INFLATION

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Inflation in Pakistan: High-Frequency Estimation and Forecasting

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ABSTRACT

I begin by motivating the utility of high-frequency inflation estimation and reviewing recent work done at the State Bank of Pakistan for inflation forecasting and now-casting GDP using machine learning (ML) tools. I also present stylised facts about the structure of historical and especially recent inflation trends in Pakistan.

However, since the available data *and* already used methods cannot achieve high-frequency forecasting, I discuss three novel techniques from recent literature, including *web scrapping*, *scanner data* and *synthetic data*. Due to a lack of access to Pakistan's scanner and web-scrapped data, I generate synthetic data using *generative* ML models (Gaussian Copula and PAR models) and *numerical analysis* (cubic spline interpolation) methods. I use cubic splines to estimate the monthly inflation rate from quarterly data and unknown high frequency, weekly inflation rate from actual monthly data. Meanwhile, I use a probabilistic autoregressive ML model to forecast future short-run inflation for Pakistan from 2020 to 2023.

I evaluate the accuracy of ML forecasts by comparing them with forecast error variances and predictions from conventional reduced form vector autoregressive models (VAR)[1]

JEL Classifications: E30, E31, E32, E37, E47, E52, E58, C53

Keywords: High Frequency Inflation Estimation and Forecasting, Forecast Accuracy, Synthetic Data, Machine Learning, Hyperinflation, Forecasts of Inflation in Pakistan, VAR Models, Web Scrapping and Scanner Data.

1 The replication code for this paper (Python, R and Julia) and the original LaTeX version is available on my github page: https://github.com/sonanmemon/High-Frequency-Inflation-Forecasting.

MOTIVATION

Accurate inflation forecasting is a concern for market players, central banks, and governments. The market participants want to update their inflation expectations in line with new information revelations to optimise their investment strategies. Meanwhile, central banks typically have mandates for price stability, (see Cukierman, et al. 1992), and they routinely collect data on inflation expectations and forecast inflation. Hyperinflation episodes dramatically hurt hand-to-mouth households. This extreme economic turmoil has political consequences for governments, especially when the election period is a few quarters away (see Binder, 2021).

Governments are incentivised to prioritise inflation control and interfere with central bank independence in these election periods. There is well-known classic literature on Nordhaus's so-called political business cycle (1975). For instance, Abrams and Butkiewicz (2012) revealed evidence from the Nixon tapes that President Nixon manipulated Arthur Burns[1] and the Federal Reserve into creating a political business cycle which helped ensure his reelection victory in 1972.

While Nixon understood the risks his monetary policy imposed, he chose to trade longer-term economic costs to the economy for his short-term political profits. At the most fundamental level, hyperinflation episodes are humanitarian and social crises which can be addressed to some extent if we develop better inflation forecasting methods and independent central banks with mandates for price stability.

While central banks collect data on consumer price indices, the collection frequency does not allow accounting for sudden swings in inflation *and* inflation expectations. Some standard measures include the HICP (Harmonized Consumer Price Index) data used in the Euro area and the CPI (consumer price index) data from the USA. Such data typically tends to be quarterly or monthly in worst cases or best cases, but results are revealed in the next month after collection.

However, for instance, in a few days and weeks, news about the Ukraine crisis changed the inflation landscape for many products; conventional price indices had little forecasting potential for the next inflation crisis. Similarly, inflation shocks can result from sudden change of central bank's governors or government change, terrorism episodes or political turmoil, especially in developing economies, where inflation tends to more volatile and central banks are less independent (see Vuletin and Zhu, 2011).

RESEARCH AT SBP

The State Bank of Pakistan's (SBP) research department has done some work on inflation forecasting by using machine learning methods (e.g. Neural Networks) and monthly year-on-year (YoY) inflation rates of Pakistan from Jan 1958 to Dec 2017 Hanif

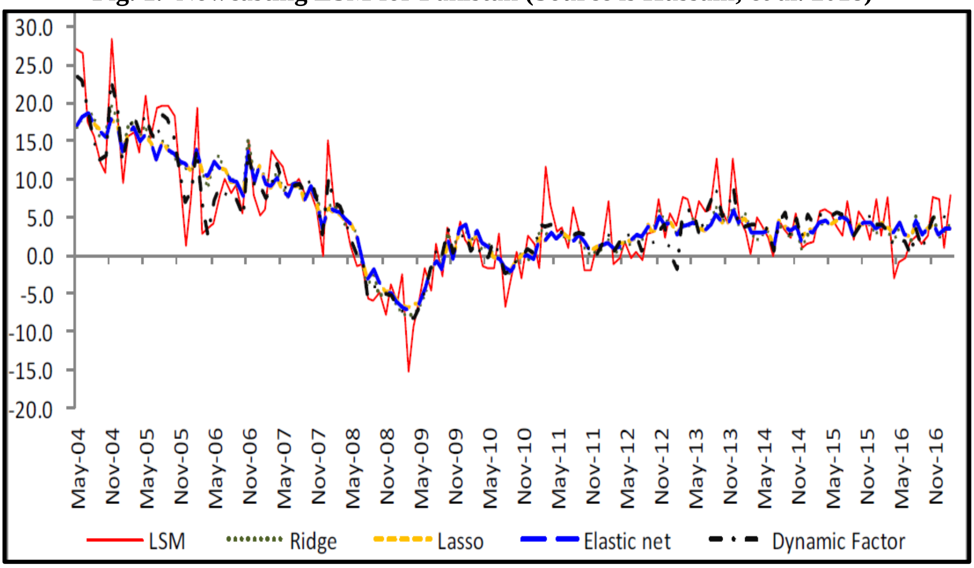
[1]Former Head of the Federal Reserve Bank in USA.

et al. (2018). The *Thick ANN* (Artificial Neural Networks) model developed in this paper is found to outperform all the 15 econometric models of Pakistan's economy previously developed in forecasting 24 months' headline inflation.

Similarly, the SBP's research team has worked on *nowcasting* GDP using data on large-scale manufacturing growth (LSM) in Pakistan (see Hussain, et al. 2018) and LASSO type[2] ML methods. The models extract unique *information* from various variables closely associated with LSM in Pakistan.

The results in Figure 1 below from Hussain, et al. (2018) reveal that the predicted LSM series closely tracks the actual LSM series. Since LSM is available at a relatively higher frequency (monthly) relative to the actual GDP (annual), it is a predictor for determinants of economic activity such as key sectors, prices, credit, interest rates and tax collection, external trade and inflows. This is in line with emerging methodologies among central banks worldwide, which are all moving toward big data and machine learning methods (see Doerr, et al. 2021).

Fig. 1. Nowcasting LSM for Pakistan (Source is Hussain, et al. 2018)



However, the lack of availability of high-frequency data on the order of weeks and months, poses a limitation on forecasting inflation. Hence, I argue that we need more *granular* data for enhancing forecasting.

Next, I discuss methods for collecting such high-frequency data, currently used at the current frontier of research on inflation in economics.

REVIEW OF MODERN FORECASTING

In this section, I will briefly review three methods, including *web scrapping* inflation data, using *scanner data* from supermarkets and *synthetic* or artificial data for high-frequency inflation forecasting.

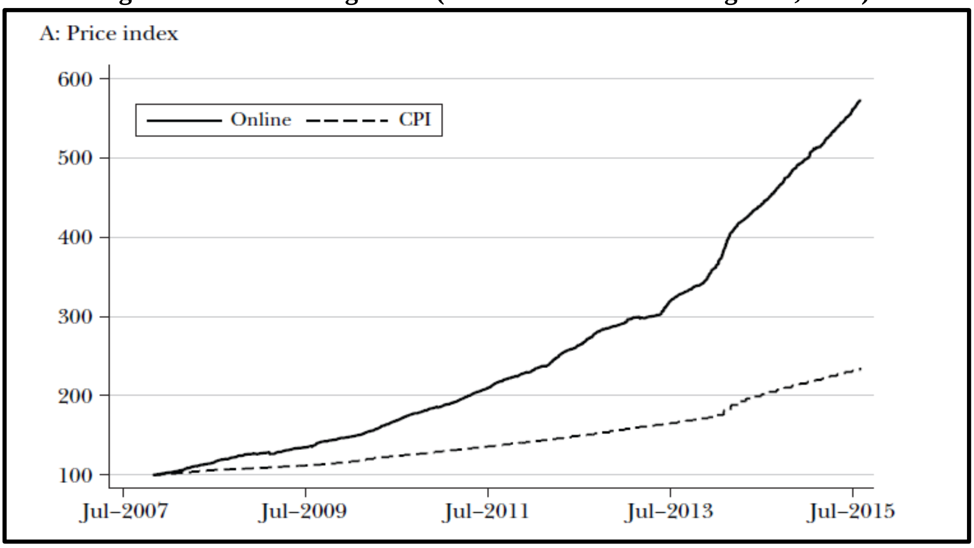
[2] Least Absolute Shrinkage Operator, Ridge Regressions and Elastic Nets.

WEB SCRAPPING

In recent literature, the daily consumer price index (CPI) produced by the Billion Prices Project (BPP CPI) of Cavallo and Rigobon (2016) offers a glimpse of the direction taken by consumer price inflation in *real-time*. For instance, Figure 2 is based on web scrapping online inflation data for Argentina (see Cavallo and Rigobon, 2016). It shows that the official CPI significantly understated actual inflation, when measured by web scrapping. An added benefit of such data is that it reveals the partisan measurement and disclosure of CPI data in developing economies such as Argentina, where central bank independence is low.

Should we expect a similar lack of correspondence between official inflation data of the SBP (State Bank of Pakistan) and non-partisan research measures? Not much is known about the political business cycle in Pakistan, and I believe that independent research not originating from the SBP, is needed to address the question. Given the low levels of central bank independence (henceforth CBI) in Pakistan and existing literature on CBI (see Vuletin and Zhu, 2011) we should expect that higher levels of price stability can be achieved if governor appointments and turnovers are not manipulated by political power.

Fig. 2. Inflation in Argentina (Source is Cavallo and Rigobon, 2016)



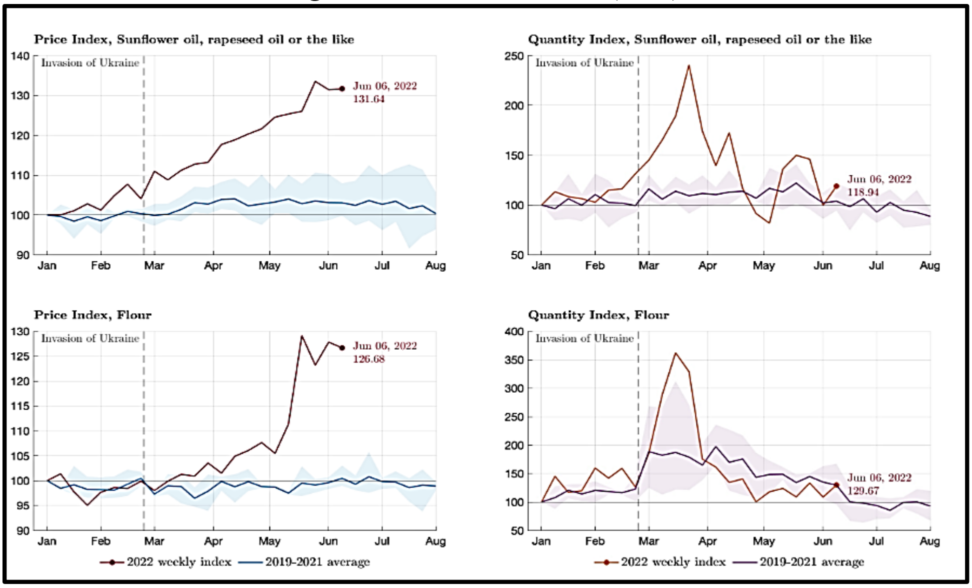
With the increasing scope of online transactions in Pakistan, *web scrapping* can also be informative, despite the absence of Amazon-type large-scale online transaction services in the country.

SCANNER DATA

Meanwhile, another branch of emerging literature uses scanner-based data (see Beck, et al. 2020) on prices rather than web-scrapping them. In Figure 3 below, recent scanner-based price indices for Germany are disclosed from the work of Beck, et al. 2022). The data shows trajectories in 2022 (red and orange solid lines below) with their historical averages from 2019 to 2021 (blue and solid purple lines), along with historical minimum and maximum values (shaded areas). The data indicates a very strong increase in sunflower oil and flour prices in light of the Ukraine conflict, accompanied by temporarily higher sales.

The price increase of sunflower oil was relatively gradual and started in early February. In contrast, flour prices increased sharply, but only more than two months after the invasion. However, in both cases, sales went far beyond their average levels, suggesting increased demand and possibly stockpiling behaviour from pessimistic consumers (see Cavallo and Kryvtsov, 2021). Concerning the more recent period up to June 2022, prices for both products seemed to have stabilised at a very high level, whereas quantities have converged to their average levels.

Fig. 3. Source is Beck, et al. (2022)



I propose high-frequency *scanner data* from supermarkets in Pakistan can improve inflation forecasting. In Karachi, Carrefour, Metro, Chase, Chase Up, Imtiaz supermarket and Bin Hashim are some major supermarkets. Similarly, Al-Fatah, Carrefour and Imtiaz supermarkets are some significant players in Lahore. However, the lack of supermarket scanner data availability is a constraint that must be overcome.

SYNTHETIC DATA

Synthetic data is artificial data (see Nikolenko, 2021) which is generated to mimic key information of the actual data and provide the ability to draw valid statistical inferences. It allows widespread access to data for analysis while overcoming privacy, confidentiality, and cost of data collection concerns (also see Raghunathan, 2021).

As proof of the pudding, relational data sets were synthetically generated and used by freelance data scientists to develop predictive models. For instance, Patki, et al. (2016) have developed the SDV (Synthetic Data Vault), which uses multivariate Gaussian Copula (see Chapter 5.1.5 of Stachurski, 2016) to calculate covariances across input columns. The distributions and covariances are sampled from the copula to produce synthetic data. The researchers found no significant difference between the results produced using the synthetic versus true data Patki, et al. (2016). For a review of other generative models using synthetic data and advanced methods such as generative adversarial networks for economists, refer to Koenecke and Varian (2020).

Synthetic data is beneficial since high-frequency inflation data is not available for Pakistan. It is also essential to state that even at a private level, State Bank of Pakistan does not have high-frequency data, so when I use the term *synthetic*, I mean artificially constructed high-frequency data from the available low-frequency data. This is in contrast with synthetic methods, which solve an information revelation problem by generating synthetic data from actual data of the same frequency (see Patki, et al. 2016).

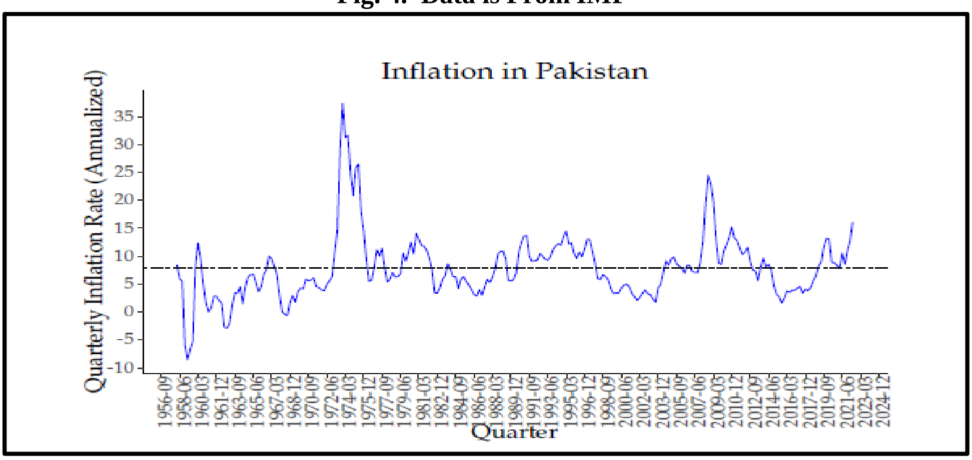
The accuracy of forecasting using *synthetic* data is questionable and using high-frequency data through scanner data, and web-scrapping is part of my long-term research agenda. This synthetic data exercise can motivate policymakers and the SBP to initiate high frequency data collection by providing a glimpse of the utility of this data.

STYLISED FACTS ON PAKISTAN'S INFLATION

In this section, I describe and visualise some historical, stylised facts about inflation in Pakistan. I present data on quarterly and monthly inflation rates from 1958 to 2022 or subsets of this maximum range. Moreover, I describe recent trends in inflation after 2018 in more detail, breaking them down across headline inflation, food inflation, core measures (non-food and non-energy), clothing, health, transport and education sectors.

In Figure 4 below, I have plotted the quarterly inflation series for Pakistan from the first quarter of 1958 to the third quarter of 2020, based on IMF data. From 1980 to 2008, the average annualised quarterly inflation rate was around 8 percent (represented by the horizontal dotted line). Pakistan had a severe hyperinflation crisis during the 1970's and other major inflation periods were during 2007-2009 (the great recession period). Relative to the disinflation observed from 2011 to 2015, an inflationary period after 2015 accelerated notably in 2018 and 2019. Thus, the upward trend of inflation had already begun before the COVID shock in 2020 and the hyperinflation crisis of 2022.

Fig. 4. Data is From IMF



The latest available consumer price index (CPI Inflation with Base Year of $2015-16=100$) data from the State Bank of Pakistan recorded inflation at 8 percent on a year-on-year basis in December 2020 (see Table 1 below) and 12.3 percent in December 2021. Moreover, the core measure of inflation, excluding food and energy (Non-food, Non-Energy (NFNE)), was recorded at 6.4 percent in December 2020 and 8.5 percent in December, 2021. Meanwhile, across various sectors, food (12.9 percent) had among the highest inflation rates, and the education sector had the lowest rates at 1.3 percent in December 2020.

In the transport sector, we observed deflation of –3.5 percent in December 2020, reflecting the demand shock due to lower mobility and the COVID crisis. On the other hand, in particular, the transport sector, followed by clothing, had among the highest inflation rates in December 2021, which reflects a massive transformation in the transport sector within one year.

Education remained among the sectors with the lowest inflation rates in December 2021.

Table 1

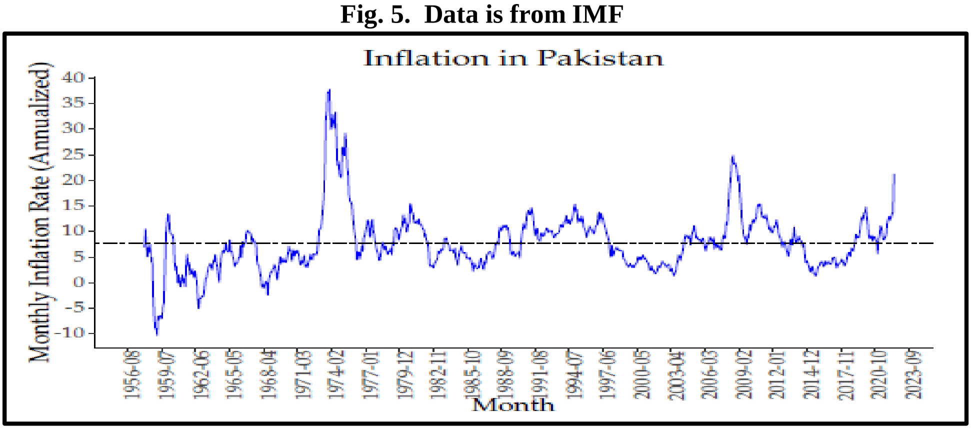
Annual National Inflation (December 2017 to December 2021)

Year on Year (%)	Dec 2017	Dec 2018	Dec 2019	Dec 2020	Dec 2021
Categories					
Headline Inflation	4.6	5.4	12.6	8	12.3
Food Inflation	3.8	0.6	17.9	12.9	10.6
Core Measure (NFNE)	5.5	7.64	7.7	6.4	8.5
Clothing	3.6	6.3	9.8	9.7	11.2
Health	10.9	7.1	11.3	8.1	9.4
Transport	4.5	18.4	14.7	–3.5	24.1
Education	12.4	9.8	6	1.3	2.8

Note: Data is from State Bank of Pakistan. Base year is 2015-2016 for all columns, apart from Dec 2017 column for which it is 2007-2008.

In Figure 5, we have monthly data for inflation in Pakistan from the IMF. The horizontal dotted line depicts mean inflation which is close to 8 percent for the whole sample. These trends are similar to the quarterly data but we can see more granular and monthly fluctuations each year from January 1958 to the sixth month (June) of 2022. With this data, we can also measure the effect of COVID and the hyperinflation crisis of 2022. As I am writing, the current inflation crisis is evolving, and Pakistani Rupee's exchange rate with respect to US Dollar is finally appreciating after a period of a sharp depreciation during August 2o22.

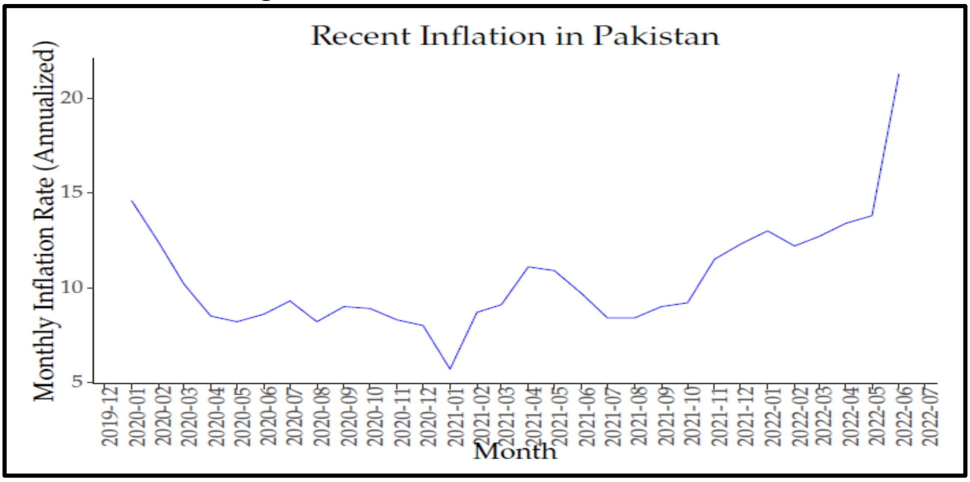
Fig. 5. Data is from IMF



Lastly, Figure 6 plots only the most recent trends in Pakistan's monthly, year-on-year inflation (headline inflation rate). The data is from the SBP and covers the months from January 2020 to June 2022. The graph indicates that during 2020, year-on-year inflation was falling despite the COVID shock. Monthly and year-on-year inflation was close to 5 percent at the beginning of 2021. However, in 2022 and especially after the debt crisis of 2022, inflation rates have skyrocketed to more than 20 percent.

In Section 1 of the appendix, I present additional data on the weighted price index, annual consumer price index and annual inflation, based on the linked GDP deflator for Pakistan.

Fig. 6. Data is from State Bank of Pakistan



METHODOLOGY

I will generate unknown, higher order synthetic series using quarterly and monthly inflation data for Pakistan from the IMF and forecast short-run, future inflation. I use machine learning methods such as multivariate *gaussian copula* (see Patki et al. (2016)) and *probability autoregressive models* for forecasting inflation. I also use *cubic spline interpolation* (numerical analysis) to estimate higher-order inflation series such as the weekly inflation series.

To evaluate these models, I use reduced form VAR[3] models to forecast future inflation in Pakistan and compare the accuracy of these forecasts with ML model forecasts. I then compare these forecasts with actual monthly inflation. Furthermore, to evaluate the accuracy of higher order interpolation from cubic splines, I first use only quarterly series to interpolate monthly inflation.

SYNTHETIC DATA FROM COPULA

A copula C in space $\mathbb{R}^n$ is a multivariate CDF (cumulative density function) supported by the unit hyperplane $[0,1]^N$ with the property that all of its marginals are

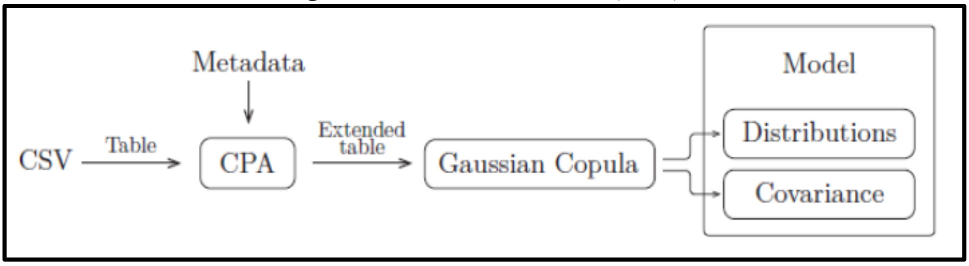
[3]Vector Autoregressive Model.

uniformly distributed on [0,1] (see Stachurski, 2016). Formally, C is the function of the form below, where $0 \leq s_n \leq 1$ and $u_n \sim U[0,1], \forall n$.

$$C(s_1, s_2, s_3, \dots, s_N) = \mathbb{P}\{u_1 \leq s_1, \dots, u_N \leq s_N\}$$

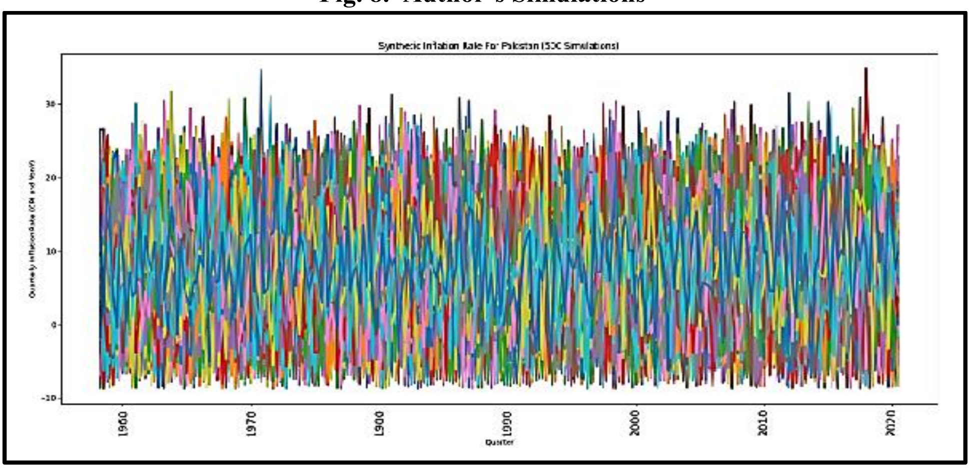
While each u_n has its marginal distribution pinned down, there can be infinitely many ways to specify the joint distribution. For instance, the independence copula, gumbel copulas and clayton copulas are some different types of joint distributions. Figure 7 represents the general structure of a generative model which uses Guassian Copula so that $F(s_1, s_2, \dots, s_N) = C(F_1(s_1), \dots F_N(s_N))$ and $F_1, F_2, \dots, F_N$ are univariate normal distributions.

Fig. 7. Source is Patki, et al. (2016)



In Figure 8 below, I use Gaussian Copula to generate synthetic time series for inflation in Pakistan for 250 quarters from quarter 1, which is 1958Q2 and ending at 2020Q3.[4] A quick comparison with Figure 4 above can reveal that the series roughly estimates actual quarterly inflation. I have generated simulations for 500 draws, but the results are robust to other simulation sizes. The simulation's average inflation closely resembles the actual average quarterly inflation of around 8 percent. However, there is a lot of variation across quarters due to the noise introduced by the gaussian copula.

Fig. 8. Author's Simulations



[4] In order to apply the gaussian copula model on my data, I use the synthetic data vault (SDV) package developed by :

SYNTHETIC DATA FROM PAR

Probability Autoregessive Model (PAR) is a synthetic data creation methodology well suited for time series models and accounts for the auto-correlation structure of time series data. The PAR class allows learning multiple types of multivariate time series data and generating new synthetic data with the same format and properties as the learned one. Salinas, et al. (2020) have done path-breaking work at the frontier by developing probabilistic forecasting models with autoregressive, recurrent *neural networks*.

Assume that we are given access to a data set $\mathcal{D}$, consisting of n-dimensional data points x. For simplicity, let us assume that the data points are binary, i.e. $x \in (0,1)^N$. By the chain rule of probability, we can factorise the joint distribution over the n-dimensions as:

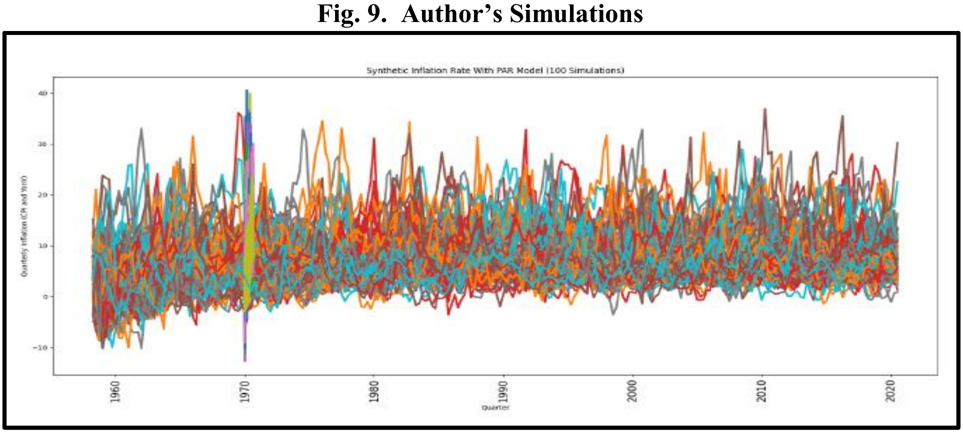
$$p(\mathrm{x}) = \prod_{i=1}^{n} p\,(x_i | x_1, x_2, x_3, \dots, x_{i-1})$$

The chain rule factorisation can be thought of as a Bayesian network. Such a Bayesian network that makes no conditional independence assumptions is said to obey the *autoregressive property*. We fix an ordering of the variables $(x_i | x_1, x_2, x_3, \dots, x_{i-1})$ and the distribution for the i^{th} random variable depends on the values of all the preceding random variables in the chosen ordering: $(x_1, x_2, x_3, \dots, x_{n-1})$. In an autoregressive generative model, the conditionals are specified as parameterised functions with a fixed number of parameters (see Salakhutdinov, 2015). That is, we assume the conditional distributions to correspond to a Bernoulli ($Bern$ in equation 3 below) random variable and learn a function that maps the preceding random variables $(x_1, x_2, x_3, \dots, x_{i-1})$ to the mean of this distribution. Hence, we get the following equation, where the number of parameters of an autoregressive generative model is given by $\sum_{i=1}^{N} |\,\theta_i|$ (see Grover, et al. 2018).[5]

$$p_{\theta_i}(x_i | x_1, x_2, x_3, \dots, x_{i-1}) = Bern(f_i(x_1, x_2, x_3, .., x_{i-1}))$$

To apply the PAR model to the data, I use the synthetic data vault package developed by Patki, et al. (2016). In Figure 9 below, I reveal my results from applying the PAR model, which includes only the inflation time series from 1958Q2 to 2020Q3 for Pakistan. A combination of 100 simulations from the fitted PAR model indicates that this model has roughly the same mean inflation of around 8 percent as the output from gaussian copula. Occasionally, the PAR model draws values above 30 percent inflation and below –5 percent.

Fig. 9. Author's Simulations



[5] For further review of PAR models, refer to .

CUBIC SPLINE INTERPOLATION

Cubic spline interpolation is an interpolation method used in numerical analysis. It uses *cubic polynomials* to connect the existing data nodes, which allows the estimation of unknown and high frequency intermediate data points. For a mathematical and formal review of cubic spline interpolation, you can refer to Burden, et al. (2015) and the appendix of this paper.

Consider the following data points (x_i, y_i) in Equation 4:

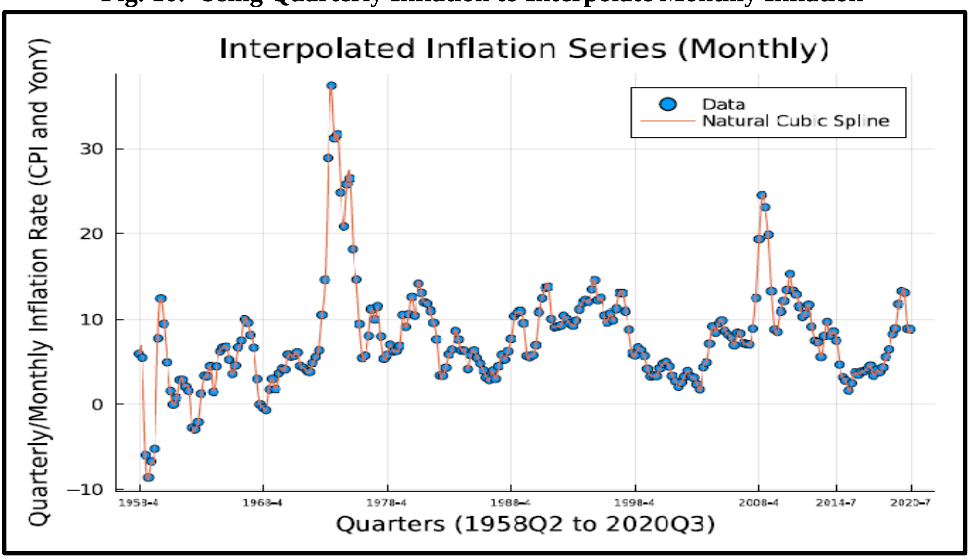
$$(x_0, y_0), (x_1, y_1), \ldots., (x_n, y_n)$$

where $x_0 < x_1 <\ldots< x_n$. In equation 5 below, the cubic polynomial's interpolating pairs of data are labeled as $S_0, \ldots, S_{n-1}$. The polynomial S_i interpolates the nodes (x_i, y_i) and (x_{i+1}, y_{i+1}). Let:

$$S_i(x) = a_i + b_i x + c_i x^2 + d_i x^3, \forall i = 0,1,2, \ldots, n-1$$

Based on the quarterly inflation series for Pakistan, I carry out cubic spline interpolation exercises, displayed in the figures below. Despite having access to only quarterly data, visible in the dots of Figure 10, the interpolation allows me access to a higher order approximation for monthly inflation rates in this period. Similarly, I can use monthly inflation data to approximate the unknown weekly inflation series (see Figure 11). The quarterly data is for the period from 1958 Q2 to 2020 Q3, whereas monthly data incorporates 774 months from January 1958 to June 2022.

Fig. 10. Using Quarterly Inflation to Interpolate Monthly Inflation



A comparison between predicted inflation from cubic spline interpolation at a monthly frequency, displayed in Figure 10 and *actual monthly inflation* data indicates that the predictions are reasonably accurate. In Table 2, I provide data on formal deviation measures between actual monthly data and interpolated/estimated monthly data from quarterly data, where the total number of observations is 748. For instance, the absolute deviation measure displayed below computes $|a_i - b_i|$ for each pair of data points, before taking averages across these absolute differences between actual monthly and interpolated monthly inflation i.e $\frac{1}{N}\sum_{i=1}^{n} | a_i - b_i |$. Similarly, RMSQ (root mean squared error) takes the average of squared deviations between actual and interpolated data before taking a root in the end i.e $\sqrt{\frac{\sum_{i=1}^{N}(a_i-b_i)^2}{N}}$. Whereas, maximum and minimum AD's are merely maximum and minimum deviations observed. Squared L_2 distance is $\sum_{i=1}^{n} | a_i - b_i |^2$, L_2 distance is $\sqrt{\sum_{i=1}^{n} | a_i - b_i |^2}$ and finally L_1 distance is $\sum_{i=1}^{n} | a_i - b_i |$.

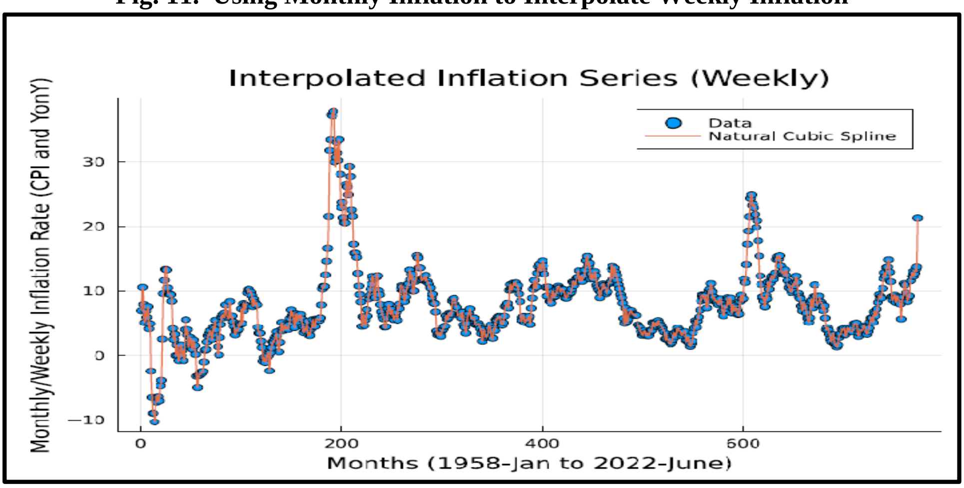
Table 2

Error Stats for Interpolated Monthly Inf Relative to Actual Monthly Inflation

Summary Stats	Mean
Measures	
Absolute Deviation (AD)	2.12
Maximum AD	19.91
Minimum AD	0.01
Root Mean Squared Error	3.25
Squared L_2 Distance	7914.3
L_2 Distance	88.96
L_1 Distance	1586.28

Table 2 reveals that even though the maximum deviation is above 19 percent, the average deviation in absolute terms and root mean squared error terms is relatively low. Hence, the interpolated series closely approximates the actual monthly inflation. Based on this estimation accuracy, I can extrapolate that the accuracy of the unknown weekly inflation forecasts from the monthly inflation series, displayed in Figure 11, is likely to be accurate. Hence, in the following graph (Figure 11), I demonstrate my estimation of Pakistan's unknown weekly inflation series from 1958 to 2022 using actual monthly data for Pakistan (the dots in the graph).

Fig. 11. Using Monthly Inflation to Interpolate Weekly Inflation



FORECASTS AND FORECAST EVALUATION

Ahmad, et al. (2019) and Syed and Lee (2021) did the latest and most recognised work on forecasting in Pakistan. The former use quarterly data (1980Q4-2017Q2) on real GDP, CPI, USD/PKR exchange rate and Call Money Rate. Since actual quarterly data of GDP is not available in Pakistan, they approximate GDP data from the work of Hanif et al. (2018), where this series is available till 2012Q2. For 2012Q2-2017Q2, they interpolate annual real GDP series to generate quarterly series. For foreign variables, they use USA's GDP, CPI and 3-months T-Bill (Treasury Bill) rate. In contrast, Syed and Lee (2021) use machine learning methodology and monthly inflation data from July 2007 to July 2017 to forecast the CPI inflation, GDP growth and the weighted average overnight repurchase rate in Pakistan. They use the naive mean and autoregressive models as benchmarks and compare their forecasting performance against the dynamic factor model (DFM) and basic machine learning methods such as ridge regressions, LASSO, elastic nets and bagging etc.

Even though my forecasting exercise is closely related to the recent work of Syed and Lee (2021), who also forecast inflation based on ML methods, but the main contribution of my paper is to use novel methods using ML *and* synthetic data as opposed to ridge regression type ML methods as in Syed and Lee (2021). To my knowledge, this combination of methodologies has not been applied in Pakistan to forecast and estimate high-frequency inflation. For the evaluation of ML predictions, I use reduced form VAR models and discuss forecast error variance decomposition results in addition to fan chart outcomes and compare them with ML forecasts.

FAN CHARTS AND FEVD

Vector Auto regressions are standard tools of empirical macroeconomics and for a review of these methods refer to Walsh (2017). The following fan charts of Figure 12 are based on forecasts from a simple reduced form VAR model for Pakistan. My model is not a structural VAR (SVAR) but only a reduced form VAR, since I am only interested in forecasting rather than causal inference. I estimate two versions of The VAR model; the first one is the Big VAR which includes variables such as CPI (Quarterly, Year on Year Inflation), short term external debt measure, M2 (a measure of money), tax revenue, imports and SR rate (short term interest rate) variables for Pakistan. As a robustness check, I also estimate a simpler VAR with the variables CPI and imports only.

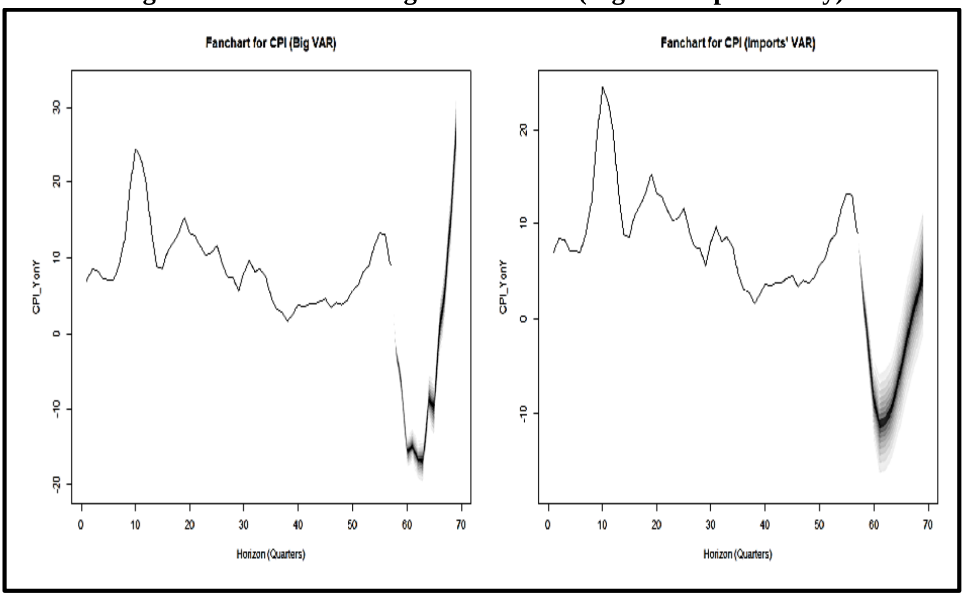
The models are estimated for the period from 2006Q2 to 2020Q2 due to restrictions on the availability of data. Standard information criteria such as AIC[6] are used to select lag order which are 7 quarter lags for the Big VAR and 6 quarter lags for small VAR with only imports. Having stationary variables is ideal in our VAR, even though not required property. Hence, I use the Phillips Perron test, which has a null hypothesis of non-stationarity. Results show that all variables in the VAR model are non-stationary, apart from imports and tax revenues. The forecasts are evaluated for 12 quarters after 2020Q2, starting from 2020 Q3 and ending at 2023 Q3. After estimating VAR, I also analyse standard diagnostic characteristics for model evaluation. All the diagnostic tests are reported in the appendix (section 8.2), since these are standard, run-of-the-mill tests in the VAR literature.

[6] Akaik Information Criteria.

We know from our stylised facts that actual inflation during 2020 Q3 was around 10 percent, reached 15 percent by the first quarter of 2022 and rose to more than 20 percent by the 2nd quarter of 2022. Meanwhile, both the VAR using only imports and the Big VAR using 5 variables predict a deflation of up to negative 13 percent and 20 percent, respectively for 2020 to 2022 (see Figure 12).

For the last 4 quarters, from 2022Q3 to 2023Q3, we get predictions of massive inflation of around 30 percent using Big VAR and around 5 percent using imports only VAR. Hence, we get grossly inaccurate forecasts using VAR methodology for the 2020 to 2022 period since the actual inflation reached above 20 percent during 2022. On the other hand, the predictions for 2022 to 2023 are yet to be examined but appear to be highly unlikely in the Big VAR case which predicts 30 percent inflation by 2023 Q3. Lastly, the imports' (small) VAR predicts a rise in inflation from negative 13 percent to almost 10 percent from 2022 to 2023. Even if the 2023 forecasts prove accurate, the overall prediction pattern from small VAR is entirely inconsistent with actual data.

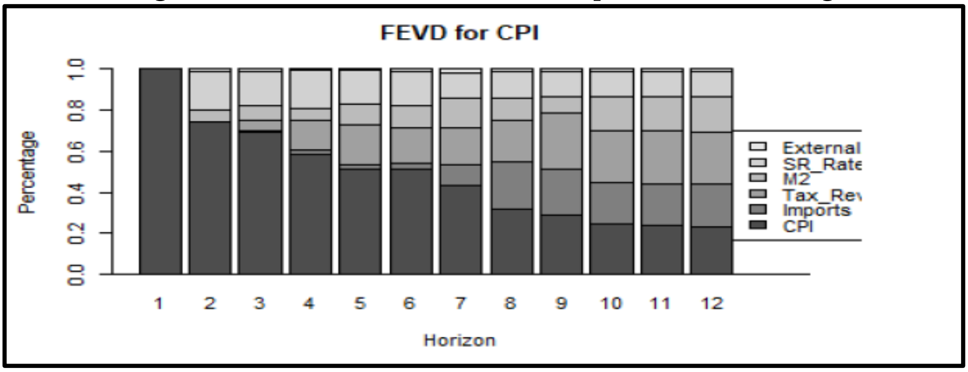
Fig. 12. Fan Charts Using VAR Models (Big and Imports Only)



I also present FEVD (forecast error variance decomposition) results[7] for the Big VAR model in Figure 13. The horizon for FEVD is 12 quarter ahead and it demonstrates that non-CPI variables drive a larger share of the forecast error at longer horizons. For instance, non-CPI variables explain more than 70 percent of the forecast error variance at the 8th quarter ahead forecasts and almost 80 percent of the error variance at 12th quarter ahead. Meanwhile, for the first three quarters, CPI explains more than 60 percent of the forecast errors.

[7] For an introduction to FEVD refer to Lütkepohl (2010).

Fig. 13. Forecast Error Variance Decomposition for VAR Big

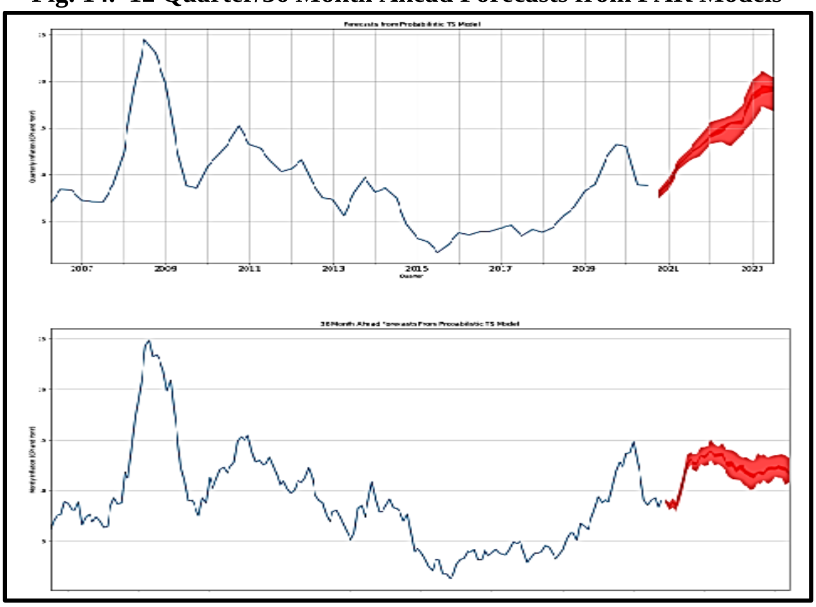


FORECASTS FROM PAR MODELS

To generate forecasts based on PAR models, I use new and currently evolving estimation techniques developed by Alexandrov et al. (2019).[8] To maintain consistency with VAR model from the last section, I apply the probabilistic autoregressive model (PAR) on the inflation series of Pakistan, starting from 2006Q2 and ending in 2020Q3. For the monthly series, I use data from April 2006 to September 2020, which corresponds with the start and end points of the frequencies for quarterly data, which ends in quarter three and roughly includes the 9th month.

The top graph in Figure 14 presents 12 quarter ahead forecasts from the PAR model, starting from 2020Q4 to 2023 Q4. Similarly, the bottom graph of Figure 14 presents 36-month ahead forecasts from October 2020 to October 2023.

Fig. 14. 12 Quarter/36 Month Ahead Forecasts from PAR Models



[8] I use Python code and packages such as *gluonts* and *pytorchts*. All code is available on my github page for replication.

A quick eye-ball comparison with previous Figure 6 reveals that the PAR model is more accurate than the VAR fan charts in the last section for both the quarterly and monthly specifications of PAR. However, the forecasts based on quarterly inflation series are more accurate predictors of the hyperinflation crisis of 2021 and 2022. For 2021 and 2022, the monthly PAR model can predict a rise in inflation which is close to 15 percent but the quarterly data predicts close to 20 percent inflation, which is closer to actual inflation.

Nevertheless, another interesting difference is that for 2022 to 2023, quarterly data continues to predict annual inflation above 15 percent. However, the monthly PAR model predicts a stabilisation and downward inflation adjustment, close to 10 percent by October 2023. Given the current trend toward mild downward stabilisation of the exchange rate and inflation in Pakistan, the monthly PAR model may be more accurate after one year. Meanwhile, the VAR model's forecast for inflation was completely off the charts for a comparable data range and predicted a disinflationary and even deflationary period during 2020Q4 to 2022Q4.

My results are robust to estimation based on the entire time series period from 1958 to 2020 for quarterly and monthly data, in addition to variations in the size of testing data cuts. My default cut for training data is 11 quarters in quarterly estimation and 69 months for monthly data. For robustness, I evaluate the forecasts for training data with 33 (11), 69 (23) and 105 (35) monthly (quarterly) cuts for monthly (quarterly) data, respectively.[9] The forecast accuracy is superior to the VAR models in all these cases.

CONCLUSION

In this working paper, I review cutting-edge methodologies for inflation forecasting while being motivated by the current inflation crisis in Pakistan. Using the high-frequency scanner, web-scrapped and synthetic data can make inflation forecasting and measurement more accurate, making policy interventions more well-informed about the ground realities. However, data from scanners and web-scrapping are currently unavailable, which leads me to use synthetic data and numerical techniques to estimate the unknown high frequency inflation series for Pakistan from 1958 to 2022.

More specifically, I mainly use *probability autoregressive models* and *cubic spline* interpolation. I find that we can approximate monthly and other low-order (weekly) inflation series for Pakistan using cubic splines. I evaluate the forecast accuracy through measures such as absolute deviation, RMSQE[10] etc., applied to a comparison between cubic spline interpolation and actual monthly inflation. In addition, I use standard, reduced form, vector autoregressive models to forecast inflation and compare the forecasting potential of VAR versus my ML model forecasts, which are based on *probability autoregressive models*. Thus, both of my methods demonstrate a lot of promise for the estimation of historically unknown high-frequency inflation as well as short-run inflation forecasting.

It may be fruitful to expand the data availability by collecting web-scrapped and scanner data on high frequencies for Pakistan. Moreover, another extension could be to use a combination of ML-based methods and VAR's (Vector Autoregressive), which

[9] I can share these additional robustness checks if requested by any reader.

[10] Root Mean Squared Error.

would be an extension of the machine learning model used in this paper, which is autoregressive but does not include other variables. Nevertheless, it is quite informative that even a probability autoregressive model can achieve more accurate inflation forecasts relative to VAR with multiple variables.

APPENDIX

MORE STYLISED FACTS

Figure 15 depicts the annual consumer price index (1958 onwards), and 16 represents the annual inflation rate based on the linked GDP deflator after 1990.

Fig. 15. Data is from IMF

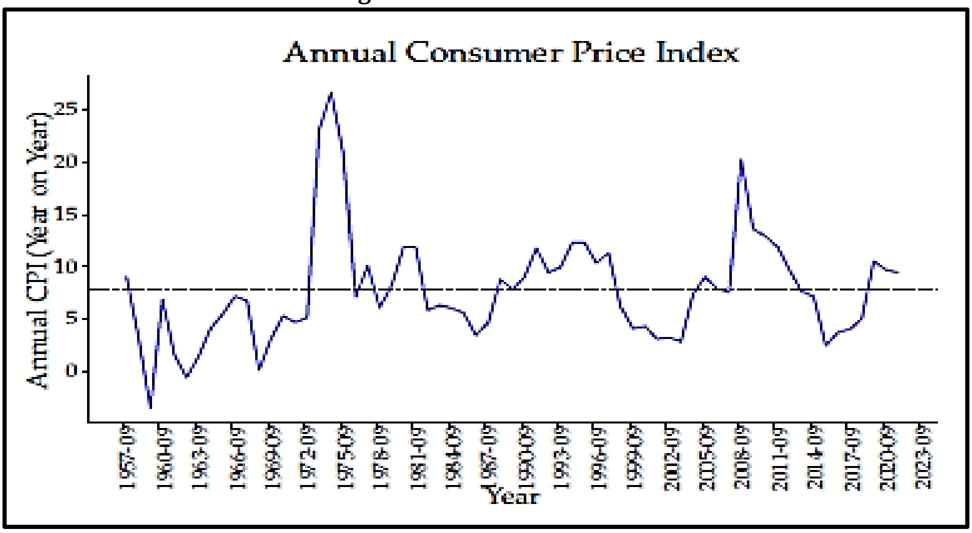
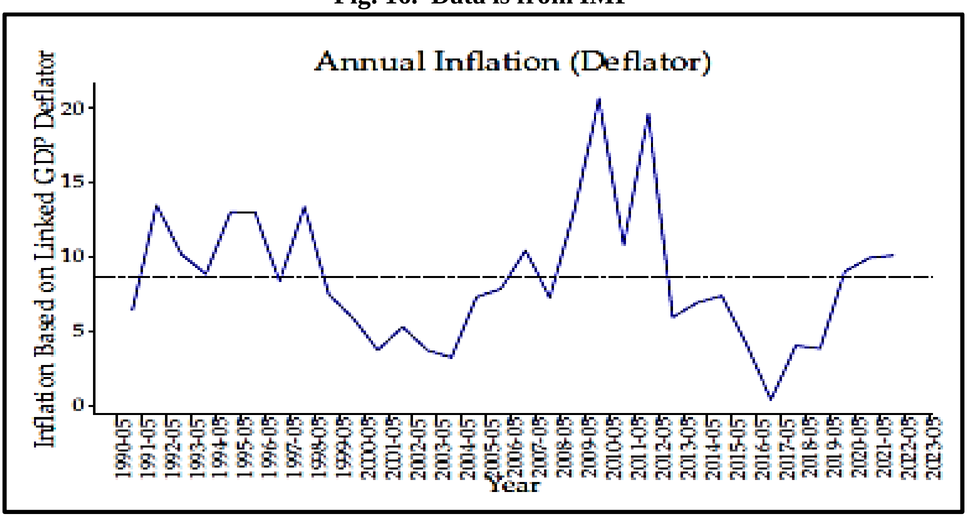


Fig. 16. Data is from IMF



VAR MODEL DIAGNOSTICS

Standard model diagnostics for VAR's are applied, including ARCH property, residual auto-correlation, normality of residual distributions and stability tests for structural breaks.

The ARCH property is firmly rejected since I cannot reject the null hypothesis of no degree of heteroscedasticity at even a 20 percent significance level. Hence, there is significant evidence for no ARCH (Autoregressive Conditional Heteroskedasticity) effects or evidence for constant volatility in the model. Another assumption is that the residuals should be non-autocorrelated. In other words, residuals should be white noise and thus uncorrelated with the previous periods. Unfortunately, the null hypothesis is rejected with a p value of less than 1 percent and model residuals are autocorrelated.

A soft requirement is also the normality of the distribution of the residuals. To test for the normality of the residuals, I use the standard jarque-bera test, the kurtosis test, and the skewness test. The model residuals fail to reject the null hypothesis of normality, based on all these three measures used, with typical p values, far above 0.7. This is an encouraging result and the requirement is satisfied.

The stability test is a test for the presence of structural breaks. We know that if we cannot test for structural breaks and if there happened to be one, the whole model becomes invalidated. Fortunately, we have a simple test for this which uses a plot of the sum of recursive residuals. It is reassuring that the stability test is satisfied since I could not detect the presence of structural breaks.

In Table 3, I summarises the model diagnostics for the Big VAR model[11]. There is no evidence for encouraging heteroskedasticity and structural breaks; on the other hand, there is evidence for normality of residual distributions, which is also positive. The only disappointing result is that there is evidence for residual autocorrelation.

Table 3

Model Diagnostics for Big VAR Model

Presence of Property	Yes/No
Properties	
Heteroskedasticity	NO
Residual Autocorrelation	YES
Normality of Residual Distributions	YES
Structural Breaks	NO

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[11] Results are similar for small, imports only VAR.

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