HOUSEHOLD BUDGET ANALYSIS FOR PAKISTAN UNDER QUADRATIC SPLINES

Eatzaz Ahmad, Professor of Economics, QAU, Islamabad
Muhammad Arshad, Ph.D. student Shangai University, Shangai

1. INTRODUCTION

- The relationship between total expenditure and the expenditure on a specific consumer good or Engel curve is the starting point for the analysis of household budgets.
- In Pakistan various studies have analyzed household budget.
- Bussink (1970), Khan (1970), Ali (1985) Malik (1982), Siddiqui (1982), Cheema and Malik (1984, 1985), Ahmad and Ludlow (1987), Malik *et.al*. (1987, 1988, 1993), Alderman (1988), Ahmed and Malik (1989), Arshad (1990), Burney and Akhtar (1990), Burney and Khan (1991), Bouis (1992), Aziz (1997), Burki (1997), Chaudhary *et al*. (1999), Shamim (1999), Shamim and Ahmad (2007)
- Recent studies employ relatively flexible functions, which however, remains confined to a turn or two in curvature.
- Large samples available in survey data allow piece-wise estimation to allow shifts in curves.
- But the Engel curves become discontinuous at shift points. For smoothness appropriate restrictions are needed.
- The use of shift dummies along with smoothing restriction results in the functions that are called Spline functions.
- If Engel equations contain enough parameters, additional smoothing can also be obtained.

- Provision of sufficient flexibility in the shape of Engel curves is the crucial benefit of Spline Engel equations.
- Spline functions have been used in economic modeling:
- Money demand function (Barth, Kraft and Kraft, 1976)
- Effects of money supply and inflation on interest rate (Suits *et. al.*, 1977)
- Household budget analysis (Karunakaran and Ahmad, 1996 and Ahmed and Karunakaran, 1997, Shamim and Ahmad, 2007)
- This study proposes a fairly comprehensive procedure for estimating Spline Engle equations.
- The Spline functions are derived for the *Quadratic Expenditure System*
- An algorithm is proposed to select the number and position of the points of shifts, known as knots.
- The empirical analysis is carried out separately for rural and urban on micro level household data of HIES 2000-01.
- Consumption expenditure is classified into 12 food categories and 10 non-food categories.

2. SPLINE ENGLE EQUATIONS

- *Quadratic Expenditure System* (Pollak and Wales, 1979):

$$E_i = \alpha_i + \beta_i TE + \gamma_i TE^2 + U_i$$

Extension to Quadratic Splines (Ahmad and Karunakaran, 1997)

- Shift dummies at m arbitrary shift points, known as knots:

  $D_i = 1$ for $TE \geq TE_i$ and $= 0$ otherwise.

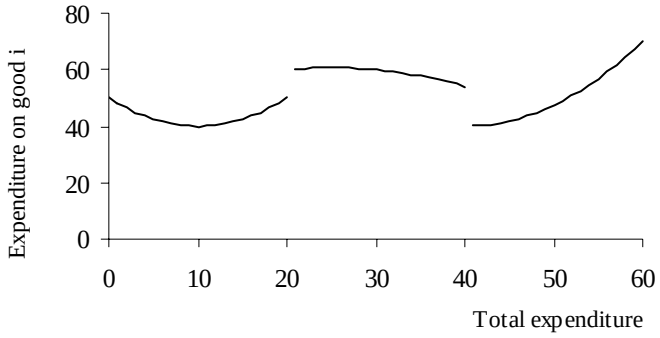
- Parameters can be varied across knots as

$$\alpha_i = \alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} D_k \ , \ \beta_i = \beta_{i0} + \sum_{k=1}^{m} \beta_{ik} D_k \ , \ \gamma_i = \gamma_{i0} + \sum_{k=1}^{m} \gamma_{ik} D_k$$

- Resulting Engle equations

$$E_i = \left(\alpha_{i0} + \sum_{k=1}^{m} \alpha_{ik} D_k \right) + \left(\beta_{i0} + \sum_{k=1}^{m} \beta_{ik} D_k \right) TE + \left(\gamma_{i0} + \sum_{k=1}^{m} \gamma_{ik} D_k \right) TE^2 + U_i$$

Figure 1: Piece-Wise Engle Equation with Discontinuous Level and Discontinuous Slope



- Conditions for continuity at knots:

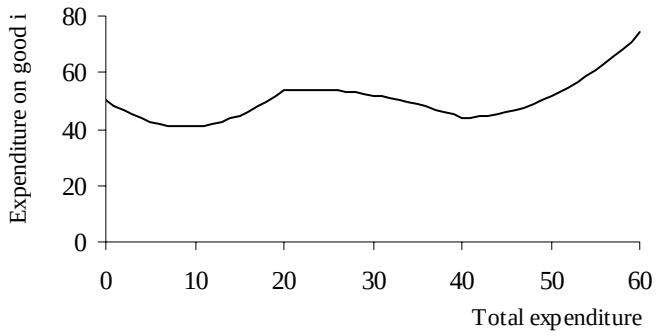
$$\underset{TE \to TE_k^-}{Limit}(E_k) = \underset{TE \to TE_k^+}{Limit}(E_k)$$

- Resulting Engle equations:

$$E_i = \left[\alpha_{i0} + \sum_{k=1}^{m} \left(-\beta_{ik} TE_k - \gamma_{ik} TE_k^{\,2} \right) D_k \right] + \left(\beta_{i0} + \sum_{k=1}^{m} \beta_{ik} D_k \right) TE + \left(\gamma_{i0} + \sum_{k=1}^{m} \gamma_{ik} D_k \right) TE^2 + U_i$$

$$= \alpha_{i0} + \beta_{i0} TE + \gamma_{i0} TE^2 + \sum_{k=1}^{m} \beta_{ik} \left(TE - TE_k \right) D_k + \sum_{k=1}^{m} \gamma_{ik} \left(TE^2 - TE_k^{\,2} \right) D_k + U_i$$

Figure 2: Engle Equation with Continuous Level But Discontinuous Slope



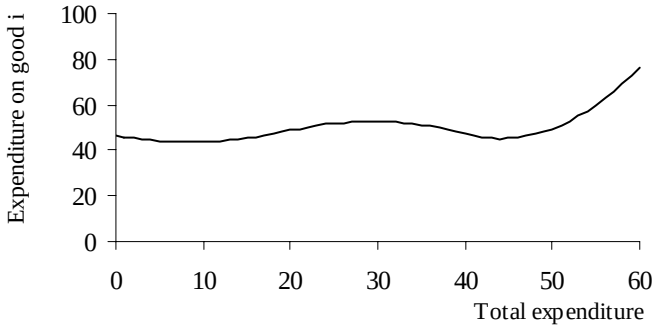
- Conditions for continuity of slopes at knots:

$$\underset{TE \to TE_k^-}{Limit}\left(\frac{\partial Ei}{\partial TE}\right) = \underset{TE \to TE_k^+}{Limit}\left(\frac{\partial Ei}{\partial TE}\right)$$

- Resulting Engle equations:

$$E_i = \alpha_{i0} + \beta_{i0} TE + \gamma_{i0} TE^2 + \sum_{k=1}^{m} \gamma_{ik} \left(TE - TE_k\right)^2 D_k + U_i$$

Figure 3: Piece-Wise Engle Equation with Continuous Level and Continuous Slope



3. DATA AND ESTIMATION PROCEDURE

- Micro level household budget data *HIES 2000-01*
- 14536 households: 9090 rural and 5446 urban

Table 1. Households' Demographic Categories

Variable Name	Age Limit	Notation
Number of babies	Age < 2	BAB
Number of toddlers	2 ≤ Age ≤ 5	TOD
Number of female children	5 ≤ Age ≤ 15	FCH
Number of male children	5 ≤ Age ≤ 15	MCH
Number of female adults	15 ≤ Age ≤ 60	FAD
Number of male adults	15 ≤ Age ≤ 60	MAD
Number of elderly	Aged > 60	ELD

- The numbers of household members belonging to various categories are included directly into the Spline functions as additional variables.
- Engle equations in expenditure share form:

$$E_i / TE_i = \alpha_{i0} TE^{-1} + \beta_{i0} + \gamma_{i0} TE + \sum_{k=1}^{m} \gamma_{ik} \left[\left(TE - TE_k \right)^2 / TE \right] D_k$$
$$+ \phi_{i1} BAB + \phi_{i2} TOD + \phi_{i3} FCH + \phi_{i4} MCH + \phi_{i5} FAD + \phi_{i6} MAD + \phi_{i7} ELD + V_i$$

- Determination of the number and locations of knots.
- The exact location of the knots is not very important because the general shape of the Spline function remains quite flexible and it adjusts in the light of the location.
- However, the number of knots chosen can matter in determining the shapes of the Engle curves obtained.
- The earlier studies (e.g. Ahmad and Karunakaran, 1997 and Karunakaran and Ahmad, 1996 and Shamim and Ahmad 2007) chose arbitrary number of knots and base the location of the knots on qualitative assessment e.g. poverty line, average per capita income, etc.
- In system framework the location of knots for all the Engle equations must be the same.

- Algorithm:

1. Sort data in ascending order with respect to total expenditure.

2. Estimate n-2 systems of Engle equations, allowing shift at the observations number m by setting m = 2,…, n-1 alternatively.

3. Consider the system that yields maximum value of log-likelihood function. If shift at the chosen point is statistically insignificant then there is no shift in the system and the algorithm ends. If the shift is significant, the search yields one knot. Let this knot be at the observation number n_1.

4. Repeat the above procedure to search for a knot from observations 2 to n_1-1 and another one from observations n_1+1 to n-1.

5. Suppose the likelihood function attains maximum value with the shifts at observations n_a and n_b, where $n_a < n_1 < n_b$. If both the shifts are insignificant, the algorithm ends and there is only one knot at the observation n_1. If the shift at n_a (n_b) is insignificant and the one at n_b (n_a) is significant then there is no knot in the range of observations from 1 to n_1 (n_1 to n) while the point n_b (n_a) represents a knot and the search continues over the ranges of observations n_1 to n_b and n_b to n (1 to n_a and n_a to n_1). In case both the shift at n_a and n_b are significant, both are considered at the knots and the search is continued in the ranges 1 to n_a, n_a to n_1, n_1 to n_b and n_b to n.

6. Continue the above procedure till no more knots are obtained.

- The search of knots at each step assumes that the knots already chosen are at the right location.
- This introduces some degree of arbitrariness and there is a need for fine-tuning.
- Thus further search is carried out in the neighborhood of each knot following the above procedure.
- If two knots come very close to each other or very close to the beginning or end of the data, attempt is made to choose one knot and more rigorously search is carried out involving the location of more than one knot together in a single sequence.

4. RESULTS AND DISCUSSION

- Among rural households total expenditure elasticities of all the 22 commodity groups are positive at all levels of income.
- The same result holds for urban households except that wheat is considered ‘inferior good’ among the middle-income class.
- Most of the Engle curves for non-food items get steeper, while most of the curves for food items flatten out as the level of total expenditure increases.
- Although in most cases urban and rural households are observed to have similar consumption pattern, there is a clear contrast of preferences when it comes to the matter of health.
- While urban households regard wheat, housing and health as absolute necessities, rural households consider wheat, housing and tobacco, besides clothing & footwear, as absolute necessities.
- Likewise dairy products, which are expected to be more abundantly available is rural areas than in urban areas, are considered a luxury consumption item for poor to middle income rural households, while in the urban sample this pattern holds for poor households only.
- The Engle curves for high protein and high-energy foods like sugar, edible oils, meats, poultry & fish and dairy products are by-and-large steeper, indicated higher total expenditure elasticities, for rural sample as compared to the urban sample.
- These results show that urban households tend to have stronger preferences for the goods that have higher health value like health care, dairy, sugar, edible oils, meats, poultry and fish.

- With the average income levels of urban households also better than those of rural households, this preference structure has undesirable implications for health of the poor.

- The flexibility of Engle curves obtained through Spline specification unveils many interesting patterns.

- Wile wheat is treated as ‘inferior’ among middle-income urban households, it turns into ‘normal’ among the rich households, though its expenditure elasticities remains quite low.

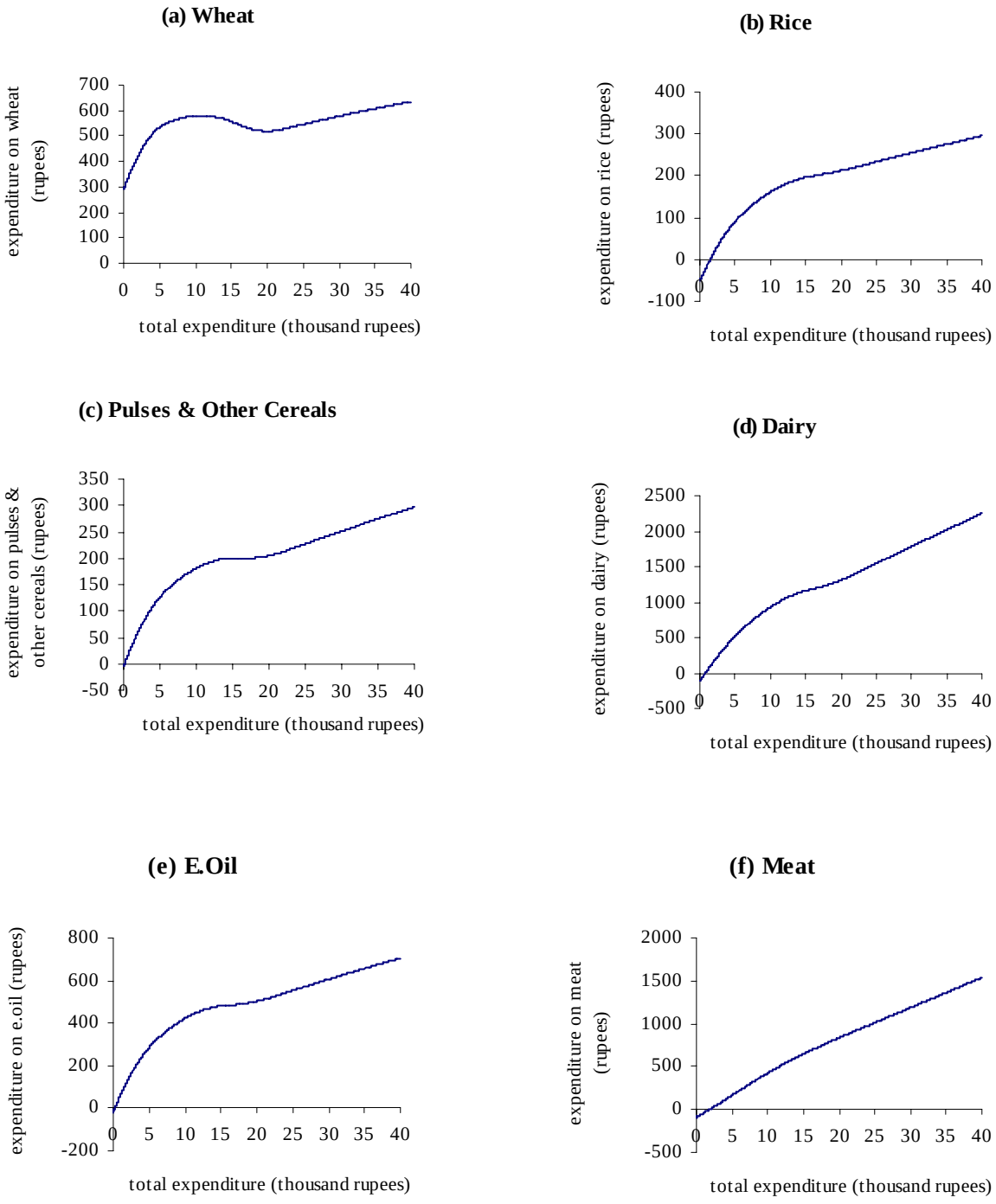
- Some of the consumption items that uncharacteristically turn from necessities into luxuries at very high levels of total expenditure include entertainment, personal care and durables among urban households and dairy among the rural households.

- The changing slopes and curvatures of Engle curves suggest that even a uniform tax structure, e.g. in the form of GST will have varying implications for budget allocation and welfare of households belonging to different income classes.

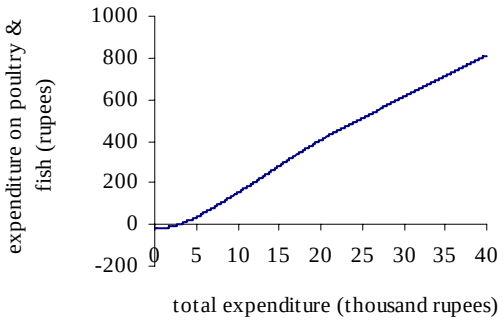
- Although practically it may be difficult to design a tax structure with varying tax rates, one may at least be informed of the distributional implications of a given tax system in order to propose supplementary transfer measures.

- A detailed analysis of taxation structure on these lines is our future research agenda.

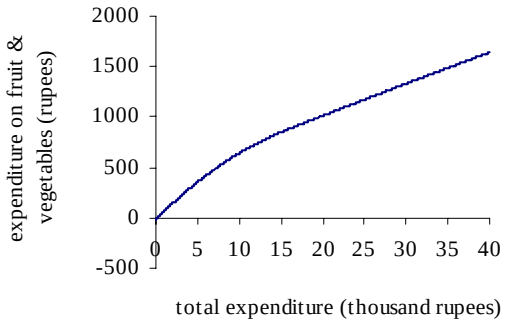
Figure 1: Spline Curves for Urban Pakistan



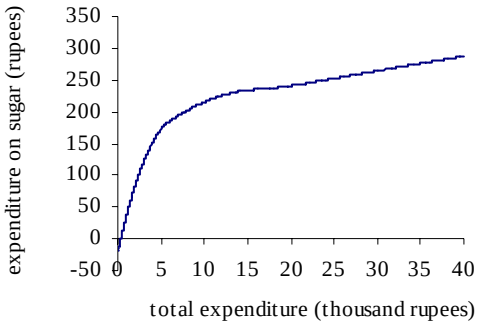
(g) Poultry & Fish



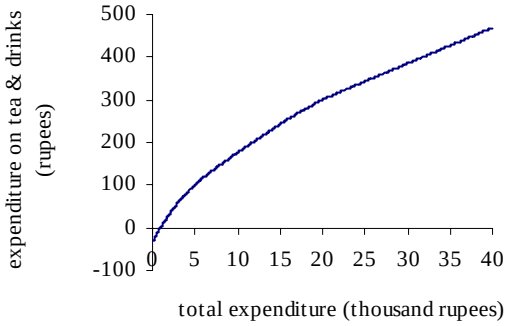
(h) Fruit & Vegetables



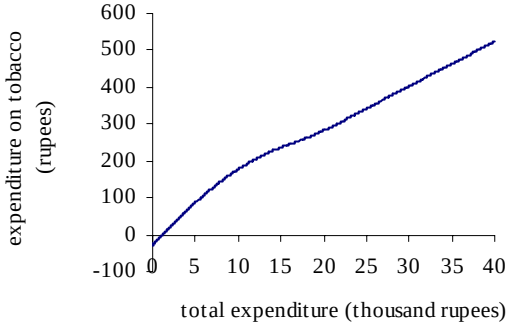
(i) Sugar



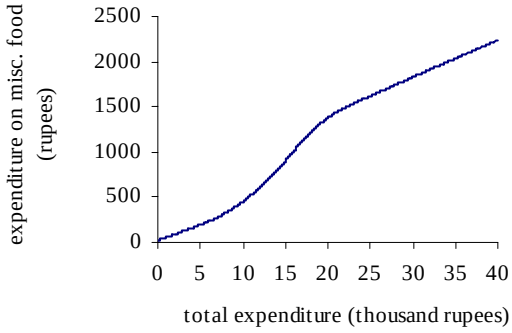
(j) Tea & Drinks



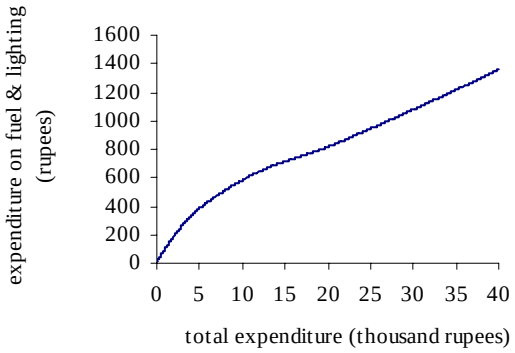
(k) Tobacco



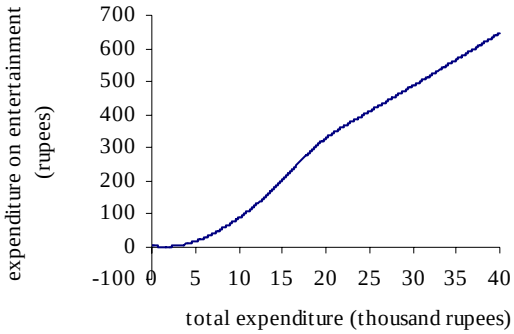
(l) Misc. Food



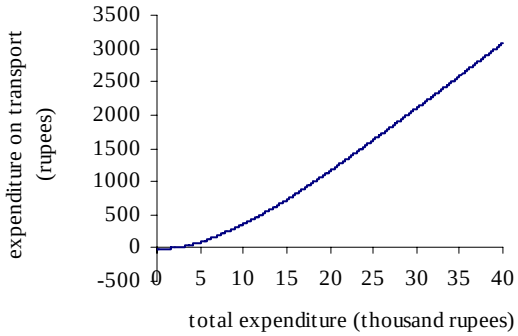
(m) Fuel & Lighting



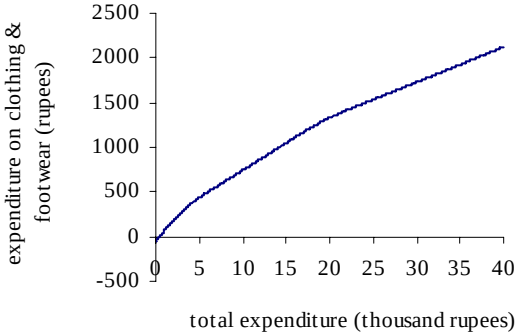
(n) Entertainment



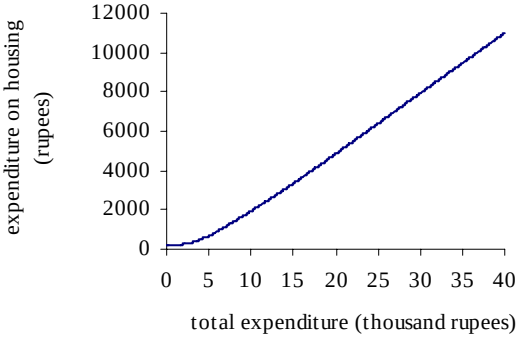
(o) Transport



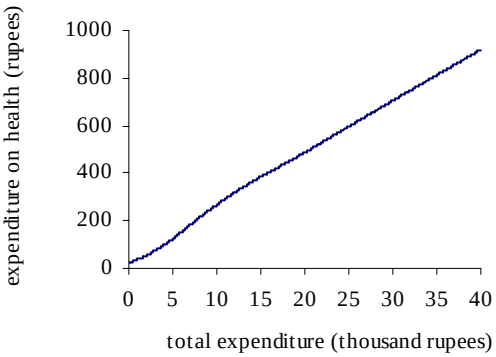
(p) Clothing & Footwear



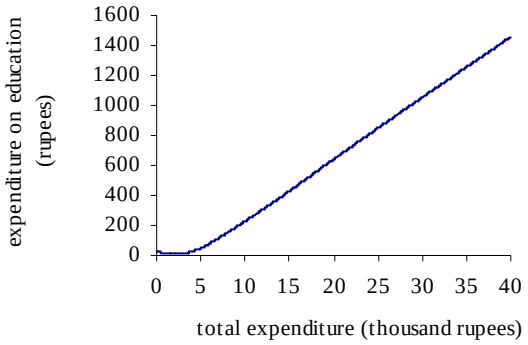
(q) Housing



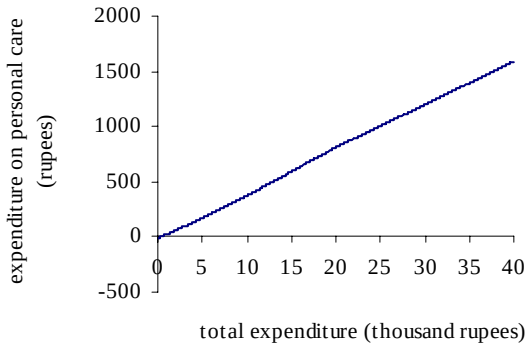
(r) Health



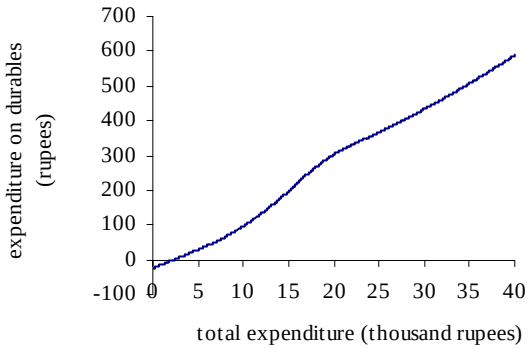
(s) Education



(t) Personal Care



(u) Durables



(v) Misc.Non-food

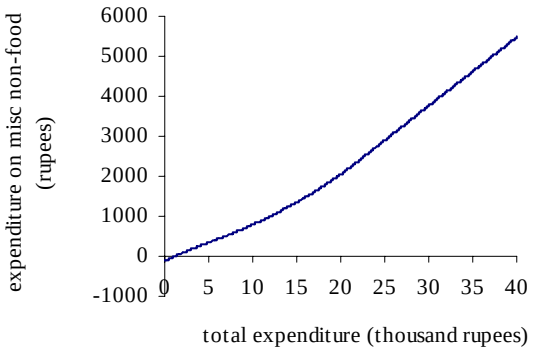
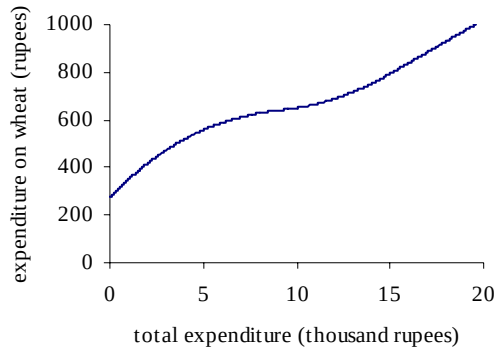
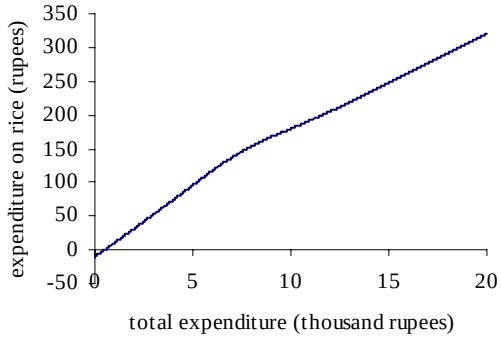


Figure 2: Spline Curves for Rural Pakistan

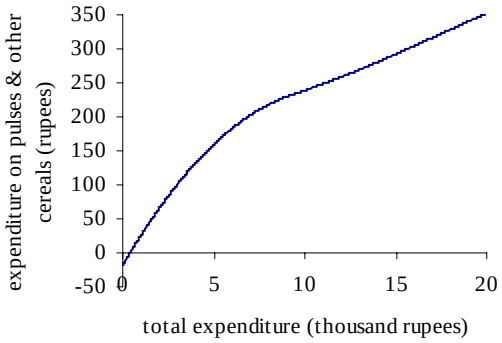
(a) Wheat



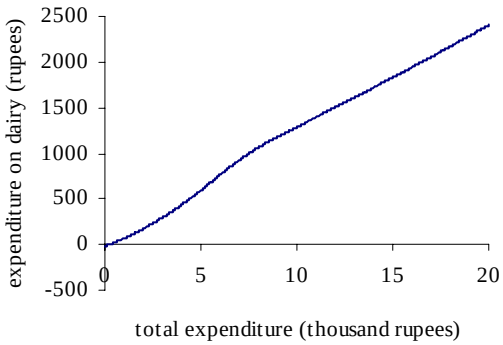
(b) Rice



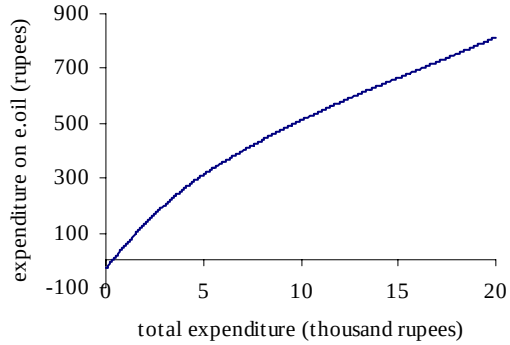
(c) Pulses & Other Cereals



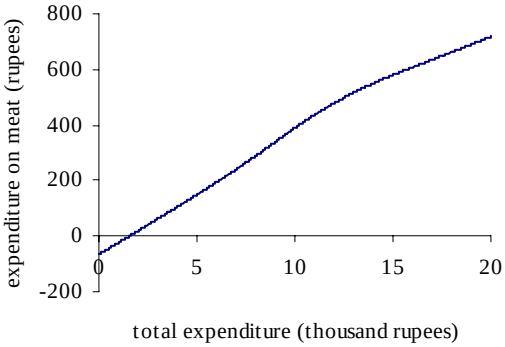
(d) Dairy



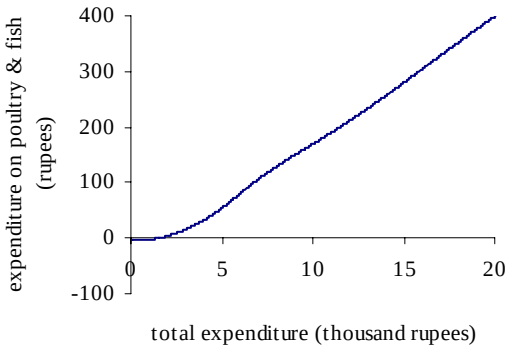
(e) E.Oil



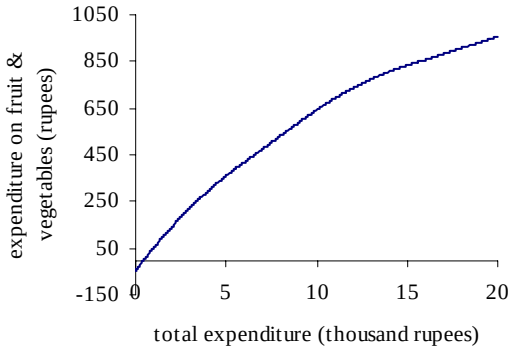
(f) Meat



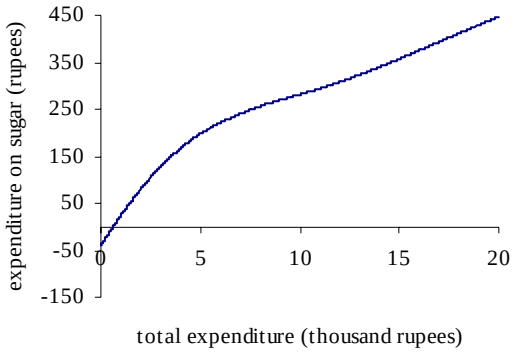
(g) Poultry & Fish



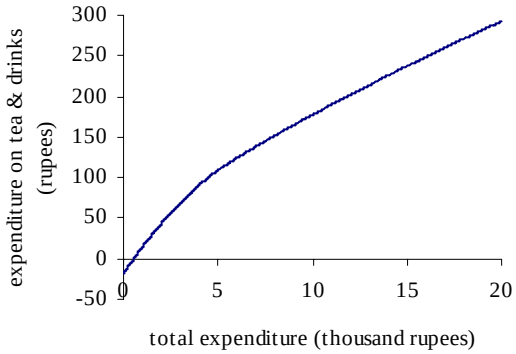
(h) Fruit & Vegetables



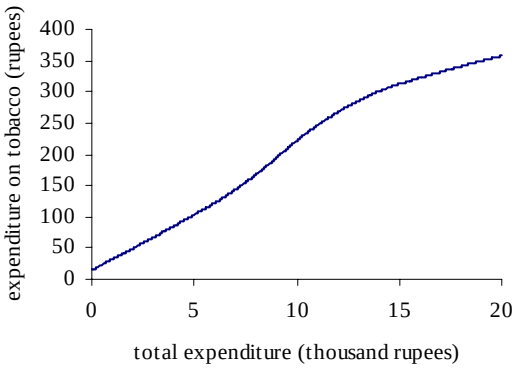
(i) Sugar



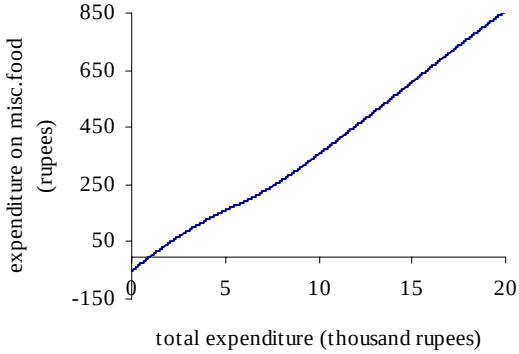
(j) Tea & Drinks



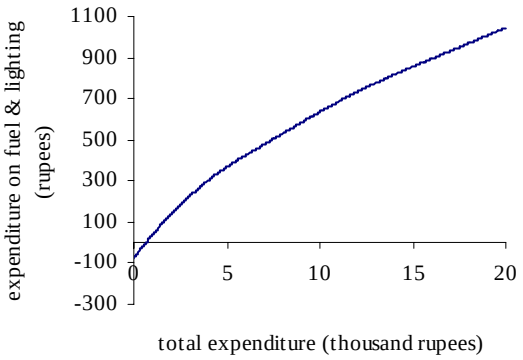
(k) Tobacco



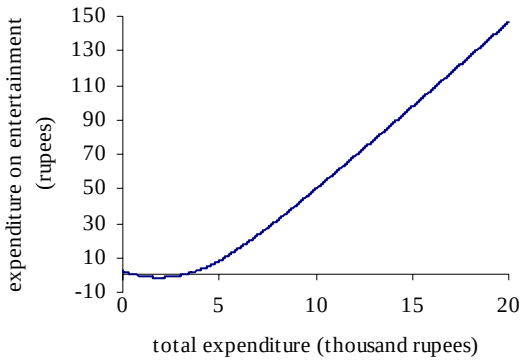
(l) Misc. food



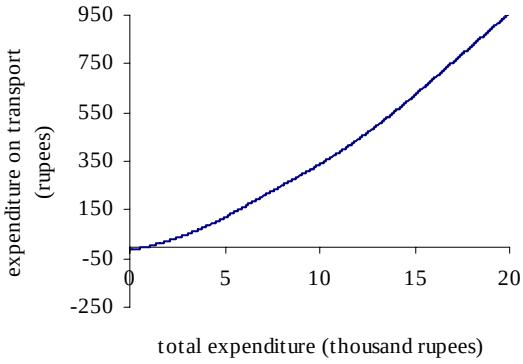
(m) Fuel & Lighting



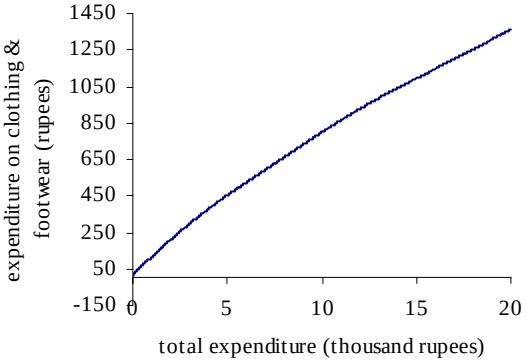
(n) Entertainment



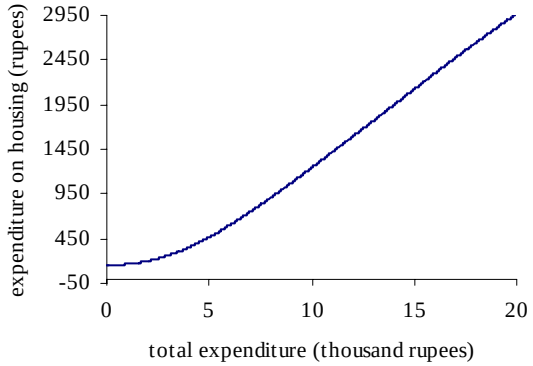
(o) Transport



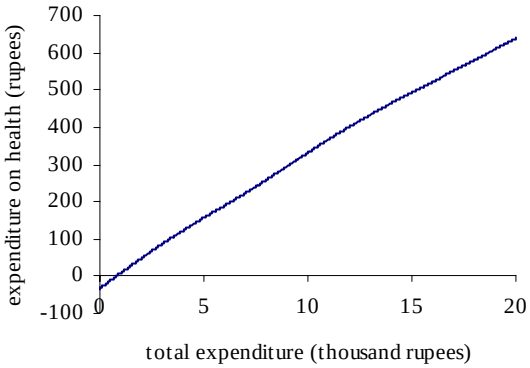
(p) Clothing & Footwear



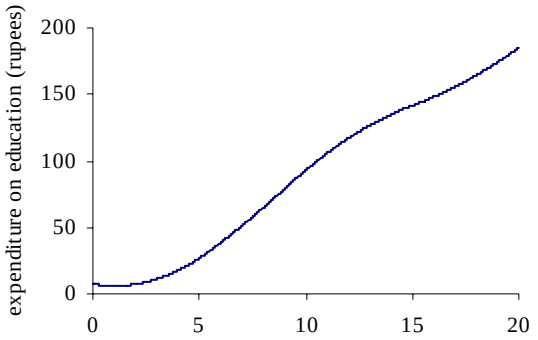
(q) Housing



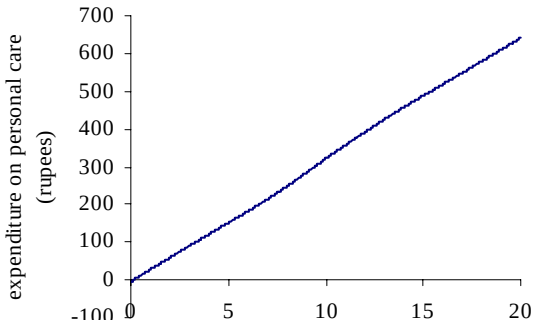
(r) Health



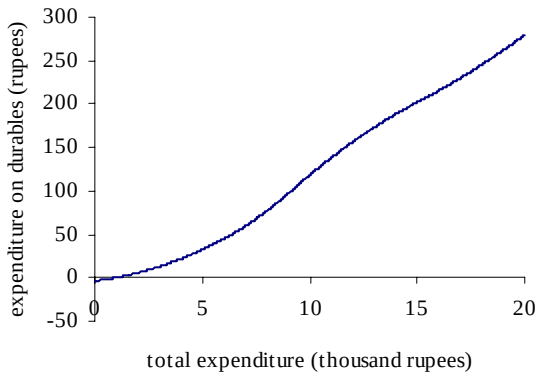
(s) Education



(t) Personal Care



(u) Durables



(v) Misc. Non-food