach Estimates of		Official Rate of Inflation	Stochastic Approach Estimates of		Extended Stochastic Approach Estimates of		Official Rate of Inflation	Stochastic Approach Estimate of		Extended Stochastic Approach Estimates of	
		inflation		Inflation			Inflation		Inflation			Inflation		Inflation	
		S.E.		S.E.			S.E.		S.E.			S.E.		S.E.	
Dec-04	-0.85	-0.97	0.70	-0.05	0.38	7.37	6.91	0.73	6.87	0.54	7.45	7.79	0.40	7.03	0.40
Jan-05	0.97	1.04	0.23	0.95	0.19	8.51	7.89	0.90	7.88	0.63	7.72	7.90	0.40	7.21	0.38
Feb-05	0.99	1.03	0.22	0.83	0.17	9.95	9.53	0.80	8.95	0.66	8.19	8.23	0.42	7.54	0.39
Mar-05	1.29	1.56	0.55	0.84	0.27	10.25	10.35	0.76	9.05	0.60	8.60	8.56	0.44	7.81	0.40
Apr-05	1.74	2.10	1.01	0.94	0.41	11.10	11.76	1.15	9.56	0.61	9.03	9.00	0.46	8.11	0.40
May-05	-0.44	-0.84	0.88	0.16	0.31	9.84	9.96	0.71	8.70	0.51	9.26	9.18	0.47	8.22	0.39
Jun-05	0.10	-0.28	0.42	0.74	0.19	8.74	8.57	0.51	8.32	0.46	9.28	9.13	0.47	8.27	0.39
Jul-05	1.62	1.55	0.39	1.06	0.18	8.99	8.54	0.53	8.17	0.48	9.25	9.04	0.46	8.28	0.38
Aug-05	0.04	0.12	0.44	0.12	0.23	8.40	7.98	0.52	7.67	0.46	9.17	8.90	0.46	8.24	0.37
Sep-05	0.50	0.44	0.35	0.34	0.15	8.53	8.17	0.53	7.73	0.48	9.13	9.12	0.47	8.21	0.36
Oct-05	0.94	0.53	0.43	0.51	0.16	8.27	7.46	0.75	7.48	0.55	9.09	9.01	0.46	8.18	0.37
Nov-05	0.76	0.94	0.53	0.93	0.18	7.89	7.41	0.70	7.36	0.54	8.97	8.84	0.45	8.08	0.38
Dec-05	-0.27	-0.33	0.44	-0.10	0.23	8.51	8.10	0.54	7.30	0.45	9.06	8.93	0.46	8.11	0.38
Jan-06	1.20	1.28	0.37	0.96	0.31	8.76	8.37	0.54	7.32	0.40	9.08	8.96	0.46	8.06	0.39
Feb-06	0.33	0.42	0.45	0.69	0.32	8.05	7.71	0.63	7.17	0.48	8.92	8.81	0.45	7.93	0.40
Mar-06	0.23	0.25	0.25	0.16	0.17	6.91	6.32	0.64	6.49	0.51	8.64	8.50	0.44	7.73	0.41
Apr-06	1.02	1.09	0.67	0.68	0.29	6.16	5.27	0.96	6.24	0.58	8.23	7.98	0.41	7.45	0.42
May-06	0.45	0.32	0.74	0.34	0.22	7.12	6.51	0.76	6.42	0.58	8.01	7.71	0.40	7.28	0.43
Jun-06	0.59	0.49	0.37	0.94	0.30	7.65	7.33	0.51	6.62	0.43	7.92	7.61	0.39	7.14	0.43
Jul-06	1.61	1.64	0.40	1.09	0.24	7.63	7.42	0.61	6.65	0.52	7.81	7.52	0.39	7.01	0.42
Aug-06	1.25	1.38	0.44	0.97	0.15	8.93	8.77	0.62	7.50	0.48	7.86	7.59	0.39	7.00	0.40
Sep-06	0.32	0.18	0.32	0.38	0.11	8.73	8.49	0.61	7.55	0.51	7.88	7.45	0.38	6.97	0.39
Oct-06	0.36	0.44	0.53	0.20	0.22	8.11	8.39	0.63	7.24	0.46	7.87	7.49	0.38	6.96	0.38
Nov-06	0.73	1.10	0.75	0.68	0.28	8.07	8.55	1.09	6.98	0.52	7.89	7.61	0.39	6.97	0.37
Dec-06	0.47	0.88	0.67	0.50	0.33	8.88	9.88	1.60	7.59	0.73	7.92	7.76	0.40	7.03	0.38
Jan-07	-0.88	-1.10	0.81	-0.36	0.52	6.64	7.29	1.04	6.26	0.57	7.74	7.68	0.39	6.96	0.37
Feb-07	1.04	1.19	0.36	0.95	0.21	7.39	8.12	1.23	6.53	0.64	7.69	7.71	0.40	6.91	0.35
Mar-07	0.49	0.27	0.51	0.18	0.31	7.67	8.14	1.02	6.55	0.66	7.75	7.85	0.40	6.93	0.34
Apr-07	0.31	0.07	0.80	-0.05	0.31	6.92	7.05	0.85	5.82	0.58	7.81	8.01	0.41	6.91	0.33
May-07	0.92	0.97	0.97	0.85	0.33	7.41	7.74	0.98	6.32	0.72	7.83	8.10	0.42	6.89	0.35
Jun-07	0.20	-0.09	0.46	0.92	0.20	7.00	7.12	0.67	6.30	0.61	7.77	8.07	0.41	6.87	0.36
Jul-07	1.01	1.08	0.39	1.10	0.26	6.37	6.53	0.67	6.31	0.62	7.66	7.99	0.41	6.84	0.36
Aug-07	1.32	1.46	0.45	1.06	0.23	6.45	6.61	0.60	6.40	0.55	7.45	7.80	0.40	6.74	0.38
Sep-07	2.13	2.32	0.57	1.54	0.23	8.37	8.89	0.79	7.57	0.57	7.43	7.84	0.40	6.75	0.41
Oct-07	1.23	1.27	0.58	0.90	0.17	9.31	9.79	0.85	8.27	0.61	7.54	7.99	0.41	6.85	0.44
Nov-07	0.14	-0.15	0.59	0.40	0.26	8.67	8.44	0.93	8.00	0.67	7.60	7.97	0.41	6.90	0.45
Dec-07	0.58	0.74	0.49	0.85	0.23	8.79	8.29	1.29	8.35	0.77	7.60	7.84	0.40	6.92	0.47
Jan-08	1.91	1.94	0.35	1.58	0.25	11.86	11.62	1.20	10.29	0.82	8.04	8.21	0.42	7.27	0.51
Feb-08	0.49	0.18	0.55	0.13	0.41	11.25	10.50	1.36	9.46	0.88	8.36	8.42	0.43	7.50	0.54
Mar-08	3.08	3.22	0.88	2.35	0.34	14.12	13.75	1.54	11.64	0.86	8.91	8.89	0.46	7.95	0.56
Apr-08	3.04	3.17	1.18	2.60	0.43	17.21	17.28	1.05	14.29	0.73	9.78	9.74	0.50	8.67	0.57
May-08	2.69	2.79	0.83	2.73	0.41	19.27	19.40	1.36	16.17	0.87	10.78	10.74	0.55	9.55	0.58
Jun-08	2.10	1.91	0.35	2.17	0.24	21.53	21.79	1.38	17.43	0.92	12.00	11.97	0.61	10.53	0.59
Jul-08	3.34	3.33	0.44	3.03	0.39	24.33	24.49	1.33	19.36	0.79	13.51	13.48	0.69	11.67	0.60
Aug-08	2.14	2.33	0.53	1.84	0.29	25.33	25.57	1.32	20.13	0.76	15.10	15.09	0.77	12.86	0.61
Sep-08	0.97	0.86	0.41	0.79	0.20	23.91	23.78	1.19	19.38	0.71	16.42	16.34	0.84	13.86	0.62
Oct-08	2.12	2.15	0.57	1.72	0.26	25.00	24.85	1.35	20.19	0.79	17.75	17.62	0.90	14.84	0.64
Nov-08	-0.12	-0.29	0.53	0.62	0.40	24.68	24.68	1.10	20.41	0.66	19.09	18.99	0.97	15.89	0.63
Dec-08	-0.50	-0.25	0.48	0.23	0.17	23.34	23.45	0.98	19.79	0.59	20.29	20.24	1.04	16.84	0.60
Jan-09	-0.42	-0.55	0.39	-0.13	0.31	20.52	20.43	0.75	18.08	0.55	20.97	20.94	1.07	17.44	0.56
Feb-09	0.95	1.22	0.34	1.15	0.23	21.07	21.69	1.13	19.10	0.70	21.75	21.84	1.12	18.19	0.55
Mar-09	1.37	1.94	0.70	0.95	0.25	19.07	20.17	1.83	17.69	0.87	22.11	22.34	1.15	18.59	0.54
Apr-09	1.41	1.10	0.85	0.98	0.29	17.19	17.76	1.10	16.07	0.75	22.04	22.32	1.14	18.68	0.52
May-09	0.23	0.04	0.82	0.40	0.40	14.39	14.61	0.88	13.74	0.69	21.55	21.81	1.12	18.39	0.48
Jun-09	0.99	0.70	0.36	1.32	0.27	13.14	13.25	0.84	12.89	0.73	20.77	21.01	1.08	17.90	0.45
Jul-09	1.54	1.61	0.38	1.30	0.22	11.17	11.37	0.85	11.17	0.72	19.60	19.84	1.02	17.14	0.44
Aug-09	1.70	1.66	0.51	1.30	0.27	10.69	10.64	0.82	10.63	0.71	18.33	18.38	0.94	16.29	0.45
Sep-09	0.45	0.38	0.38	0.39	0.19	10.12	10.11	0.78	10.23	0.68	17.15	17.05	0.87	15.49	0.48
Oct-09	0.95	0.89	0.45	0.66	0.15	8.87	8.74	0.66	9.17	0.63	15.79	15.61	0.80	14.56	0.51
Nov-09	1.39	1.27	0.48	1.35	0.22	10.51	10.45	0.69	9.90	0.55	14.65	14.35	0.74	13.69	0.51
Dec-09	-0.49	-0.36	0.51	0.04	0.23	10.52	10.32	0.72	9.72	0.56	13.65	13.19	0.68	12.87	0.51
Jan-10	2.42	2.47	0.31	2.28	0.29	13.68	13.68	0.81	12.13	0.58	13.15	12.59	0.65	12.42	0.52
Feb-10	0.39	0.37	0.28	0.58	0.22	13.04	12.73	0.77	11.56	0.58	12.57	11.84	0.61	11.87	0.52
Mar-10	1.25	1.29	0.22	0.99	0.16	12.91	12.01	1.09	11.61	0.66	12.12	11.15	0.57	11.42	0.51
Apr-10	1.73	2.07	0.71	1.85	0.42	13.26	13.09	0.81	12.48	0.65	11.84	10.76	0.55	11.16	0.51
May-10	0.06	-0.19	0.76	0.17	0.20	13.07	12.83	0.79	12.24	0.64	11.75	10.57	0.54	11.06	0.51
Jun-10	0.65	0.34	0.37	0.78	0.17	12.69	12.43	0.74	11.70	0.62	11.73	10.43	0.54	11.01	0.51

Source: Authors' calculations, except the official inflation rate for which the source is Pakistan Bureau of Statistics.

Table A2

Group-wise Change in Relative Price (July 01–June 10)

Group	Weight in CPI Basket	Month on Month Inflation			Headline (Year on Year)			12-month moving average		
		Change in Relative Price (%)	Change in Relative Price (%)	S.E.	Change in Relative Price (%)	Change in Relative Price (%)	S.E.	Change in Relative Price (%)	Change in Relative Price (%)	S.E.
Food Beverages & Tobacco	0.403	0.14	0.18	0.001	1.62	1.72	0.01	1.47	1.72	0.01
Apparel, Textile & Footwear	0.061	-0.26	-0.21	0.004	-3.38	-3.15	0.09	-3.18	-3.22	0.07
House Rent	0.234	-0.06	-0.01	0.001	-0.51	-0.36	0.02	-0.55	0.04	0.02
Fuel & Lighting	0.073	0.03	-0.18	0.003	0.73	0.37	0.08	0.99	-0.44	0.06
Household Furniture & Equipment	0.033	-0.24	-0.18	0.008	-2.89	-2.37	0.18	-2.59	-2.16	0.14
Transport & Communication	0.073	-0.06	-0.14	0.003	-0.34	-1.06	0.08	-0.09	-1.37	0.06
Recreation & Entertainment	0.008	-0.45	-0.51	0.035	-6.20	-6.33	0.75	-6.17	-6.78	0.55
Education	0.035	-0.16	-0.20	0.008	-1.77	-1.48	0.75	-1.87	-1.58	0.13
Cleaning, Laundry & Personal Appearance	0.059	-0.17	-0.17	0.005	-2.08	-2.21	0.01	-2.04	-2.14	0.07
Medicare	0.021	-0.28	-0.27	0.140	-3.77	-2.86	0.29	-3.34	-3.42	0.22

Source: Authors' calculations.

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